



Calculation of rf current drive in tokamaks

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... to Lucas, born yesterday afternoon !

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- current drive is a key tool for real-time control of plasma performances and stability of tokamak plasmas
- keywords for current drive are *localization* and *efficiency*.
- high frequency rf waves are attractive tools for a long time:
 - **Lower Hybrid** (1-10GHz): $D_{QL} \parallel B$
 - **Electron cyclotron** (~100 GHz): $D_{QL} \perp B$



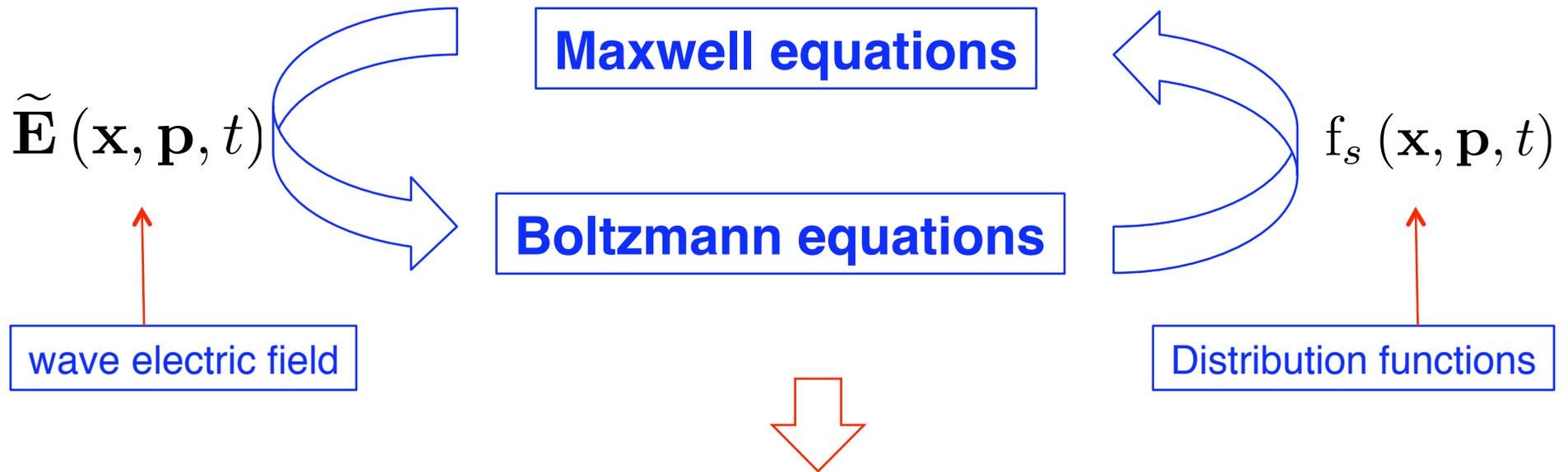
- integrated tokamak modeling put high constraints on the level of accuracy that should be reached by simulations of rf current drive:
 - *interpretation* of the observed phenomenology
 - reliable *prediction* capability
- **physics**: unified multi-wave description (synergism), consistent momentum/configuration space dynamics, neoclassical corrections (high ∇p), locally non-axisymmetric magnetic configuration,...
- **numerics**: high modularity, fast and robust algorithms



Outline

- background of rf current drive theory
- the current drive module
 - **C3PO** ray-tracing
 - **LUKE** Fokker-Planck
- rf current drive in tokamaks
 - **LH** wave: *ITER, Tore Supra*
 - **EC** wave: *TCV*
- conclusion and prospects

rf current drive theory



$$\mathbf{J}(\mathbf{x}, t) = |e| Z_s \iiint d^3 \mathbf{p} f_s(\mathbf{x}, \mathbf{p}, t) \mathbf{p} / \gamma$$



- Boltzmann equation (ion dynamics ignored ($m_e/m_i \ll 1$))

$$\left\{ \begin{array}{l} \frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \nabla_{\mathbf{x}} f + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} f = C(f, f_s) \\ \dot{\mathbf{p}} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ \dot{\mathbf{x}} = \mathbf{v} = \mathbf{p}/\gamma \end{array} \right.$$

↓
Fokker-Planck
collision operator
 $\mathcal{O}(1/\log \Lambda)$

- Maxwell's equations

$$\left\{ \begin{array}{l} \nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} - c^{-2} \partial \mathbf{E} / \partial t \end{array} \right.$$

