Gyrokinetic turbulence: a nonlinear route to dissipation through phase space

arXiv:0806.1069

Alexander Schekochihin
Imperial College, London

with
Steve Cowley *(Imperial & Culham)*
Bill Dorland, Tomo Tatsuno *(Maryland)*
Greg Hammett *(Princeton)*
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Reprints/references on http://www2.imperial.ac.uk/~aschekoc/

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(1851-1925)

Leverhulme Trust International Network for Magnetised Plasma Turbulence
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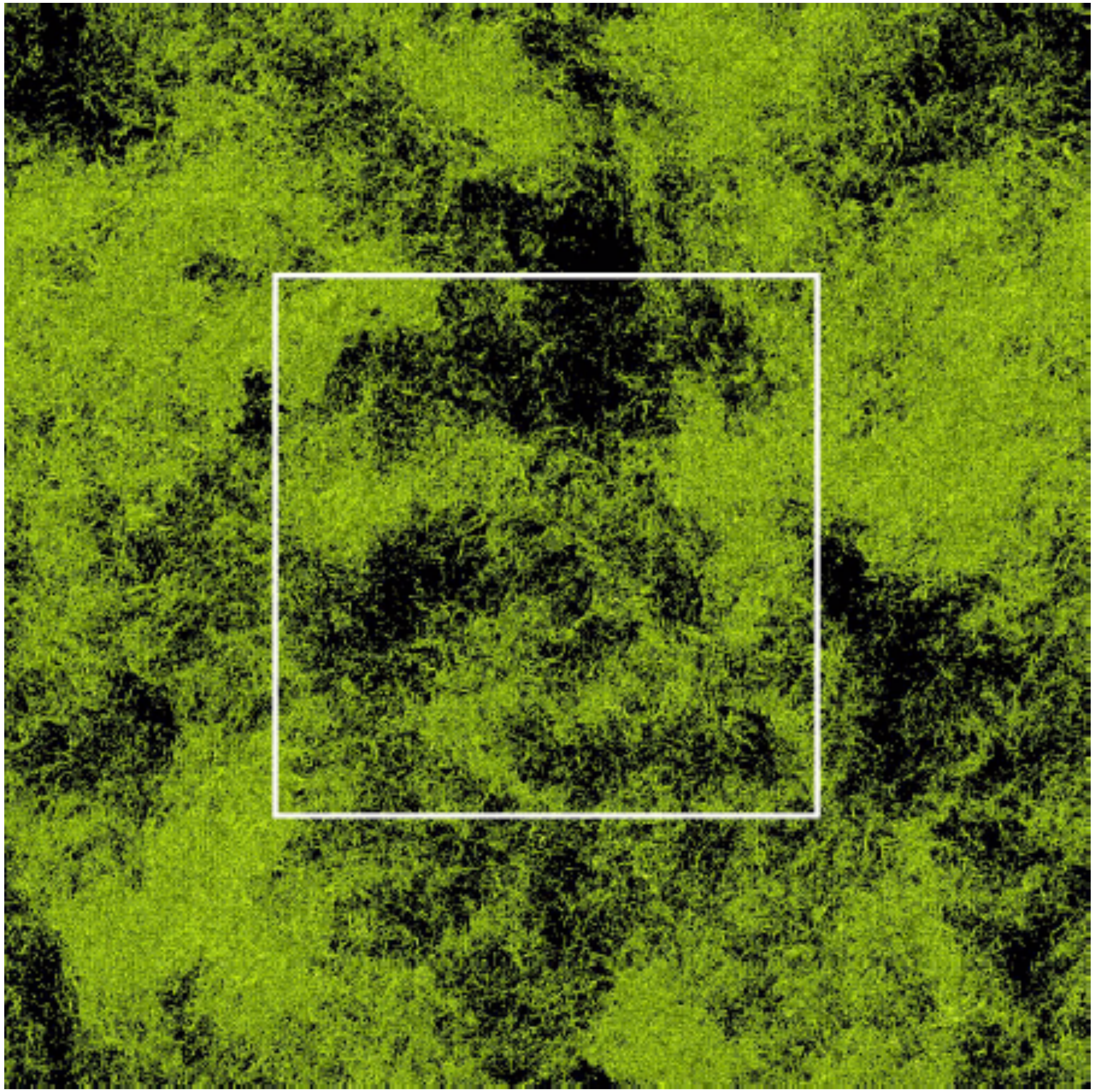
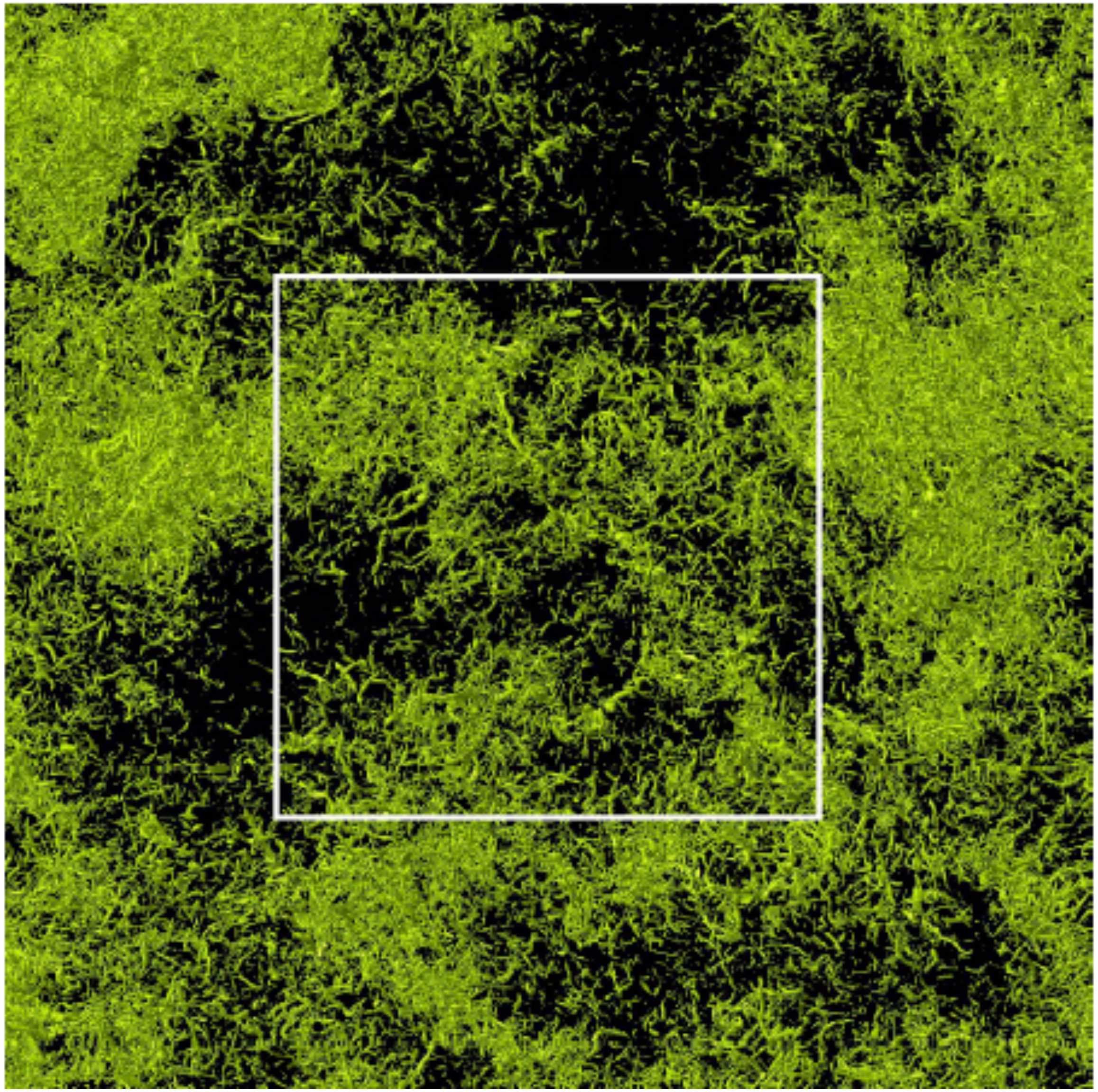
Turbulence is Multiscale Disorder



[Image: Y. Kaneda *et al*., **Earth Simulator, isovorticity surfaces, 4096^3**]

Turbulence is Multiscale Disorder



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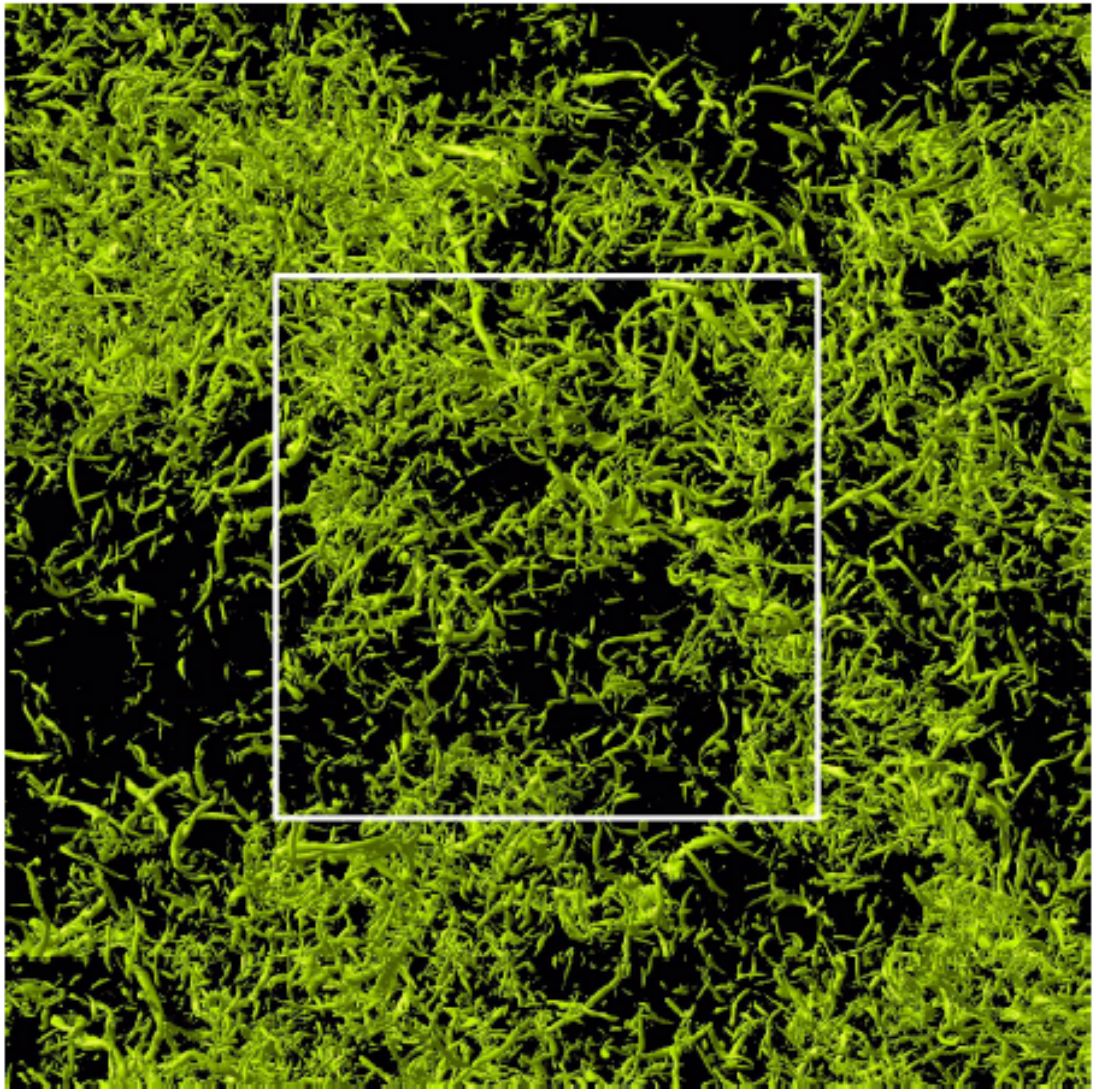

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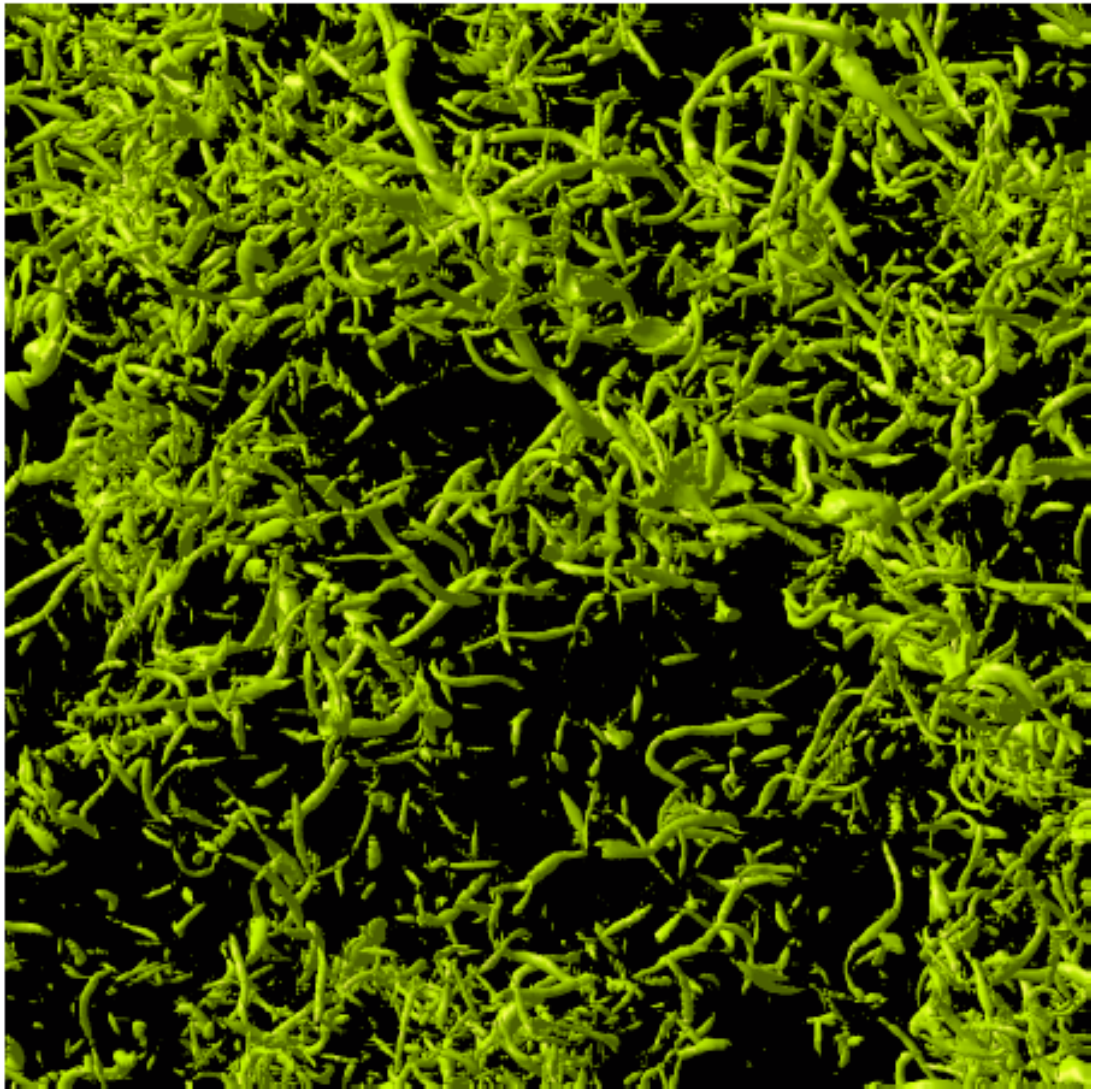


[Image: Y. Kaneda *et al*., **Earth Simulator, isovorticity surfaces, 4096^3**]

