A boundary integral equation solver for stepped pressure equilibria in stellarators

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Abstract

We present BIEST (Boundary Integral Equation Solver for Taylor states), a new numerical solver for computing stepped-pressure equilibria in stellarators. BIEST decomposes the calculation of stepped-pressure equilibria into the same two iterative steps as the SPEC solver (Stepped Pressure Equilibrium Code) [1] does: 1) The location of ideal MHD barriers is first prescribed, and Taylor states are computed in the different regions separated by these ideal MHD surfaces; 2) the ideal MHD barriers are then moved in order to satisfy force balance at each interface. However, BIEST computes Taylor states in a fundamentally different way from SPEC. Specifically, in BIEST Taylor states are formulated in terms of boundary integral equations, in which the unknowns are only defined on the surfaces corresponding to ideal MHD barriers, and the volume never needs to be discretized. This approach has several advantages: 1) the number of unknowns is greatly reduced as compared to SPEC, thus reducing the memory requirements; 2) BIEST avoids issues with the coordinate singularity which occurs when parameterizing the volume of genus-one domains, and has greater geometric flexibility; 3) the nature of the integral equations in BIEST leads to favorable conditioning as compared to volume based discretization schemes.

We tested the performance of BIEST for several stellarator geometries (W7-X, QAS3), and numerically confirmed the favorable comparison with SPEC, in terms of both accuracy (BIEST reaches 9-digit accuracy without encountering conditioning issues) and speed (factor of 10 speedup as compared to SPEC). Furthermore, for the needs of BIEST, we developed a new and efficient quadrature scheme for singular integrals on stellarator surfaces which can be used to accelerate the evaluation of integrals in several stellarator codes, such as BNORM, NESCOIL/REGCOIL, DIAGNO, EXTENDER, etc.

References

[1] S. R. Hudson, R. L. Dewar, G. Dennis, M. J. Hole, M. McGann, G. von Nessi, and S. Lazerson. Computation of multi-region relaxed magnetohydrodynamic equilibria. Physics of Plasmas, 19(11):112502, Nov 2012.

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