Space- and time-scale ordering

- Small parameter expansion: $\delta^2 \simeq \rho/R \simeq t_b/\Omega$
- In tokamaks, Coulomb collisions $\nu/\Omega \leq \delta^2$

$$\left\{ \begin{array}{l} \partial/\partial t = \boxed{\partial/\partial t_{\omega,\Omega}} + \boxed{\delta^2 \partial/\partial t_b} \\ \nabla_{\mathbf{x}} = \boxed{\nabla_{\mathbf{x}_\rho}} + \boxed{\delta \nabla_{\mathbf{x}_T}} + \boxed{\delta^2 \nabla_{\mathbf{x}_R}} \end{array} \right.$$

Gyro-motion ← Radial transport → Orbits

- Expansion in power of δ

$$f = f_0 + \delta f_1 + \delta^2 f_2 + \dots$$

$$\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}_1 + \delta^2 \mathbf{B}_2 + \dots$$

$$\mathbf{E} = \mathbf{E}_0 + \delta \mathbf{E}_1 + \delta^2 \mathbf{E}_2 + \dots$$

$$\mathbf{J} = \mathbf{J}_0 + \delta \mathbf{J}_1 + \delta^2 \mathbf{J}_2 + \dots$$

$$C = \delta^2 C_2 + \dots$$

- Magnetic equilibrium: $\mathbf{E}_0 = 0$



- to order δ^0

$$\cancel{\partial f_0 / \partial t}_{\omega, \Omega} + \mathbf{v} \cdot \cancel{\nabla}_{\mathbf{x}_\rho} f_0 + \boxed{\Omega \partial f_0 / \partial \varphi} = 0$$

gyro-independent ←

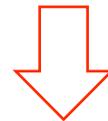
Equilibrium magnetic field and current:


$$\left\{ \begin{array}{l} \nabla_{\mathbf{x}_R} \times \mathbf{B}_0 = \mu_0 \mathbf{J}_0 \\ \mathbf{J}_0(\mathbf{x}, t) = e \iiint \mathbf{v} f_0(\mathbf{x}, \mathbf{p}, t) d^3 \mathbf{p} \end{array} \right.$$



- to order δ^1

$$\begin{aligned} \partial f_1 / \partial t_{\omega, \Omega} + \mathbf{v} \cdot \nabla_{\mathbf{x}_\rho} f_1 + \Omega \partial f_1 / \partial \varphi = \\ -e [\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1] \cdot \nabla_{\mathbf{p}} f_0 - \mathbf{v} \cdot \nabla_{\mathbf{x}_T} f_0 \end{aligned}$$



$$\mathbf{J}_1(\mathbf{x}, t) = e \iiint \mathbf{v} f_1(\mathbf{x}, \mathbf{p}, t) d^3 \mathbf{p}$$

$$= \boxed{\mathbb{S}}(f_0) \cdot \mathbf{E}_1$$

← constitutive relation

conductivity tensor 



Maxwell equation linear in \mathbf{E}_1 :

$$\nabla_{\mathbf{x}_\rho} \times \nabla_{\mathbf{x}_\rho} \times \mathbf{E}_1 + \mu_0 \mathcal{S}(f_0) \cdot \partial \mathbf{E}_1 / \partial t_{\Omega, \omega} + c^{-2} \partial^2 \mathbf{E}_1 / \partial^2 t_{\Omega, \omega} = 0$$



- to order δ^2

$$\begin{aligned} & \partial f_2 / \partial t_{\omega, \Omega} + \mathbf{v} \cdot \nabla_{\mathbf{x}_\rho} f_2 + \Omega \partial f_2 / \partial \varphi + \\ & \boxed{\partial f_0 / \partial t_b} + \mathbf{v} \cdot \nabla_{\mathbf{x}_R} f_0 + \mathbf{v} \cdot \nabla_{\mathbf{x}_T} f_1 + \\ & e [\mathbf{E}_2 + \mathbf{v} \times \mathbf{B}_2] \cdot \nabla_{\mathbf{p}} f_0 + \\ & e [\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1] \cdot \nabla_{\mathbf{p}} f_1 = C(f_0) \end{aligned}$$

Using linear relation between f_0 and f_1 (order δ^1) + averaging over fast time scales \rightarrow **slow time scale evolution of f_0 .**