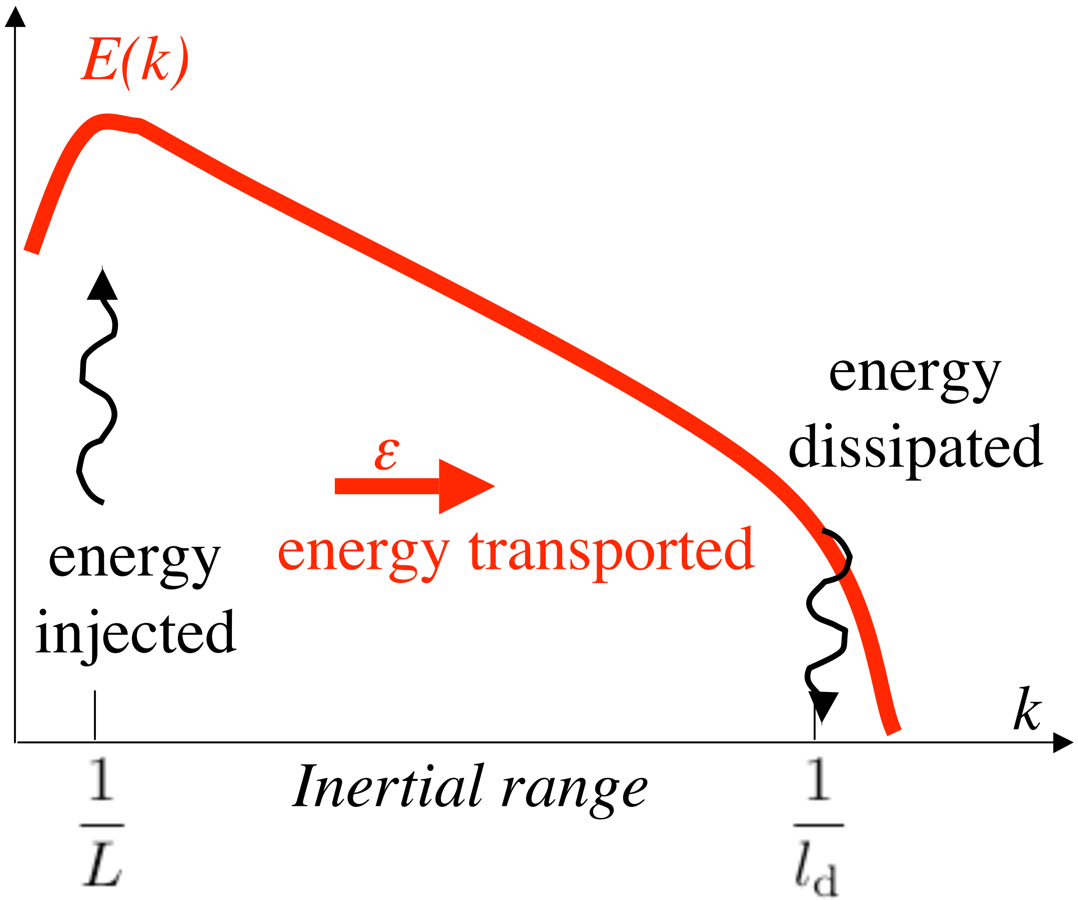
Turbulence: A Nonlinear Route to Dissipation

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2} = \varepsilon - \nu\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}|\boldsymbol{\nabla}\boldsymbol{u}|^2$$

$$l_\nu \sim (\nu^3/\varepsilon)^{1/4} \sim L\mathrm{Re}^{-3/4}$$

$$\varepsilon = (1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{u}\cdot\boldsymbol{f}$$



Turbulence: A Nonlinear Route to Dissipation

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

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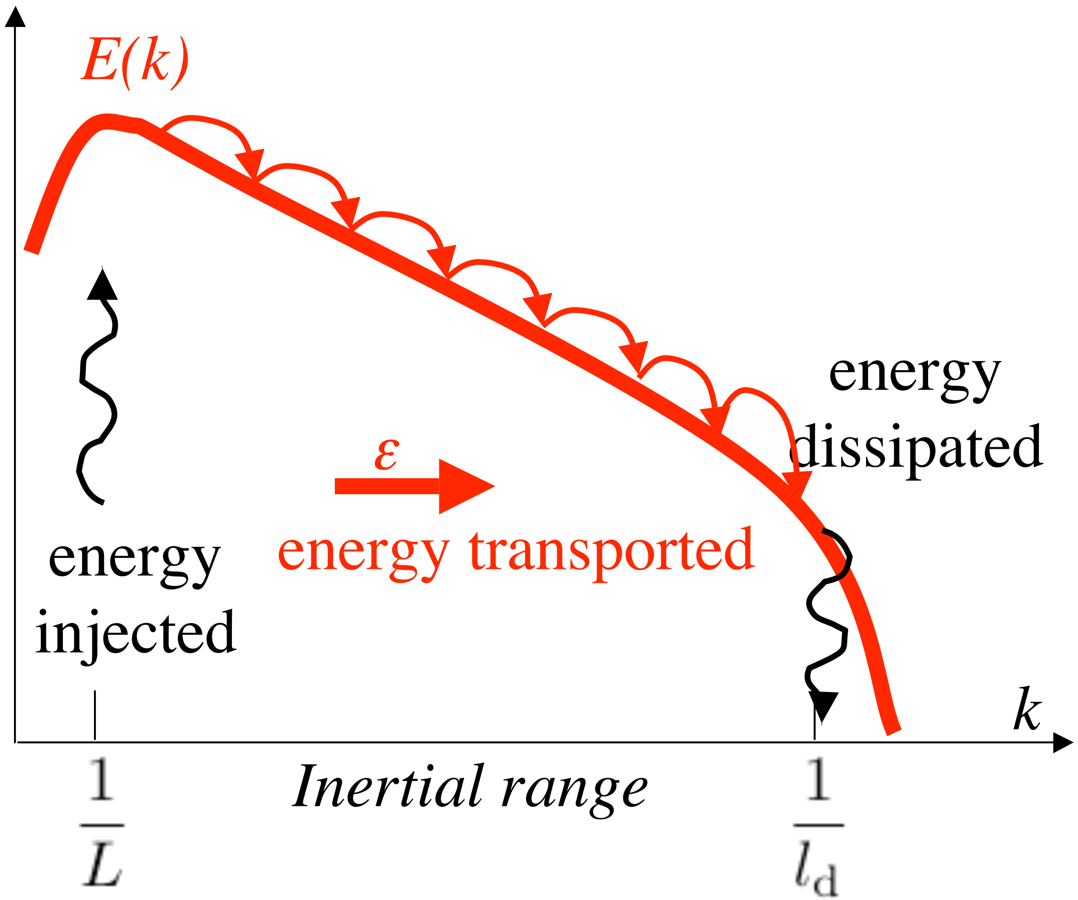
$$\varepsilon = (1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{u}\cdot\boldsymbol{f}$$

If cascade is local, intermediate scales fill up

Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.

L. F. Richardson 1922



Turbulence: A Nonlinear Route to Dissipation

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \quad \nabla\cdot\boldsymbol{u} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2} = \varepsilon - \nu\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}|\nabla\boldsymbol{u}|^2 \qquad l_\nu \sim (\nu^3/\varepsilon)^{1/4} \sim L\mathrm{Re}^{-3/4}$$

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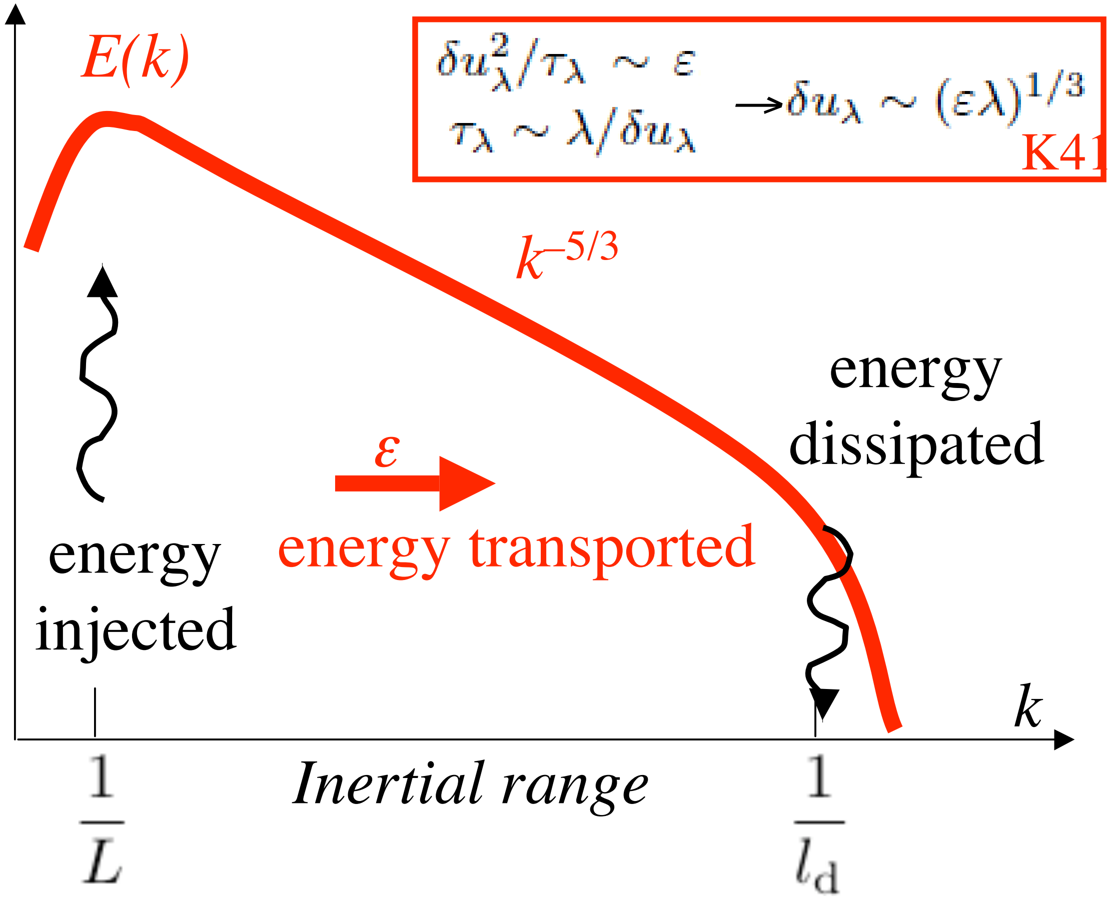

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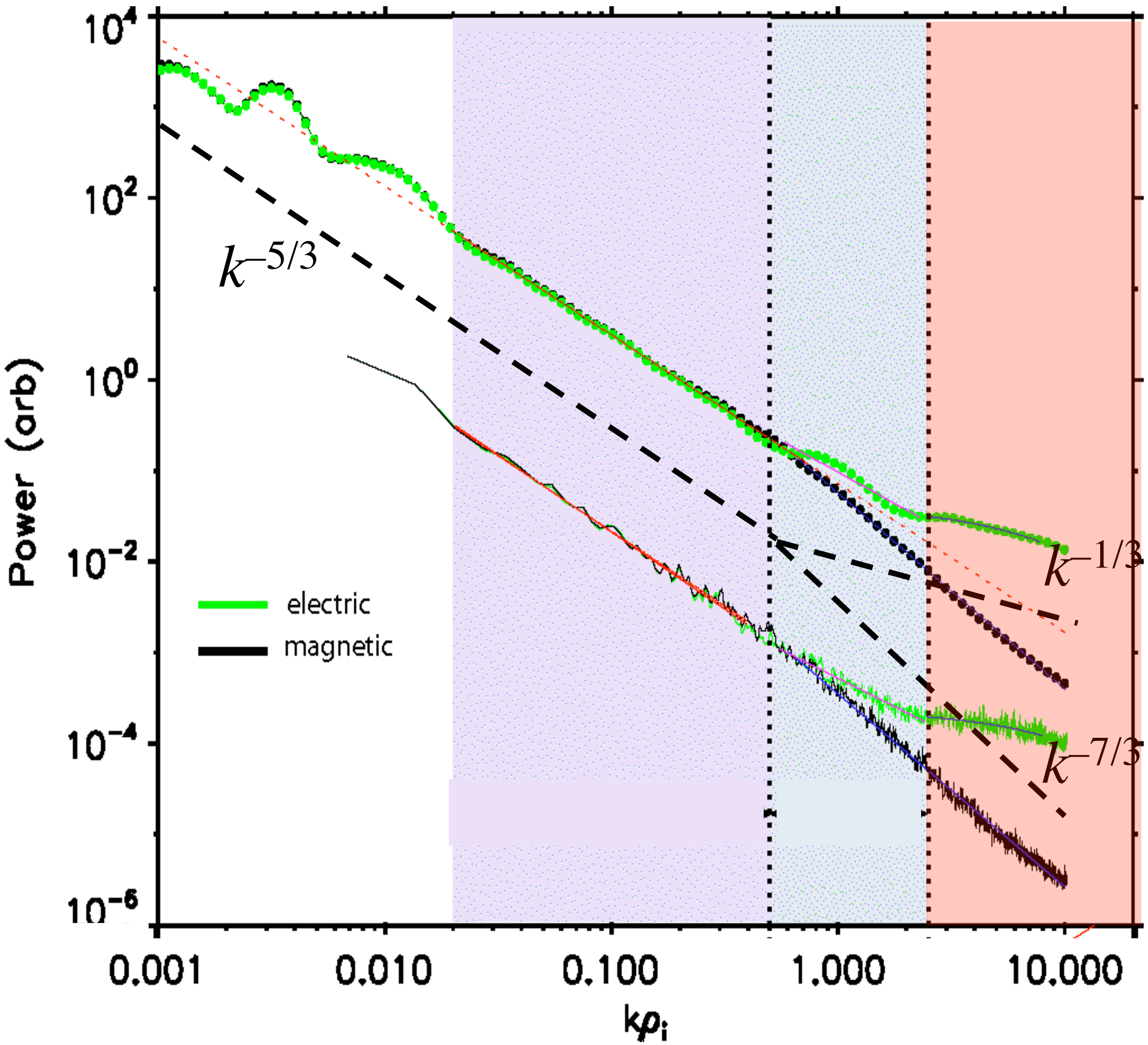
Plasma Turbulence: Analogous?

Turbulence in the solar wind

[Bale et al. 2005, *PRL* **94**, 215002]

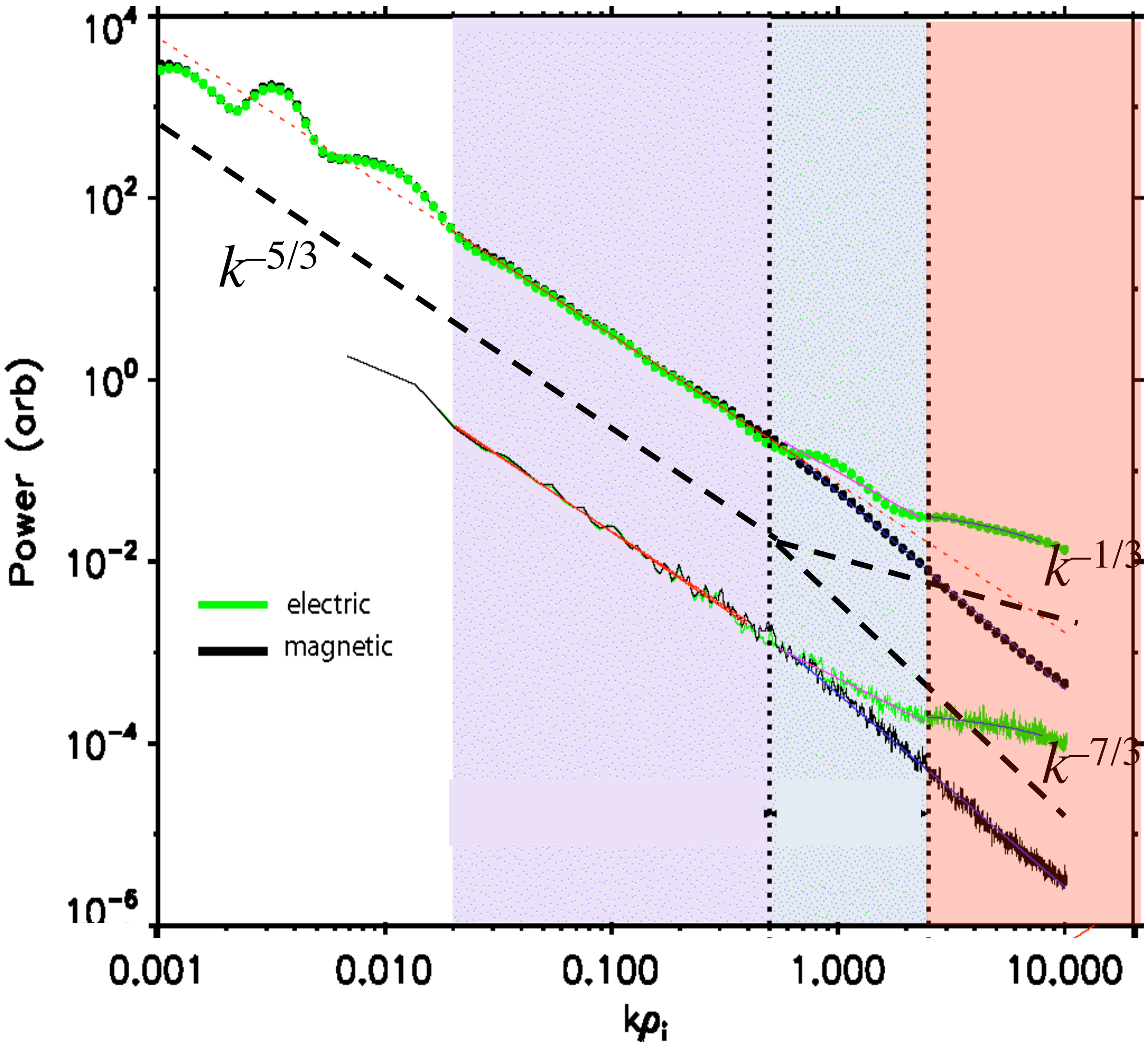


Plasma Turbulence Extends to Collisionless Scales

$\lambda_{\rm mfp} \sim 10^8$ km (~1 AU)
$\rho_i \sim 10^2$ km

Turbulence in the solar wind

[Bale et al. 2005, *PRL* **94**, 215002]



Plasma Turbulence Extends to Collisionless Scales

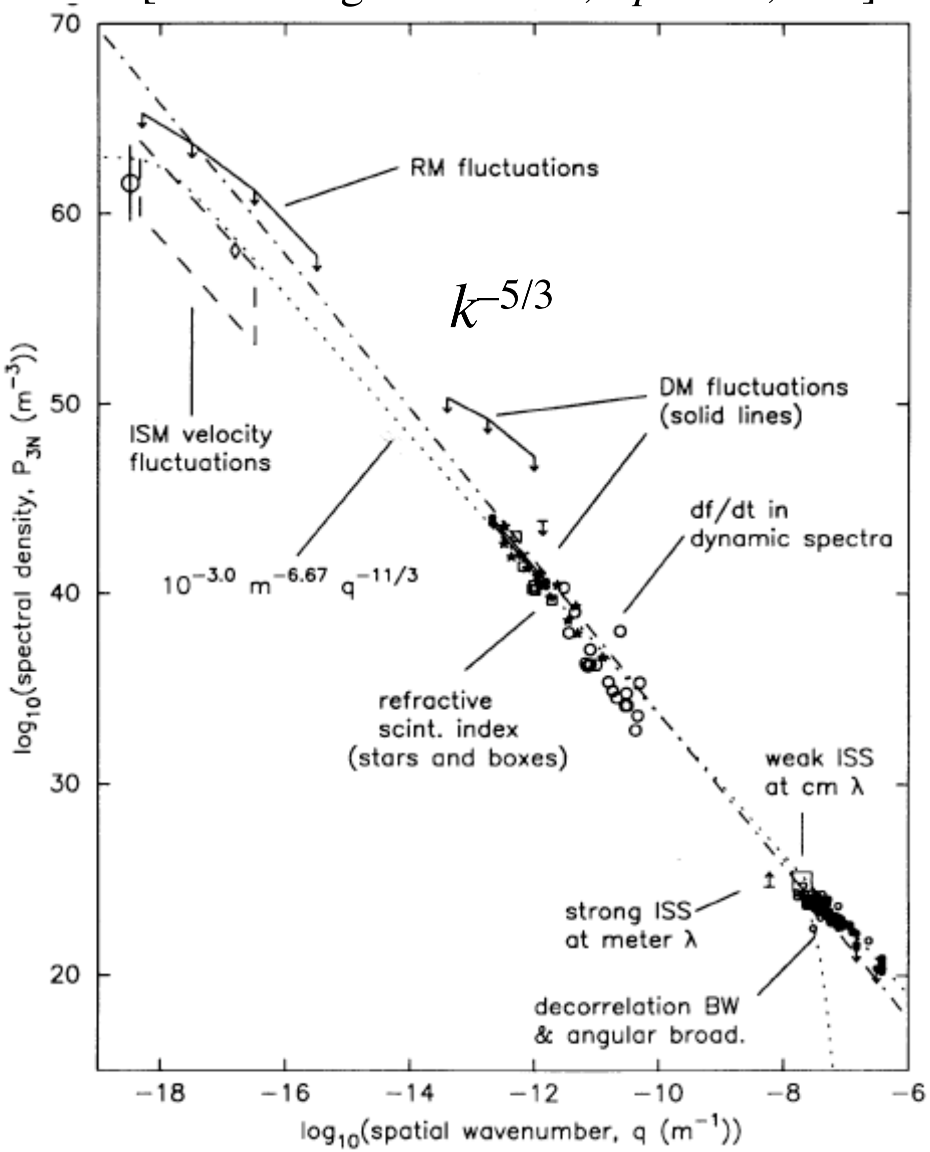
Interstellar medium: "Great Power Law in the Sky"

[Armstrong *et al*. 1995, *ApJ* **443**, 209]

$L \sim 10^{13}$ km (~100 pc)

$\lambda_{\rm mfp} \sim 10^{7}$ km

$\rho_i \sim 10^{4}$ km

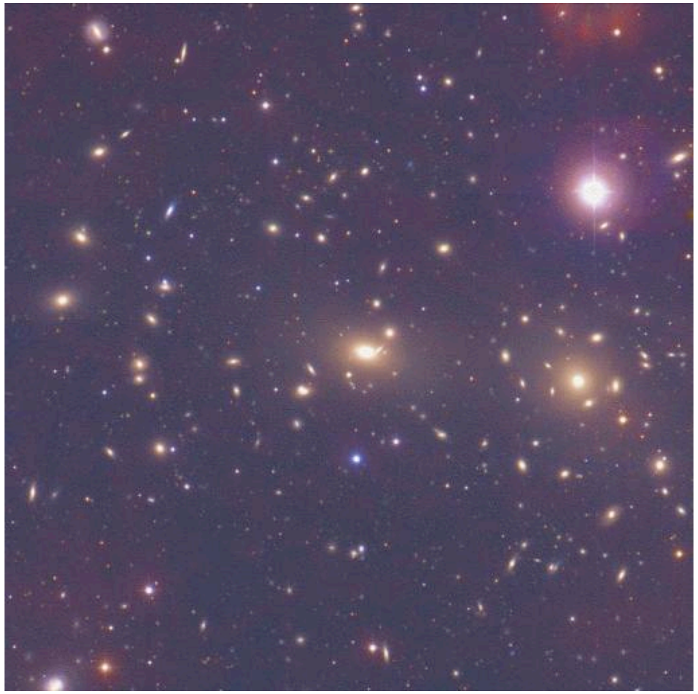


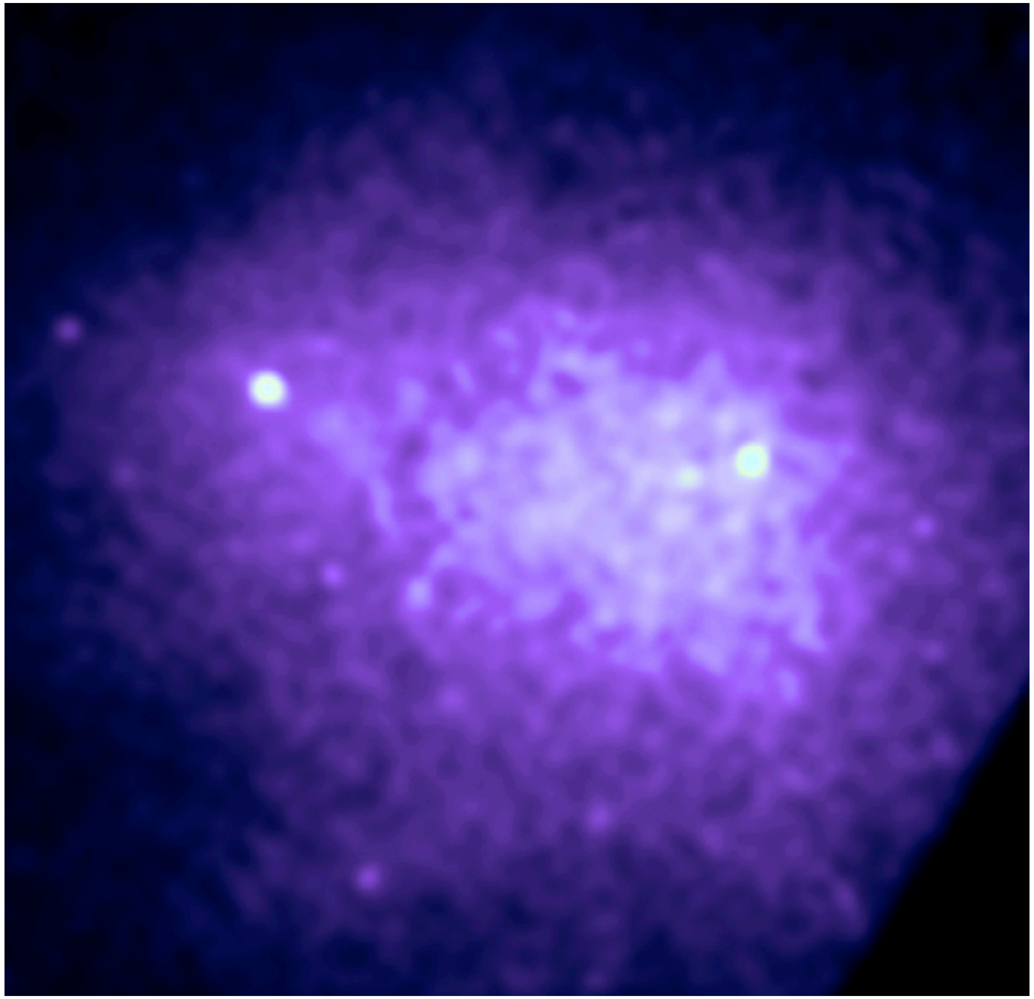
Plasma Turbulence Extends to Collisionless Scales

Intracluster (intergalactic) medium

[Coma cluster: Chandra X-ray,
A. Vikhlinin *et al.*/NASA/CXC/SAO]

$L \sim 10^{19}$ km (~1 Mpc)
$\lambda_{\rm mfp} \sim 10^{16}$ km (~1 kpc)
$\rho_i \sim 10^4$ km



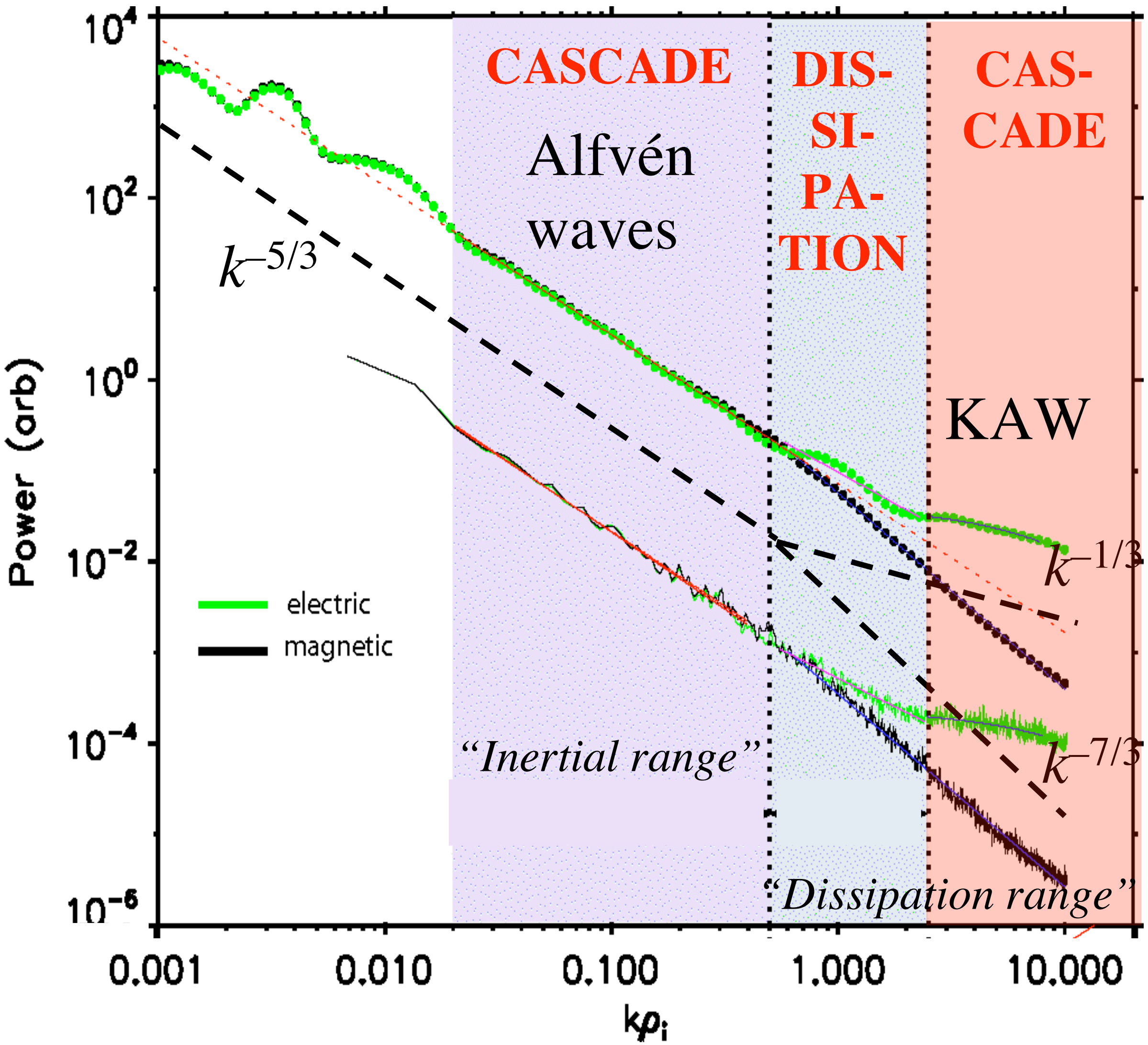


Plasma Turbulence Is Kinetic

- What is cascading in kinetic turbulence? (What is conserved?)
- Dissipation is "collisionless" (via Landau damping) How does that heat particles? (ions, electrons, minority ions)

Turbulence in the solar wind

[Bale et al. 2005, *PRL* **94**, 215002]