Quasilinear kinetic equation

$$\begin{aligned}\partial f_0 / \partial t_b + \mathbf{v}_{cg} \cdot \nabla_{\mathbf{x}_R} f_0 \\ = C(f_0) + Q(f_0) + T(f_0) + E(f_0)\end{aligned}$$

$$\left\{ \begin{aligned} E(f_0) &= -\nabla_{\mathbf{p}} \left(e \langle \mathbf{E}_2 \rangle_{\Omega, \omega} \cdot f_0 \right) \\ Q(f_0) &\equiv \nabla_{\mathbf{p}} \cdot (\mathbb{D}_{ql} \cdot \nabla_{\mathbf{p}} f_0) \rightarrow \mathbb{D}_{ql} \propto ||\mathbf{E}_1||^2 \\ T(f_0) &\equiv \nabla_{\mathbf{x}_T} \cdot (\mathbb{D}_{\mathbf{x}} \cdot \nabla_{\mathbf{x}_T} f_0) \end{aligned} \right.$$



- quasilinear approximation valid for small wave field amplitude
- no electron trapping in the rf wave field (collisions)
- Guiding center approximation

$$\mathbf{v}_{cg} \simeq p_{\parallel} \hat{\mathbf{b}} / \gamma + \mathbf{v}_D \rightarrow (\text{order } \delta^2)$$
$$\mathbf{p} = p_{\parallel} \hat{\mathbf{b}} + \mathbf{p}_{\perp}$$

Expansion to δ^4 for self-consistent rf & bootstrap current calculations

- pitch-angle cosine: $\xi = p_{\parallel} / p$

 $f_0(\psi, \theta, \phi, p_{\parallel}, p_{\perp})$ is function of five coordinates



Bounce-averaging

- axisymmetric configuration \longrightarrow averaging over ϕ
- New ordering: low collision or « banana » regime