Plasma Turbulence *Ab Initio*

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_s + \frac{q_s}{m_s}\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)\cdot\frac{\partial f_s}{\partial \boldsymbol{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{c}}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 4\pi\sum_s q_s n_s, \qquad n_s = \int \mathrm{d}^3\boldsymbol{v}\, f_s,$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\left(\boldsymbol{j} + \boldsymbol{j}_{\mathrm{ext}}\right), \qquad \boldsymbol{j} = \sum_s q_s \int \mathrm{d}^3\boldsymbol{v}\, \boldsymbol{v} f_s,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{\nabla}\times\boldsymbol{E}, \qquad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0.$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon=-(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

$$\nabla\cdot\boldsymbol{E}=4\pi\sum_s q_s n_s,\qquad n_s=\int\mathrm{d}^3\boldsymbol{v}\,f_s,$$

$$\nabla\times\boldsymbol{B}-\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}=\frac{4\pi}{c}\left(\boldsymbol{j}+\boldsymbol{j}_{\mathrm{ext}}\right),\qquad \boldsymbol{j}=\sum_s q_s\int\mathrm{d}^3\boldsymbol{v}\,\boldsymbol{v}f_s,$$

$$\frac{\partial\boldsymbol{B}}{\partial t}=-c\nabla\times\boldsymbol{E},\qquad \nabla\cdot\boldsymbol{B}=0.$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon=-(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Entropy produced:

$$\frac{\mathrm{d}S_s}{\mathrm{d}t}\equiv\frac{d}{dt}\left[-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,f_s\ln f_s\right]=-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\ln f_s\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{c}}\geq 0$$

Boltzmann 1872

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s = \int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j} = \varepsilon - \frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon = -(1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Entropy produced:

$$T_{0s}\frac{\mathrm{d}S_s}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,(F_{0s}+\delta f_s) - \int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}\right]$$

$$= -\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}} - n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

$$f_s = F_{0s} + \delta f_s$$

$$F_{0s} = n_{0s}(\pi v_{\mathrm{th}s}^2)^{-3/2}\exp(-v^2/v_{\mathrm{th}s}^2)$$

$$v_{\mathrm{th}s} = (2T_{0s}/m_s)^{1/2}$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

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Entropy produced:

$$T_{0s}\frac{\mathrm{d}S_s}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\left[\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,(F_{0s}+\delta f_s)-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}\right]$$

$$=-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}-n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon=-(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Heating:

$$\frac{3}{2}\,n_{0s}\frac{\mathrm{d}T_{0s}}{\mathrm{d}t}=-\overline{\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}-n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

Fluctuation energy budget:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

Plasma Turbulence: Generalised Energy Cascade

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\underset{-T\delta S}{\frac{T_{0s}\delta f_s^2}{2F_{0s}}} + \underset{\text{energy}}{\frac{E^2+B^2}{8\pi}}\right]$$

$$= \varepsilon + \underset{\text{heating}}{\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}$$

Generalised energy = free energy of the particles + fields

Fowler 1968
Krommes & Hu 1994
Krommes 1999
Sugama et al. 1996
Hallatchek 2004
Howes et al. 2006
Schekochihin et al. 2007
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Landau damping is a redistribution between e-m fluctuation energy and (negative) perturbed entropy (free energy). It was pointed out already by Landau 1946 that δf_s *does not decay: "ballistic response"* $\delta f_s \propto \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}t}$

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Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

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small scales in 6D phase space

$$\partial_t\boldsymbol{u}+\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}=-\boldsymbol{\nabla}p+\nu\nabla^2\boldsymbol{u}+\boldsymbol{f}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2}=\varepsilon-\nu\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}|\boldsymbol{\nabla}\boldsymbol{u}|^2$$

small scales in 3D physical space

arXiv:0806.1069

Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy

heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

small scales in 6D phase space

$$\nu_{ii}v_{\mathrm{th}i}^2\left(\frac{\partial}{\partial v}\right)^2\sim\omega\;\Rightarrow\;\frac{\delta v}{v_{\mathrm{th}i}}\sim\left(\frac{\nu_{ii}}{\omega}\right)^{1/2}\sim\frac{1}{\sqrt{k_\parallel\lambda_{\mathrm{mfp}}}}\ll 1$$

In gyrokinetic turbulence, the velocity-space and x-space cascades are intertwined, giving rise to a single phase-space cascade

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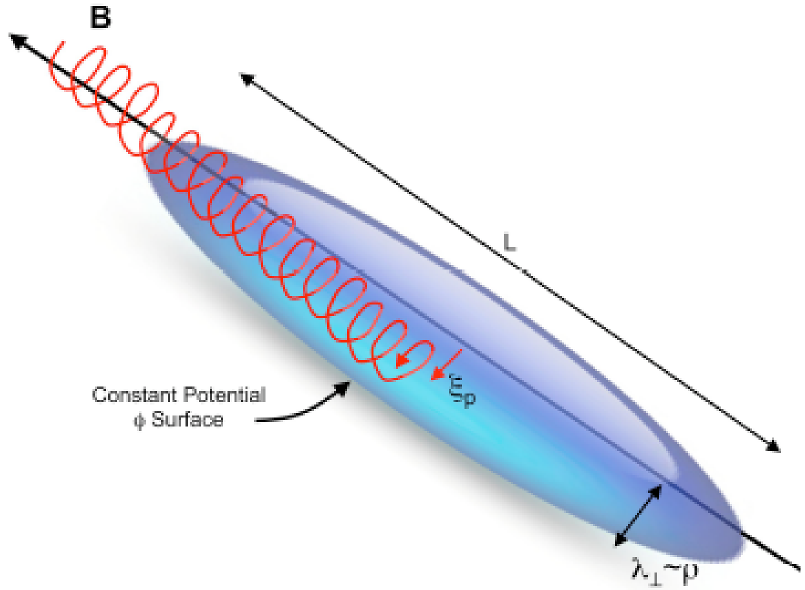
Gyrokinetics: Kinetics of Larmor Rings

$$\epsilon \sim \frac{k_\parallel}{k_\perp} \sim \frac{\omega}{\Omega_i} \sim \frac{\delta f_s}{F_{0s}}, \quad k_\perp \rho_i \sim 1$$



- Anisotropy
- Low frequency
- Small fluctuations
- FLR

$$\delta f_s = -q_s \varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel) \qquad \boldsymbol{R}_s = \boldsymbol{r} + \boldsymbol{v}_\perp \times \hat{\boldsymbol{z}}/\Omega_s$$

$$\frac{\partial h_s}{\partial t} + v_\parallel \frac{\partial h_s}{\partial z} + \frac{c}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s}, h_s\} = \frac{q_s F_{0s}}{T_{0s}} \frac{\partial \langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t} + \left(\frac{\partial h_s}{\partial t}\right)_c$$

$$\chi = \varphi - \boldsymbol{v}\cdot\boldsymbol{A}/c, \; \boldsymbol{B} = B_0\hat{\boldsymbol{z}} + \delta\boldsymbol{B}, \; \delta\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$$

$$\sum_s \frac{q_s^2 \varphi}{T_{0s}} n_{0s} = \sum_s q_s \int \mathrm{d}^3\boldsymbol{v}\, \langle h_s\rangle_{\boldsymbol{r}}$$

+ Ampère’s law

Taylor & Hastie 1968
Rutherford & Frieman 1968
Frieman & Chen 1982

Generalised Energy in Gyrokinetics

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

$$\delta f_s=-q_s\varphi F_{0s}/T_{0s}+h_s(t,\boldsymbol{R}_s,v_\perp,v_\parallel)\qquad \boldsymbol{R}_s=\boldsymbol{r}+\boldsymbol{v}_\perp\times\hat{\boldsymbol{z}}/\Omega_s$$

$$\frac{\partial h_s}{\partial t}+v_\parallel\frac{\partial h_s}{\partial z}+\frac{c}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s},h_s\}=\frac{q_sF_{0s}}{T_{0s}}\frac{\partial\langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t}+\left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{c}}$$

$$\chi=\varphi-\boldsymbol{v}\cdot\boldsymbol{A}/c,\ \boldsymbol{B}=B_0\hat{\boldsymbol{z}}+\delta\boldsymbol{B},\ \delta\boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A}$$

$$\sum_s\frac{q_s^2\varphi}{T_{0s}}n_{0s}=\sum_s q_s\int\mathrm{d}^3\boldsymbol{v}\,\langle h_s\rangle_{\boldsymbol{r}}$$

+ Ampère's law

Taylor & Hastie 1968
Rutherford & Frieman 1968
Frieman & Chen 1982

Generalised Energy in Gyrokinetics

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\left(\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\langle h_s^2\rangle_{\boldsymbol{r}}}{2F_{0s}} - \frac{q_s^2\varphi^2 n_{0s}}{2T_{0s}}\right) + \frac{|\delta\boldsymbol{B}|^2}{8\pi}\right]$$

$-T\delta S$ energy

$$= \varepsilon + \sum_s\int\mathrm{d}^3\boldsymbol{v}\int\frac{\mathrm{d}^3\boldsymbol{R}_s}{V}\,\frac{T_{0s}h_s}{F_{0s}}\left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{c}}$$

arXiv:0704.0044

$$\delta f_s = -q_s\varphi F_{0s}/T_{0s} + h_s(t,\boldsymbol{R}_s,v_\perp,v_\parallel) \qquad \boldsymbol{R}_s = \boldsymbol{r} + \boldsymbol{v}_\perp\times\hat{\boldsymbol{z}}/\Omega_s$$

$$\frac{\partial h_s}{\partial t} + v_\parallel\frac{\partial h_s}{\partial z} + \frac{c}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s}, h_s\} = \frac{q_s F_{0s}}{T_{0s}}\frac{\partial\langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t} + \left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{c}}$$

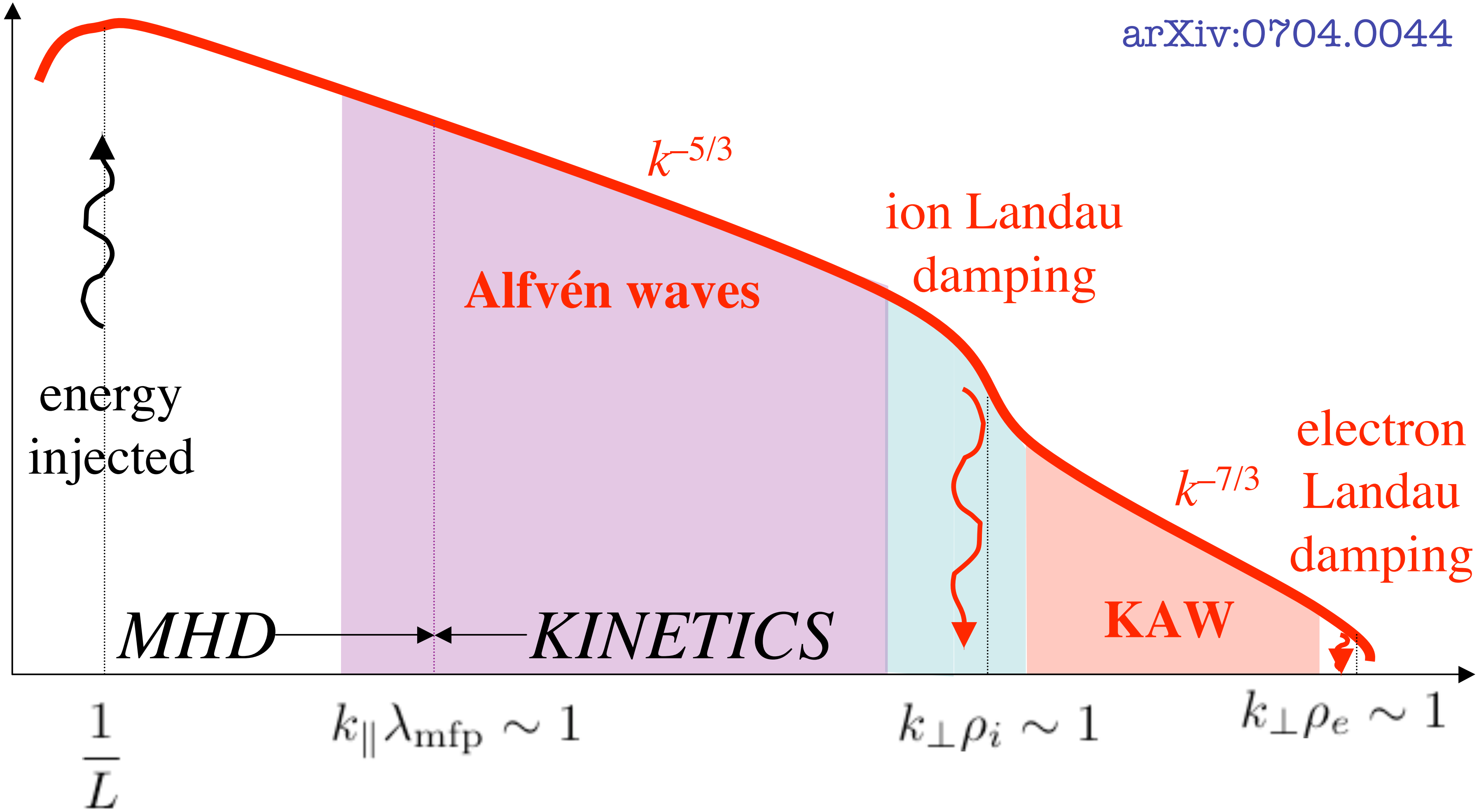
$$\chi = \varphi - \boldsymbol{v}\cdot\boldsymbol{A}/c,\ \boldsymbol{B} = B_0\hat{\boldsymbol{z}} + \delta\boldsymbol{B},\ \delta\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$$

$$\sum_s\frac{q_s^2\varphi}{T_{0s}}n_{0s} = \sum_s q_s\int\mathrm{d}^3\boldsymbol{v}\,\langle h_s\rangle_{\boldsymbol{r}}$$

+ Ampère's law

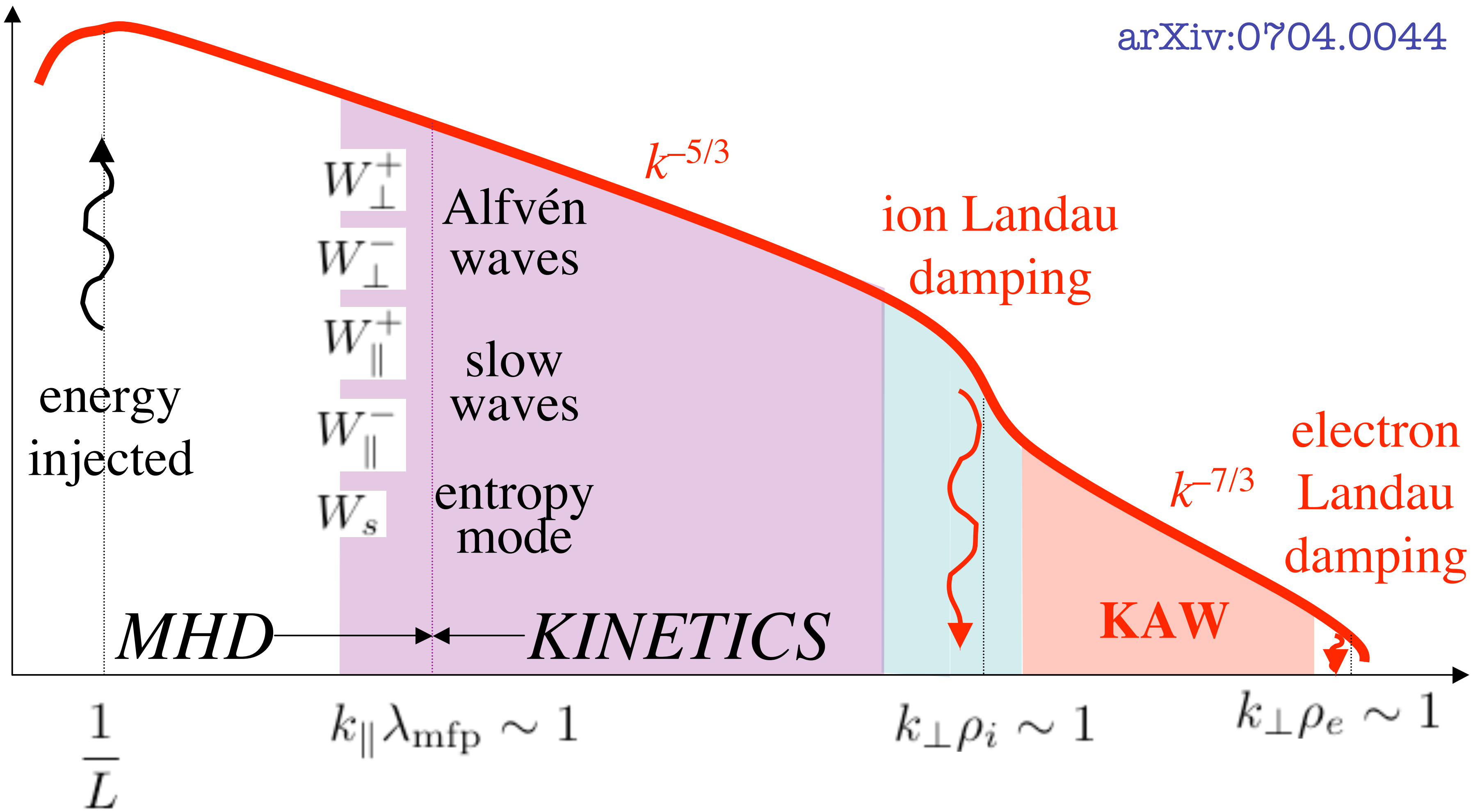
Taylor & Hastie 1968
Rutherford & Frieman 1968
Frieman & Chen 1982

The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left(\sum_s \int d^3\mathbf{v} \frac{T_{0s}\delta f_s^2}{2F_{0s}} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right)$$

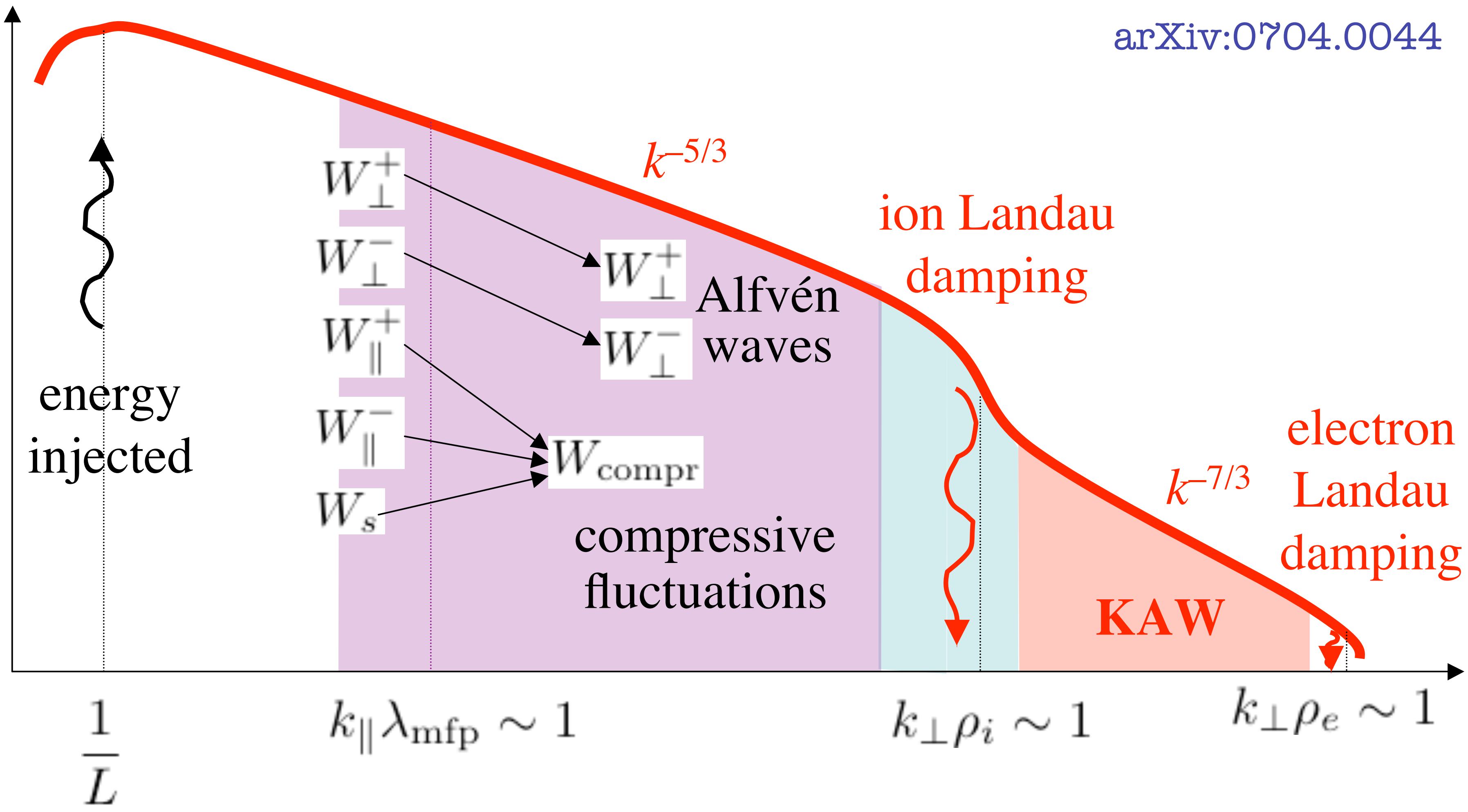
The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left[\frac{m_i n_{0i}}{2} \left(|\nabla \zeta^+|^2 + |\nabla \zeta^-|^2 \right) + \frac{m_i n_{0i}}{2} \left(|z_\parallel^+|^2 + |z_\parallel^-|^2 \right) + \frac{3}{4} n_{0i} T_{0i} \frac{1 + Z/\tau}{5/3 + Z/\tau} \frac{\delta s^2}{s_0^2} \right]$$

Alfvén waves **slow waves** **entropy fluctuations**

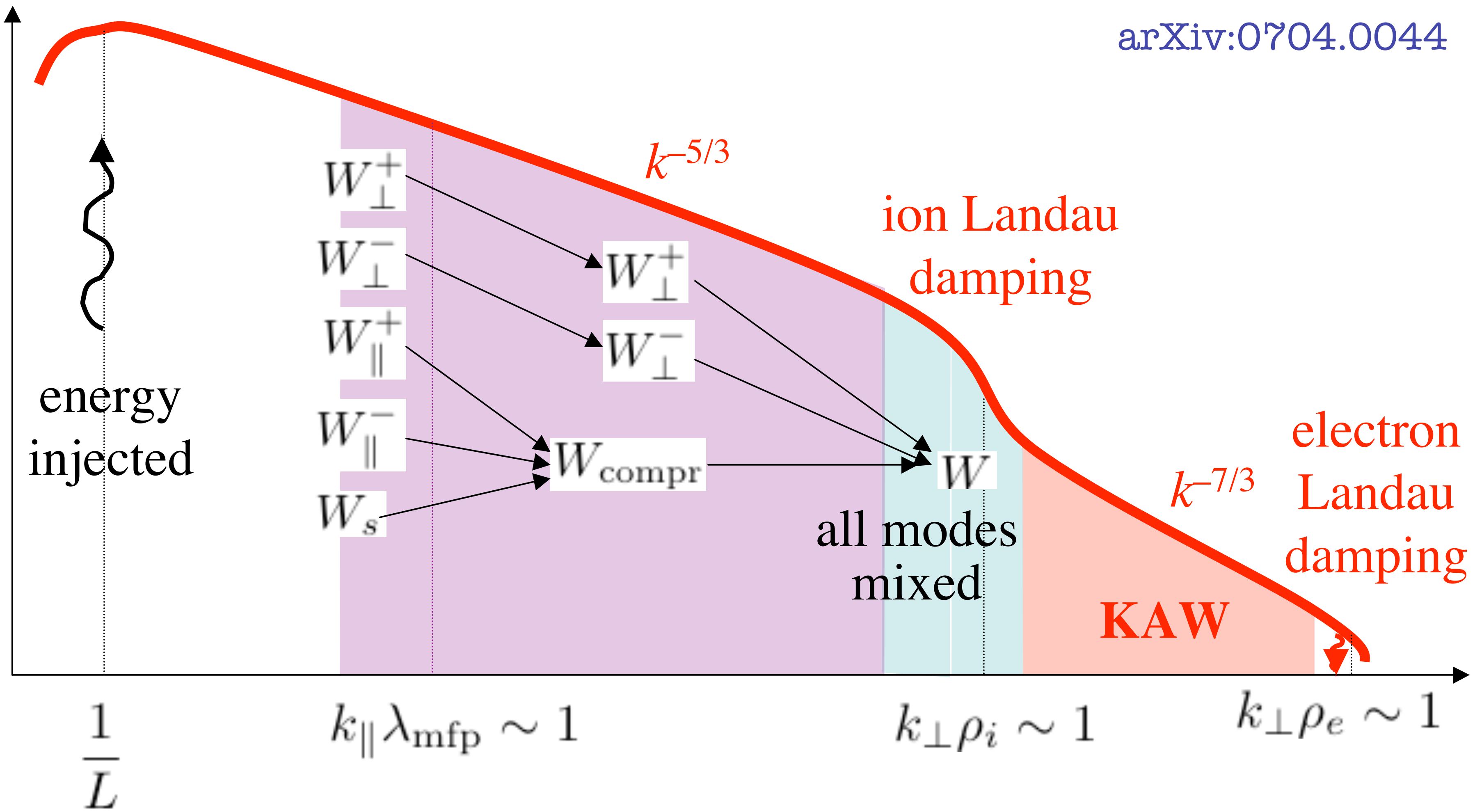
The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left[\frac{m_i n_{0i}}{2} \left(|\nabla \zeta^+|^2 + |\nabla \zeta^-|^2 \right) + \frac{n_{0i} T_{0i}}{2} \left(\frac{Z}{\tau} \frac{\delta n_e^2}{n_{0e}^2} + \frac{2}{\beta_i} \frac{\delta B_\parallel^2}{B_0^2} + \frac{1}{n_{0i}} \int d^3\mathbf{v} \frac{T_{0i} \delta \tilde{f}_i^2}{2F_{0i}} \right) \right]$$

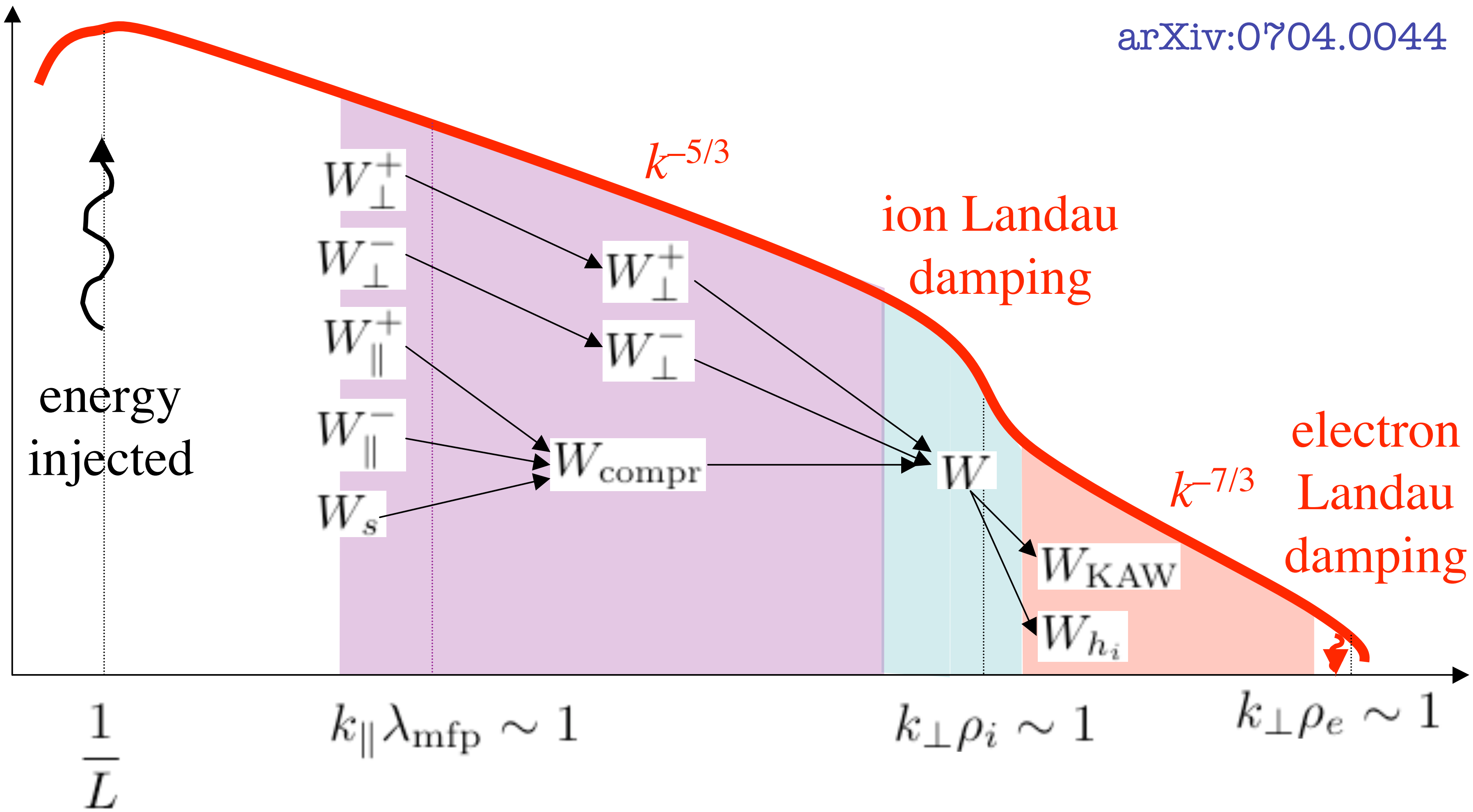
Alfvén waves **compressive fluctuations**

The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left(\int d^3\mathbf{v} \frac{T_{0i}\delta f_i^2}{2F_{0i}} + \frac{n_{0e}T_{0e}}{2}\frac{\delta n_e^2}{n_{0e}^2} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right)$$

The Grand Kinetic Cascade

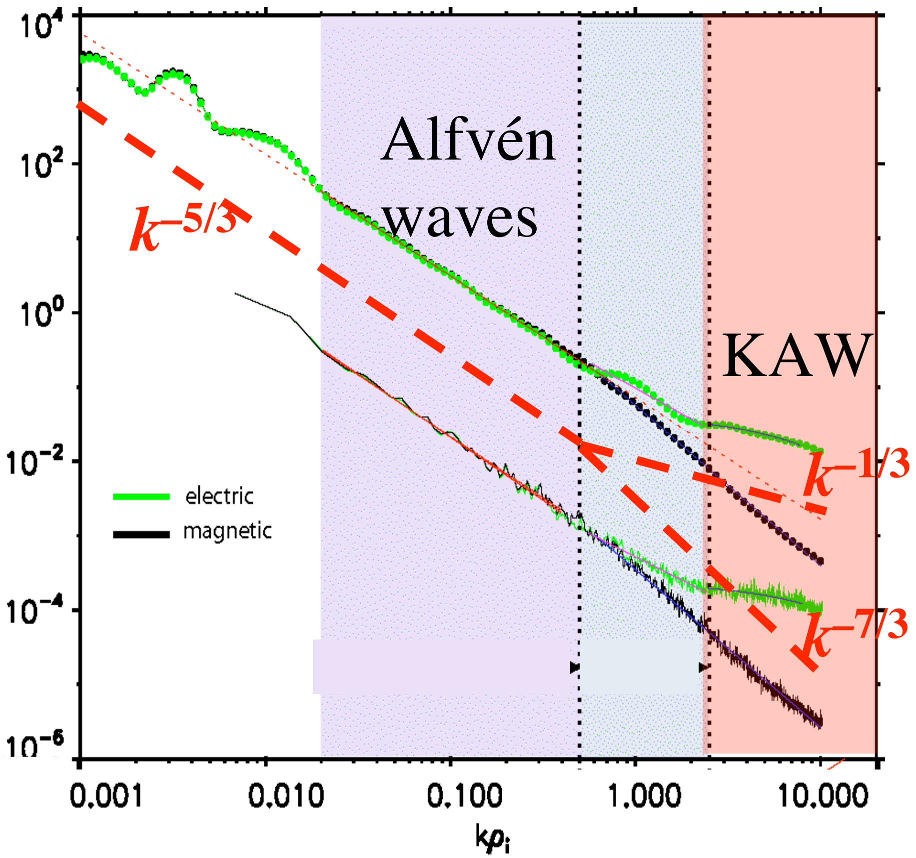


$$W = \int d^3\mathbf{r} \int d^3\mathbf{v} \frac{T_{0i} h_i^2}{2F_{0i}} + \int d^3\mathbf{r} \left\{ \frac{\delta B_\perp^2}{8\pi} + \frac{n_{0i} T_{0i}}{2} \left(1 + \frac{Z}{\tau}\right) \left[1 + \frac{\beta_i}{2}\left(1 + \frac{Z}{\tau}\right)\right] \left(\frac{Ze\phi}{T_{0i}}\right)^2 \right\}$$