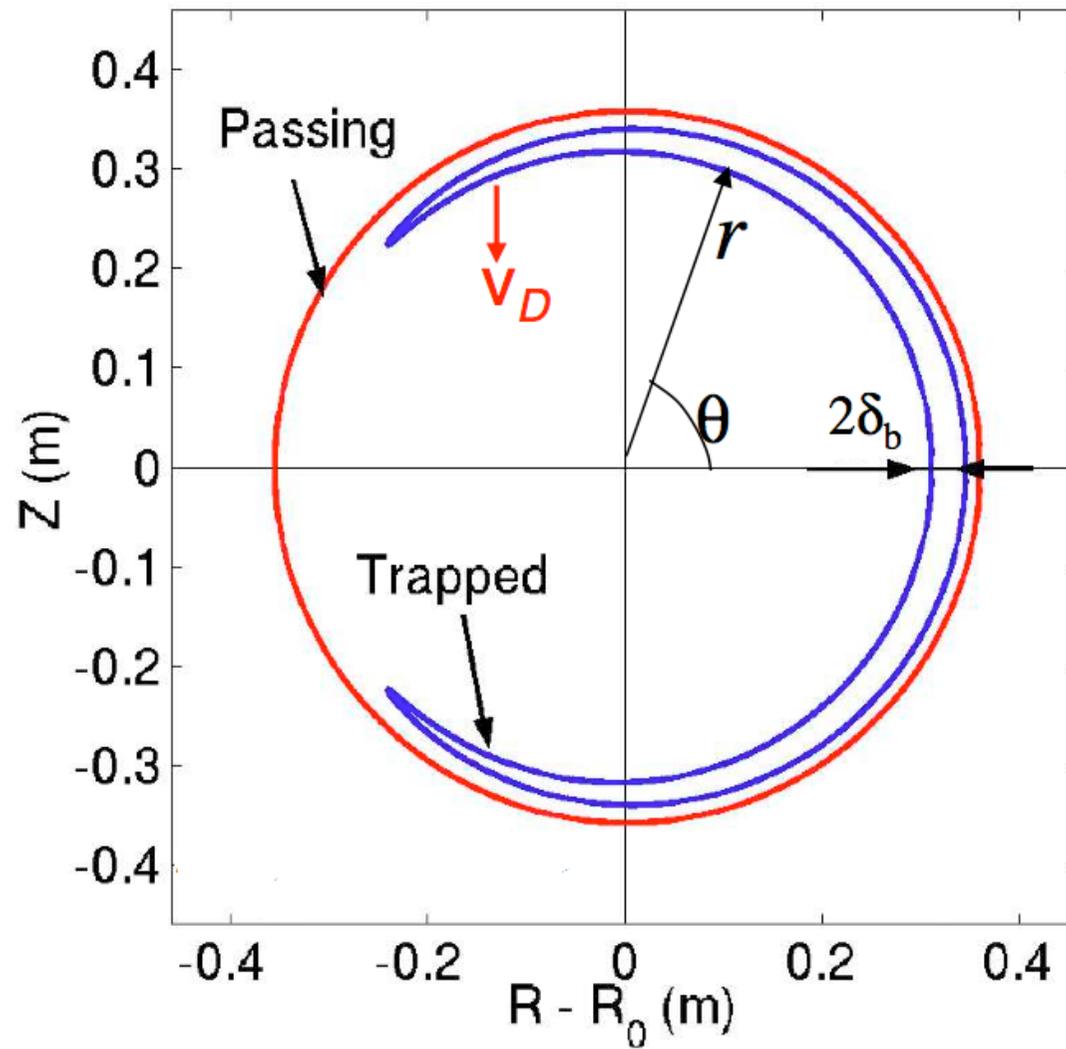
$$\delta^2 \ll \nu^* \ll 1$$

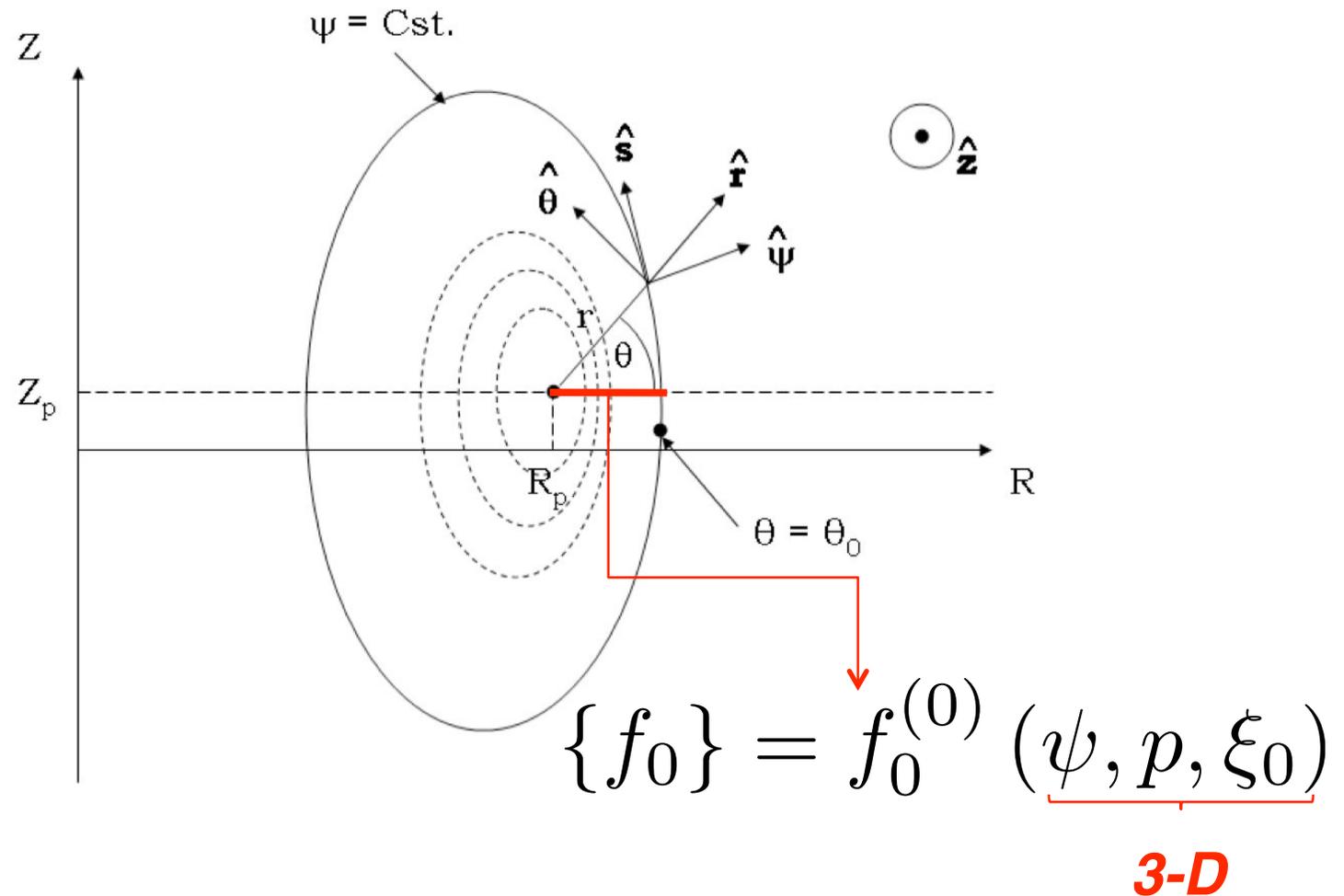
- Thin banana width approximation $\|\mathbf{v}_D\| / \|\mathbf{v}_{cg}\| \ll 1$