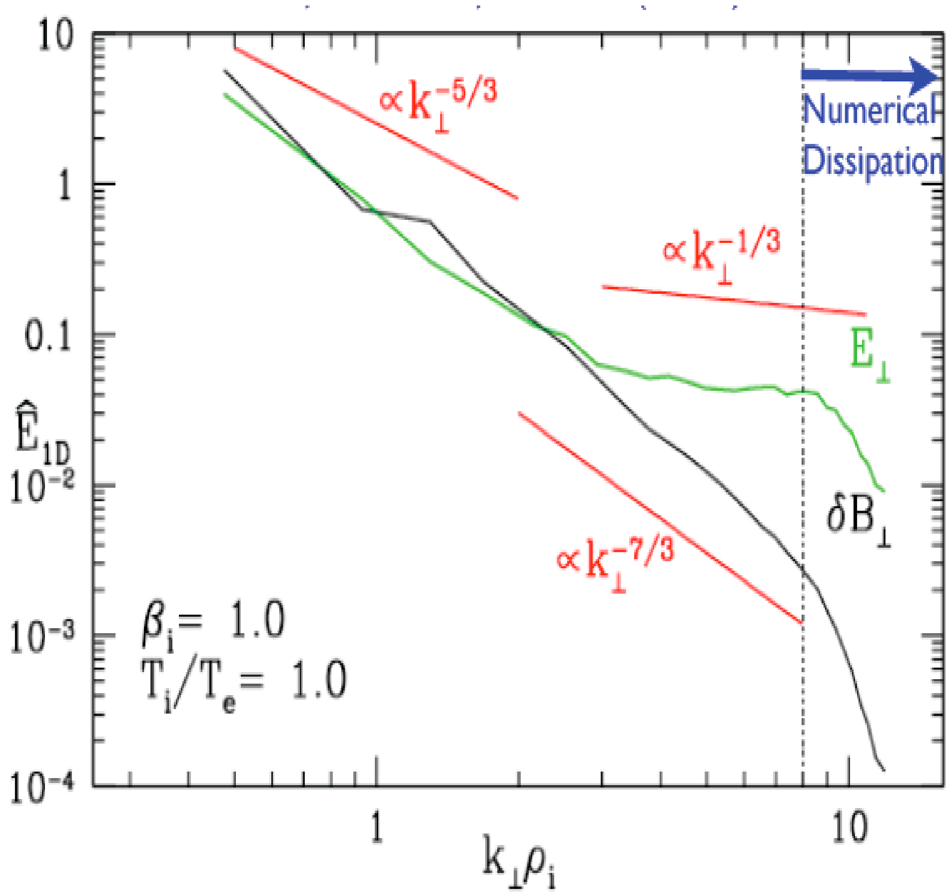
ENTROPY CASCADE **kinetic Alfvén waves**

Alfvén-Wave Turbulence: GK DNS by G. Howes

Alfvén-wave turbulence in the solar wind
[by Bale et al. 2005, *PRL* 94, 215002]



Alfvén-wave turbulence using GS2
[by Howes et al. 2008, *PRL* 100, 065004]



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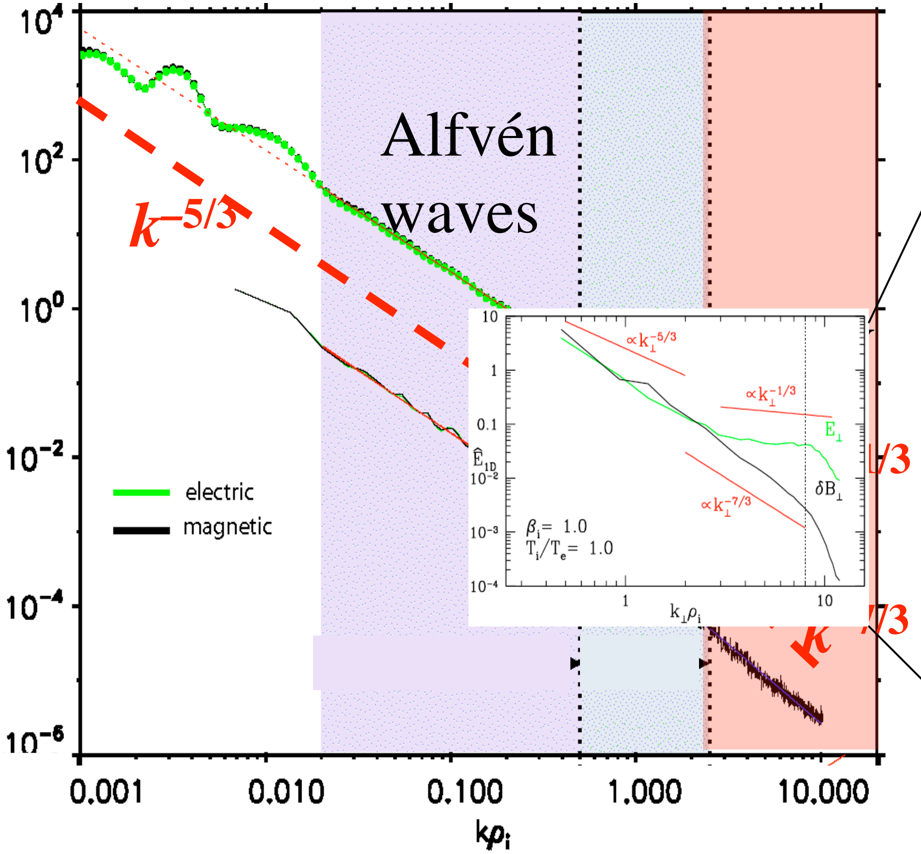
[by Bale et al. 2005, *PRL* **94**, 215002]



Alfvén-wave turbulence using GS2
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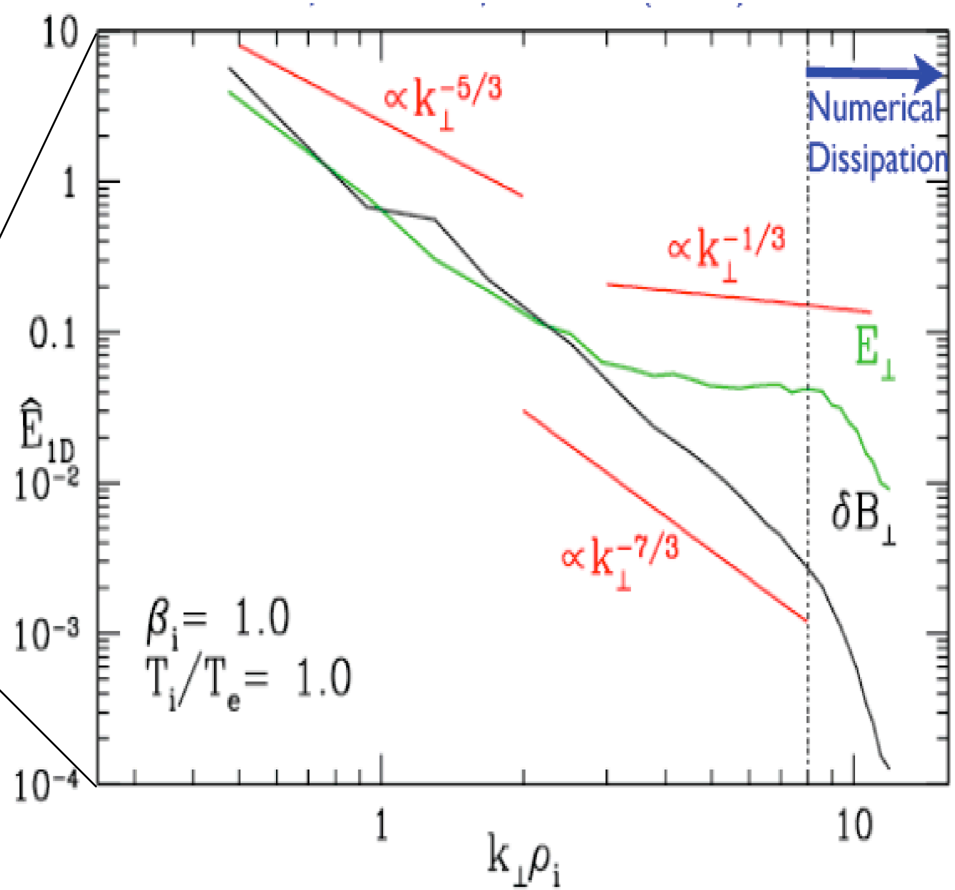

Nonlinear Perpendicular Phase Mixing

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_c = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1 + \frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int d^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

This comes from gyroaveraging

NB: In fluid models (like EMHD) these fluctuations are invisible

arXiv:0806.1069

Nonlinear Perpendicular Phase Mixing

$$\frac{\partial h_i}{\partial t} + v_{\|} \frac{\partial h_i}{\partial z} + \frac{c}{B_0} \{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t} \frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

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Low-frequency electrostatic fluctuations

- Potential mixes h_i via this term, so h_i developes small (perpendicular) scales in the gyrocenter space: $k_\perp \rho_i \gg 1$

arXiv:0806.1069

Nonlinear Perpendicular Phase Mixing

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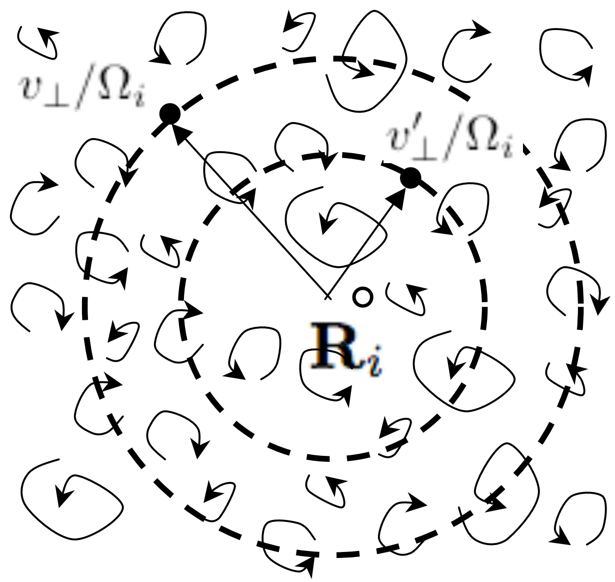
Low-frequency electrostatic fluctuations

- Potential mixes h_s via this term, so h_s developes small (perpendicular) scales in the gyrocenter space: $k_\perp \rho_i \gg 1$
- Two values of the gyroaveraged potential $\langle\varphi\rangle_{\mathbf{R}_i}(\mathbf{v})$ and $\langle\varphi\rangle_{\mathbf{R}_i}(\mathbf{v}')$ come from spatially decorrelated fluctuations if

$$\frac{v_\perp}{\Omega_i} - \frac{v'_\perp}{\Omega_i} \sim \frac{1}{k_\perp} \quad \Rightarrow \quad \frac{\delta v_\perp}{v_{\mathrm{th}i}} \sim \frac{1}{k_\perp \rho_i}$$



[The perpendicular nonlinear phase-mixing mechanism was anticipated in the work of Dorland & Hammett 1993]

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

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Low-frequency electrostatic fluctuations

- Electrostatic fluctuations come from ion-entropy fluctuations:

$$\frac{Ze\varphi(\boldsymbol{k})}{T_{0i}} \sim \frac{v_{\mathrm{th}i}^3}{n_{0i}} \frac{1}{\sqrt{k_\perp \rho_i}} \left(\frac{\delta v_\perp}{v_{\mathrm{th}i}}\right)^{1/2} h_i(\boldsymbol{k}) \sim \frac{v_{\mathrm{th}i}^3}{n_{0i}} \frac{h_i(\boldsymbol{k})}{k_\perp \rho_i}$$

- Entropy is conserved, so use const-flux argument:

$$\frac{m_i v_{\mathrm{th}i}^8}{n_{0i}} \frac{h_{i\lambda}^2}{\tau_\lambda} \sim \varepsilon$$

- Nonlinear decorrelation time:

$$\tau_\lambda \sim \left(\frac{\rho_i}{\lambda}\right)^{1/2} \frac{\lambda^2}{c\varphi_\lambda / B_0}$$

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1+\frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

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$$c\varphi_\lambda/B_0 \sim v_{\mathrm{th}i}^4 h_{i\lambda}\lambda/n_{0i}$$

- Nonlinear decorrelation time:

$$\tau_\lambda \sim \left(\frac{\rho_i}{\lambda}\right)^{1/2} \frac{\lambda^2}{c\varphi_\lambda/B_0} \sim \frac{\rho_i^{1/2}\lambda^{1/2} n_{0i}}{v_{\mathrm{th}i}^4 h_{i\lambda}}$$

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

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Low-frequency electrostatic fluctuations

We get the following set of scaling relations:

$l_0 = m_i n_{0i} v_{\mathrm{th}i}^3/\varepsilon$

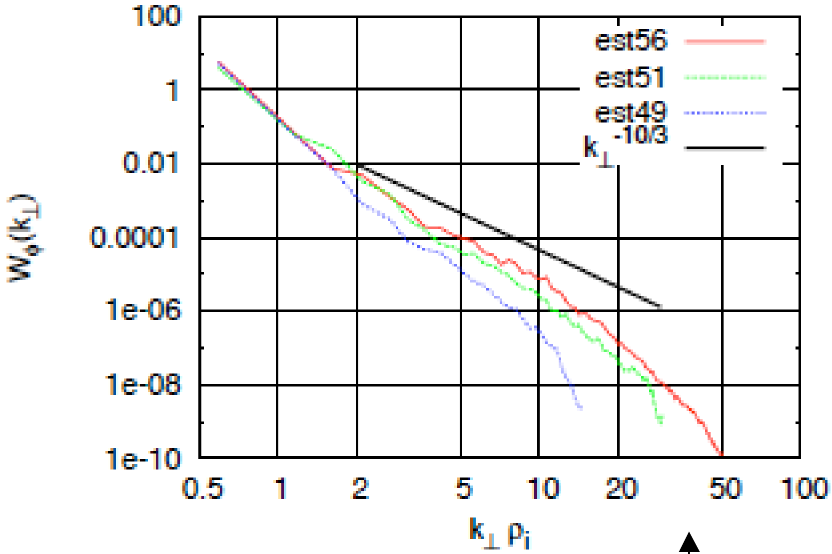
$$\frac{Ze\varphi_\lambda}{T_{0i}} \sim \frac{\lambda^{7/6}}{\rho_i^{5/6} l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-10/3}$$

$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\mathrm{th}i}^3}\frac{\rho_i^{1/6}\lambda^{1/6}}{l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-4/3}$$

$$\tau_\lambda \sim \frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_{\mathrm{th}i}}$$

arXiv:0806.1069

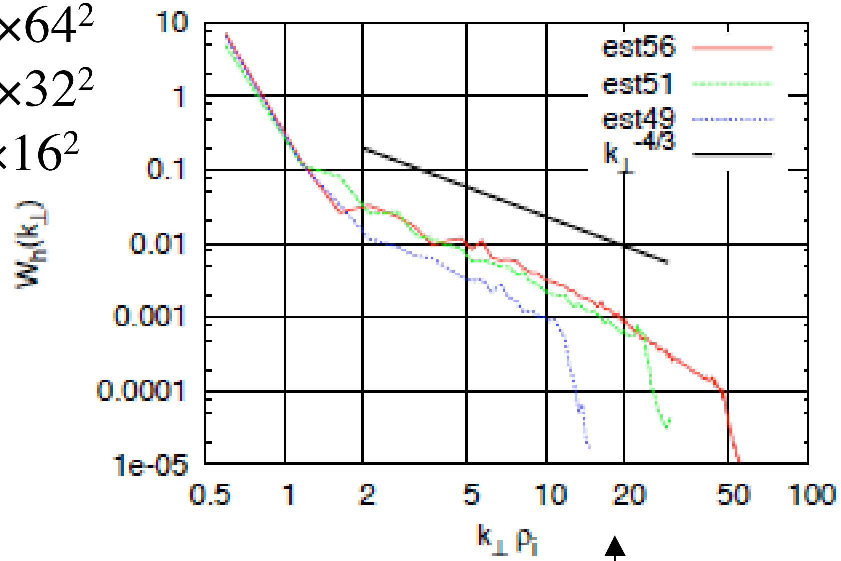
Entropy Cascade: GK 4D DNS by T. Tatsuno



$256^2 \times 64^2$

$128^2 \times 32^2$

$64^2 \times 16^2$



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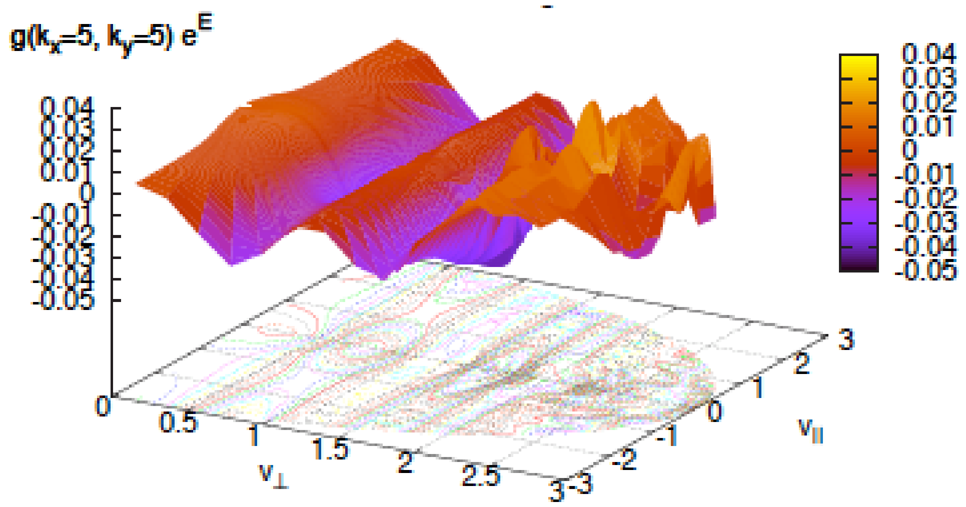
$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\mathrm{th}i}^3} \frac{\rho_i^{1/6}\lambda^{1/6}}{l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-4/3}$$

$$\tau_\lambda \sim \frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_{\mathrm{th}i}}$$

arXiv:0806.1069

Entropy Cascade: GK 4D DNS by T. Tatsuno

Distribution function develops small-scale structure in velocity space
(G. Plunk has developed v-space spectral formalism to diagnose that: work in progress)



$$l_0 = m_i n_{0i} v_{\mathrm{th}i}^3/\varepsilon$$

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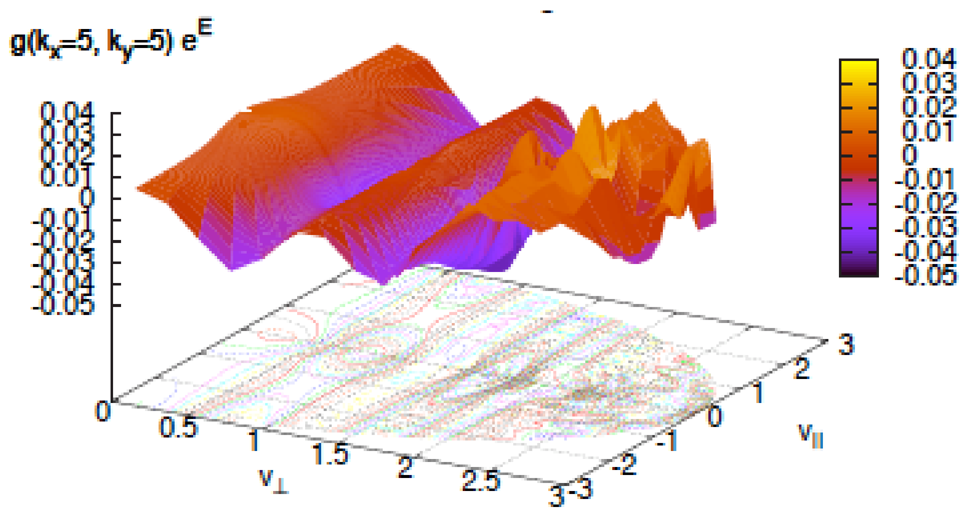
arXiv:0806.1069

Phase-Space Cutoff

Distribution function develops small-scale structure in velocity space
(G. Plunk has developed *v*-space spectral formalism to diagnose that: work in progress)



$$\frac{\delta v_\perp}{v_{\mathrm{th}i}} \sim \left(\frac{\nu_{ii}}{\omega}\right)^{1/2}$$

$$\tau_\lambda \sim \frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_{\mathrm{th}i}}$$

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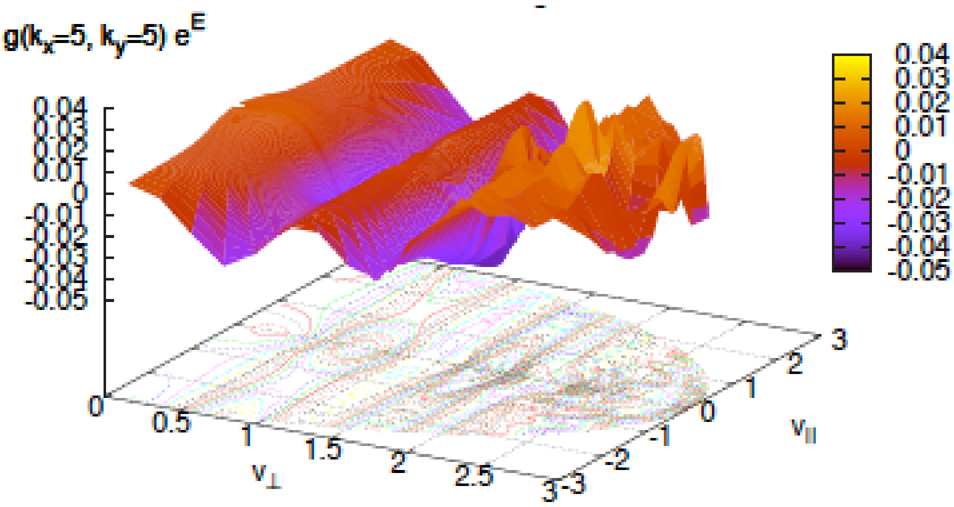


$$\frac{\delta v_{\perp c}}{v_{\rm thi}} \sim \frac{1}{k_{\perp c}\rho_i} \sim (\nu_{ii}\tau_{\rho_i})^{3/5} \sim \frac{l_0^{1/5}\rho_i^{2/5}}{\lambda_{\rm mfp}^{3/5}}$$

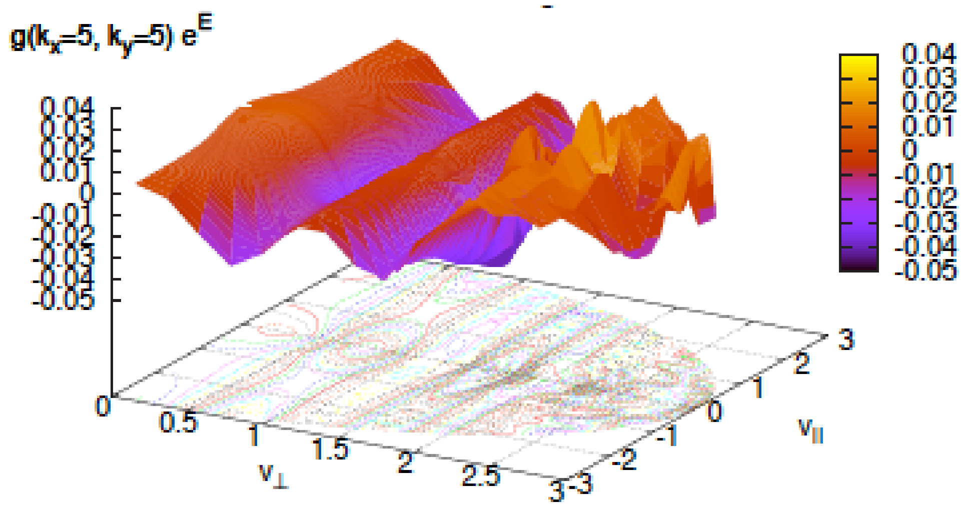
$$\tau_{\rho_i} \sim (m_i n_{0i}\rho_i^2/\varepsilon)^{1/3}$$ characteristic time at the ion gyroscale

$$l_0 = m_i n_{0i} v_{\rm thi}^3/\varepsilon$$

arXiv:0806.1069

Phase-Space Cutoff

Distribution function develops small-scale structure in velocity space
(G. Plunk has developed v-space spectral formalism to diagnose that: work in progress)



$$\frac{\delta v_{\perp c}}{v_{\mathrm{th}i}} \sim \frac{1}{k_{\perp c}\rho_i} \sim \mathrm{Do}^{3/5}$$

x- and v-space resolution are related

$$\mathrm{Do} = \nu_{ii}\tau_{\rho_i} \qquad \tau_{\rho_i} \sim (m_i n_{0i}\rho_i^2/\varepsilon)^{1/3}$$

characteristic time at the ion gyroscale

Dorland Number

cf. $k_c L \sim \mathrm{Re}^{3/4}$ in Kolmogorov fluid turbulence

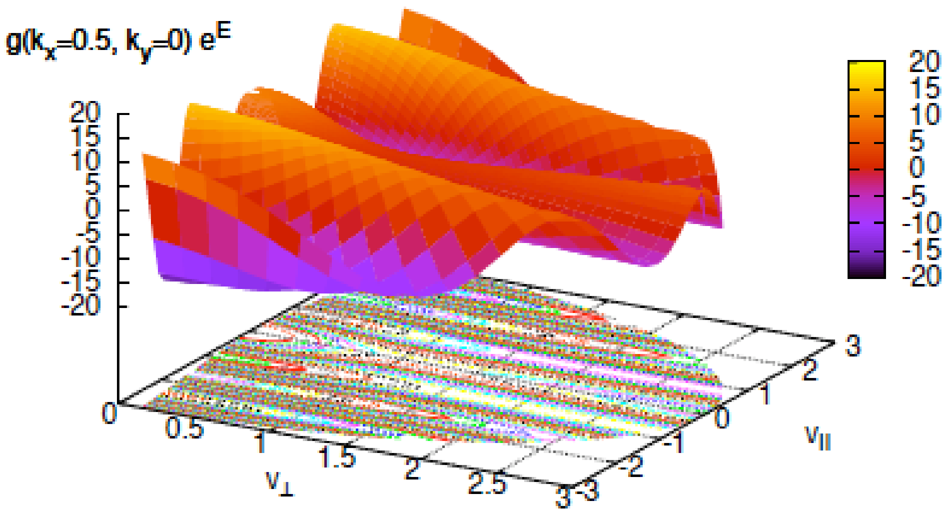
arXiv:0806.1069

Linear Parallel Phase Mixing

Parallel phase mixing is due to the "ballistic response":

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \cdots = 0$$

$$h_i \propto e^{ik_\parallel v_\parallel t}$$



$$\frac{\delta v_\parallel}{v_{\mathrm{th}i}} \sim \frac{1}{k_\parallel v_{\mathrm{th}i} t} \sim 1$$

after $t \sim \tau_\lambda$

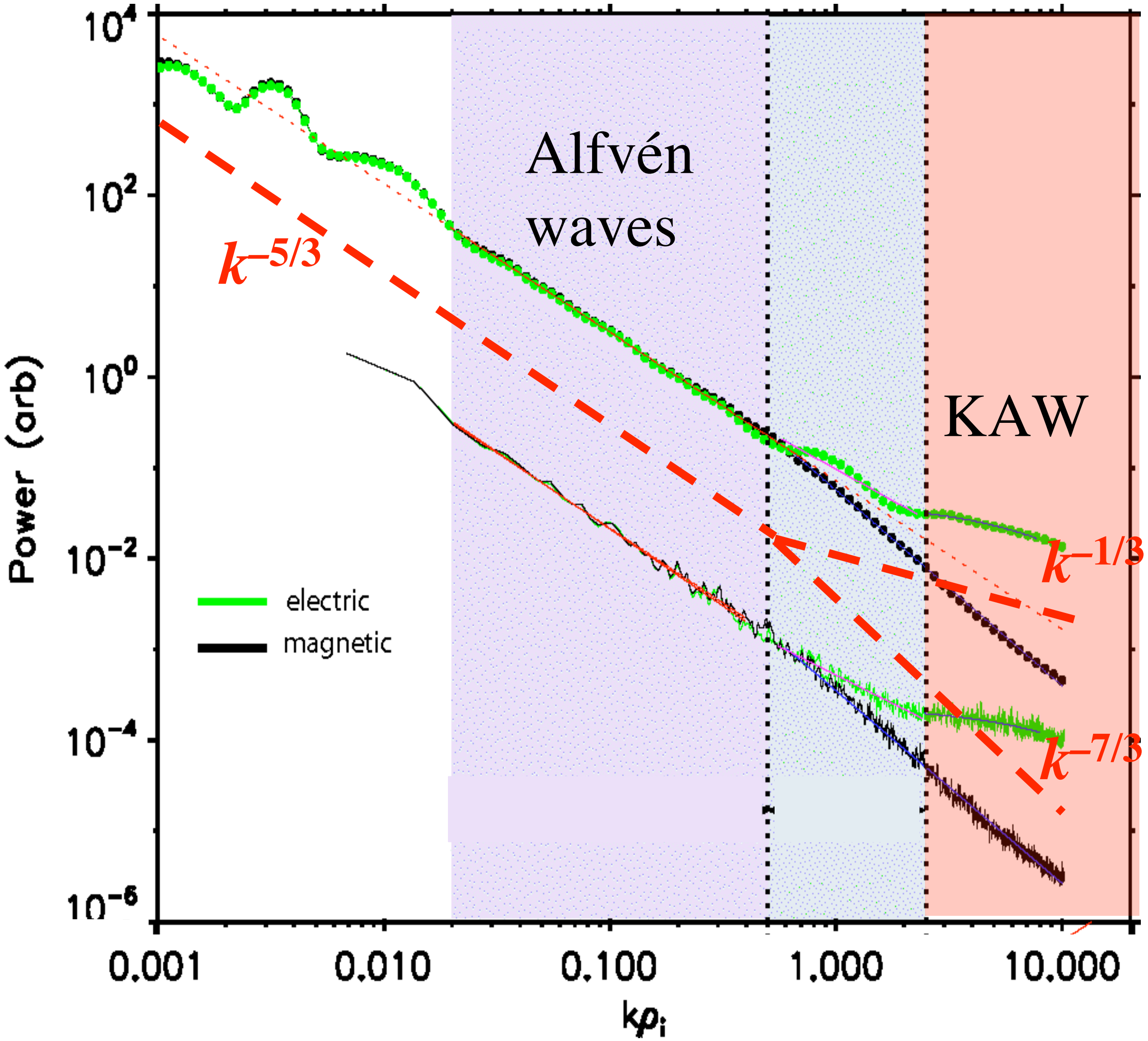
if linear propagation time ~ nonlinear decorrelation time

("critical balance")

So the nonlinear perpendicular phase mixing dominates

arXiv:0806.1069

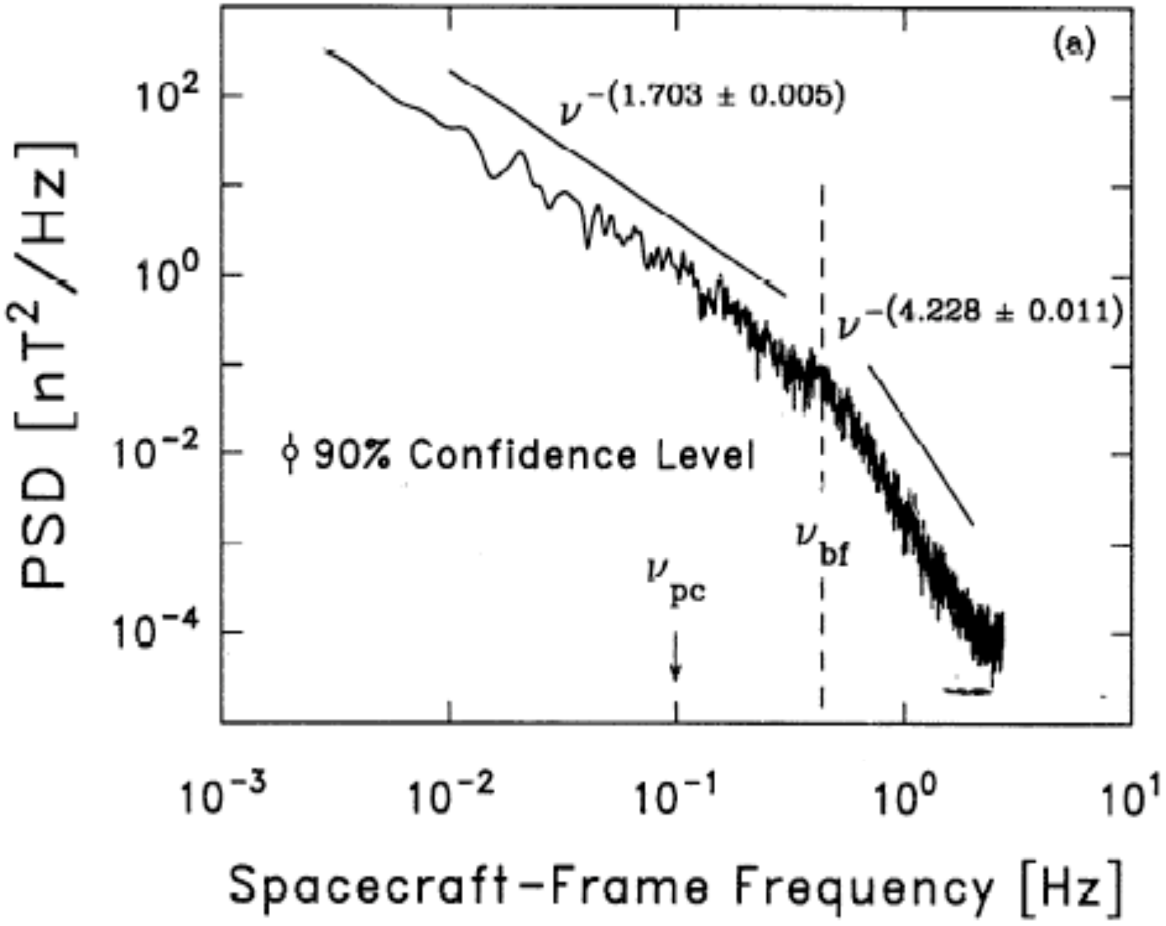
Dissipation Range of the SW: KAW?



Magnetic- and electric-field fluctuations in the solar wind at ~1 AU (19 Feb. 2002)

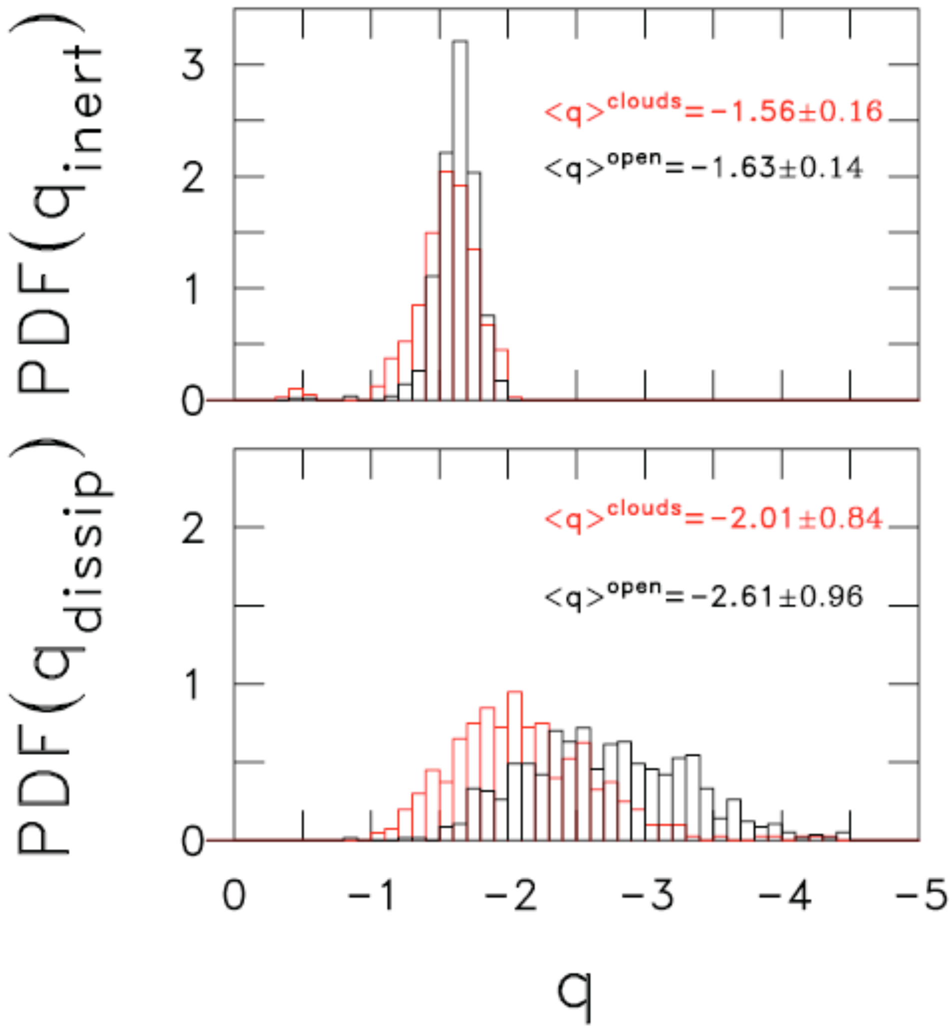
[Bale *et al*. 2005, *PRL* **94**, 215002]

Dissipation Range of the SW: No KAW?



Magnetic-field fluctuations in the solar wind at ~1 AU (19 Feb. 2002)
[Leamon *et al*. 1998, *JGR* **103**, 4775]

Dissipation Range of the SW: ???



Spectral indices in the inertial and dissipation ranges

[Smith *et al*. 2006, *ApJ* **645**, L85]

Dissipation Range With and Without KAW

With KAW

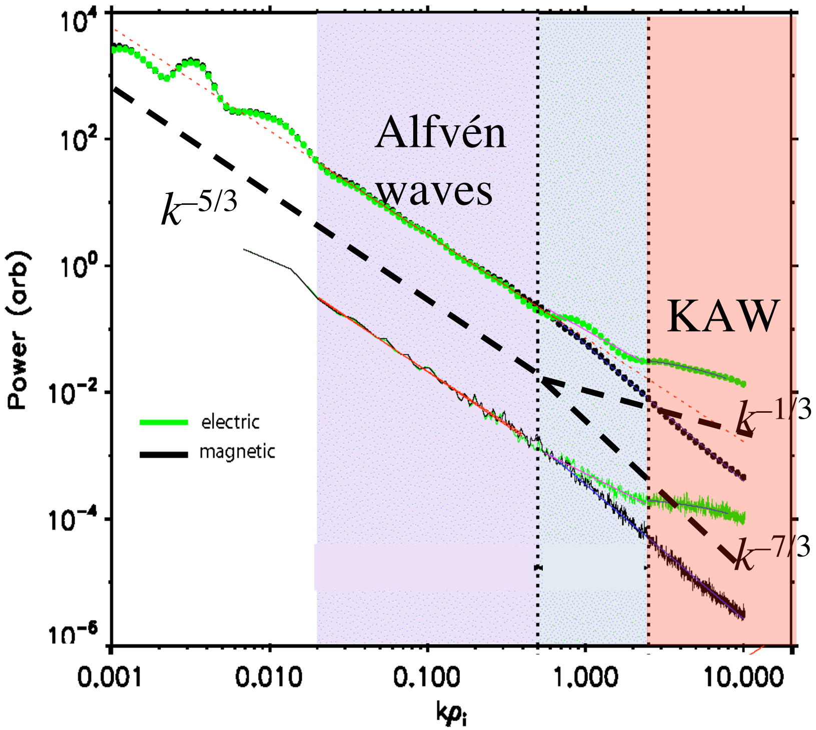
$$E_E(k_\perp) \propto k_\perp^{-1/3}$$
$$E_B(k_\perp) \propto k_\perp^{-7/3}$$
$$E_n(k_\perp) \propto k_\perp^{-7/3}$$

High-frequency, electromagnetic, fluid-like (EMHD)

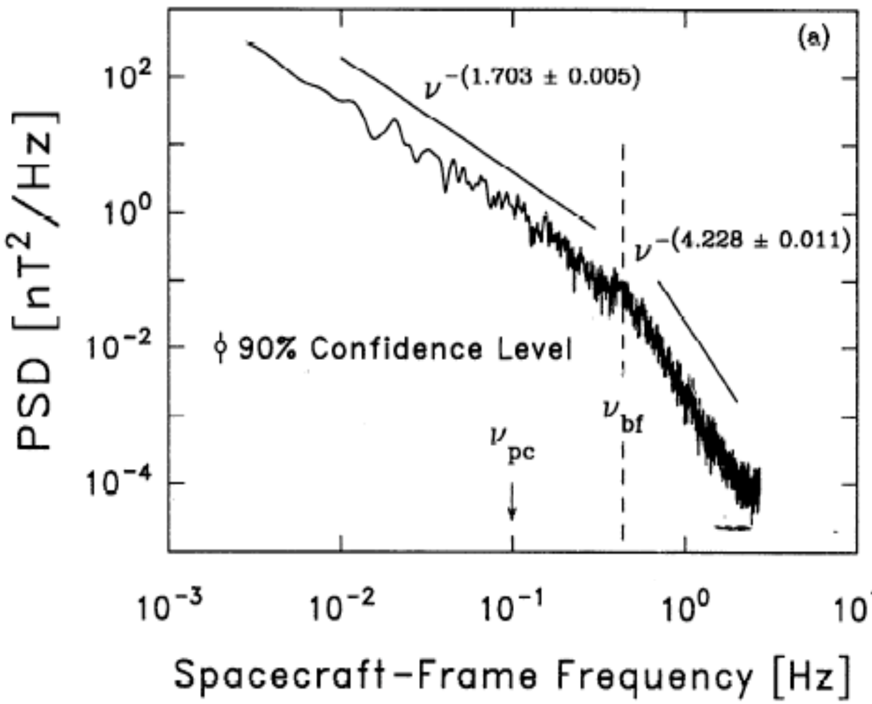
Without KAW

Low-frequency, electrostatic, purely kinetic (GK ions)

$$E_E(k_\perp) \propto k_\perp^{-4/3}$$
$$E_B(k_\perp) \propto k_\perp^{-16/3}$$
$$E_n(k_\perp) \propto k_\perp^{-10/3}$$



[Bale *et al.* 2005, *PRL* **94**, 215002]



[Leamon *et al.* 1998, *JGR* **103**, 4775]

Dissipation Range With and Without KAW

With KAW		**Without KAW**	
$E_E(k_\perp) \propto k_\perp^{-1/3}$ $E_B(k_\perp) \propto k_\perp^{-7/3}$ $E_n(k_\perp) \propto k_\perp^{-7/3}$	High-frequency, electromagnetic, fluid-like (EMHD)	Low-frequency, electrostatic, purely kinetic (GK ions)	$E_E(k_\perp) \propto k_\perp^{-4/3}$ $E_B(k_\perp) \propto k_\perp^{-16/3}$ $E_n(k_\perp) \propto k_\perp^{-10/3}$

Dissipation Range of the Solar Wind

With KAW

$$E_E(k_\perp) \propto k_\perp^{-1/3}$$
$$E_B(k_\perp) \propto k_\perp^{-7/3}$$
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Without KAW

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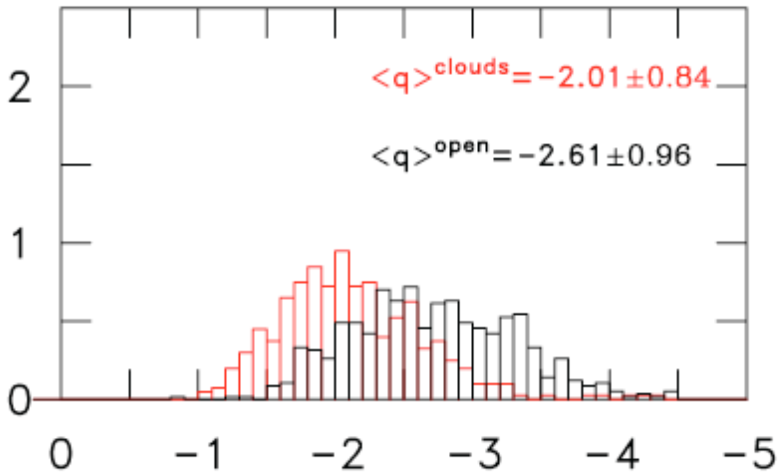
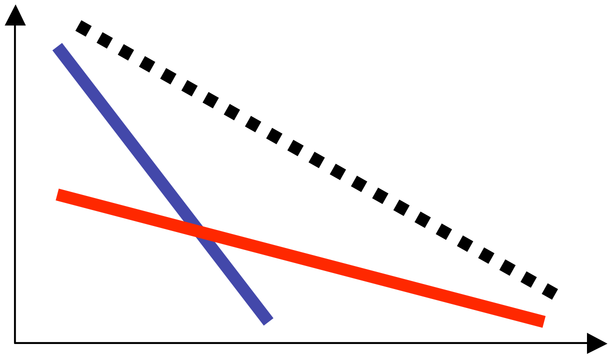
$$E_B(k_\perp) \propto k_\perp^{-16/3}$$
$$E_n(k_\perp) \propto k_\perp^{-10/3}$$



[Smith *et al*. 2006, *ApJ* **645**, L85]



Variable spectral index in the dissipation range may be due to superposition of KAW and no KAW cascades

arXiv:0704.0044

Conclusions

- **Kinetic turbulence is a generalised energy cascade in phase space towards collisional scales**
- In gyrokinetic turbulence, a fast nonlinear perpendicular phase-mixing mechanism allows small scale structure to emerge simulataneously in physical and velocity space
- This takes the form an entropy cascade, giving rise to power law spectra of electrostatic fluctuations and fluctuations of the distribution function
- This cascade has observable signatures — e.g., in the dissipation range of the solar wind turbulence

Details are in these preprints: `arXiv:0704.0044, 0806.1069`

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- **Annoying practical lessons for kinetic turbulence simulations:**
  — pay attention to velocity-space resolution!
  — need physical model for collisions!
  [see poster by Ian Abel & Michael Barnes]

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