$$\partial \{f_0\} / \partial t = \{C(f_0)\} + \{Q(f_0)\} + \{E(f_0)\}$$

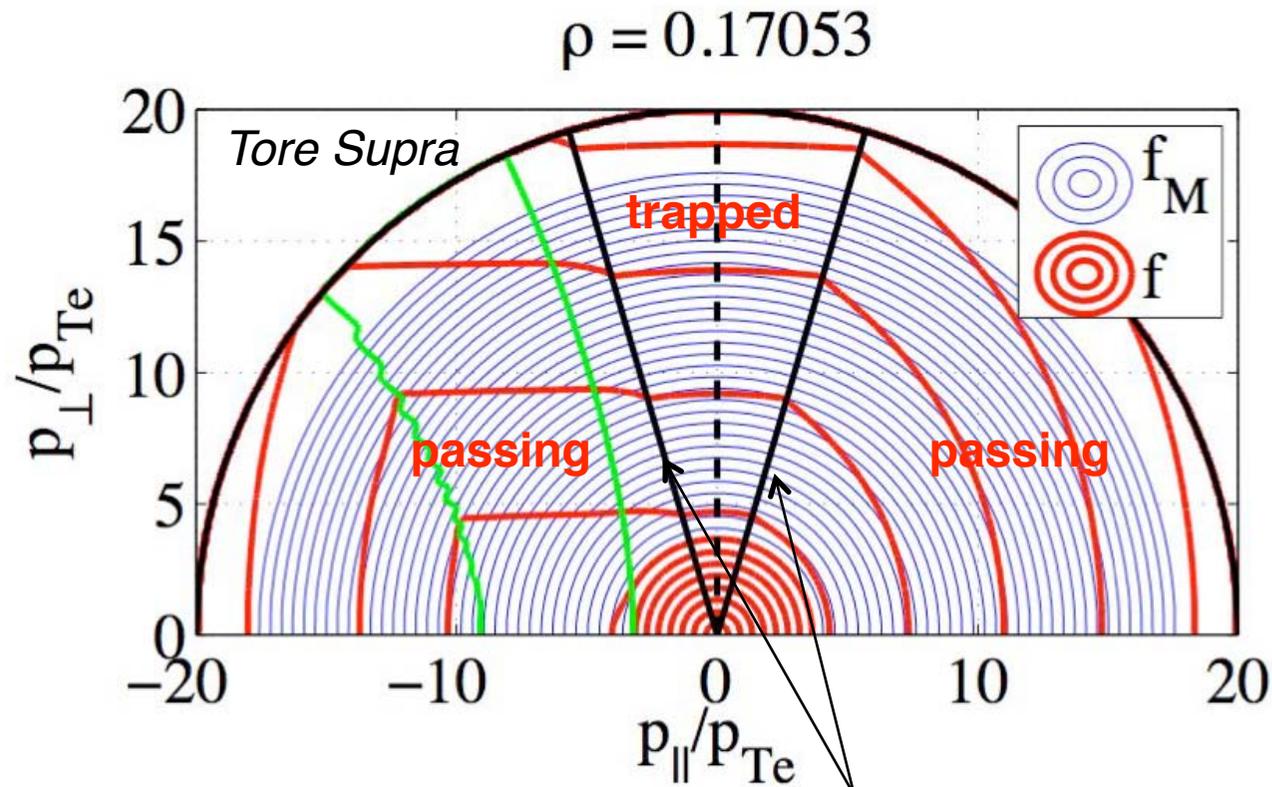
$$\{\mathbf{v}_{cg} \cdot \nabla_{\mathbf{x}_R} f_0\} = 0$$

$$\{\mathcal{O}\} \equiv \frac{1}{\lambda \tilde{q}} \left[\frac{1}{2} \sum \sigma \right]_T \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{2\pi} \frac{1}{|\hat{\psi} \cdot \hat{\mathbf{r}}|} \frac{r}{a_p} \frac{B}{B_P} \frac{\xi_0}{\xi} \mathcal{O}$$



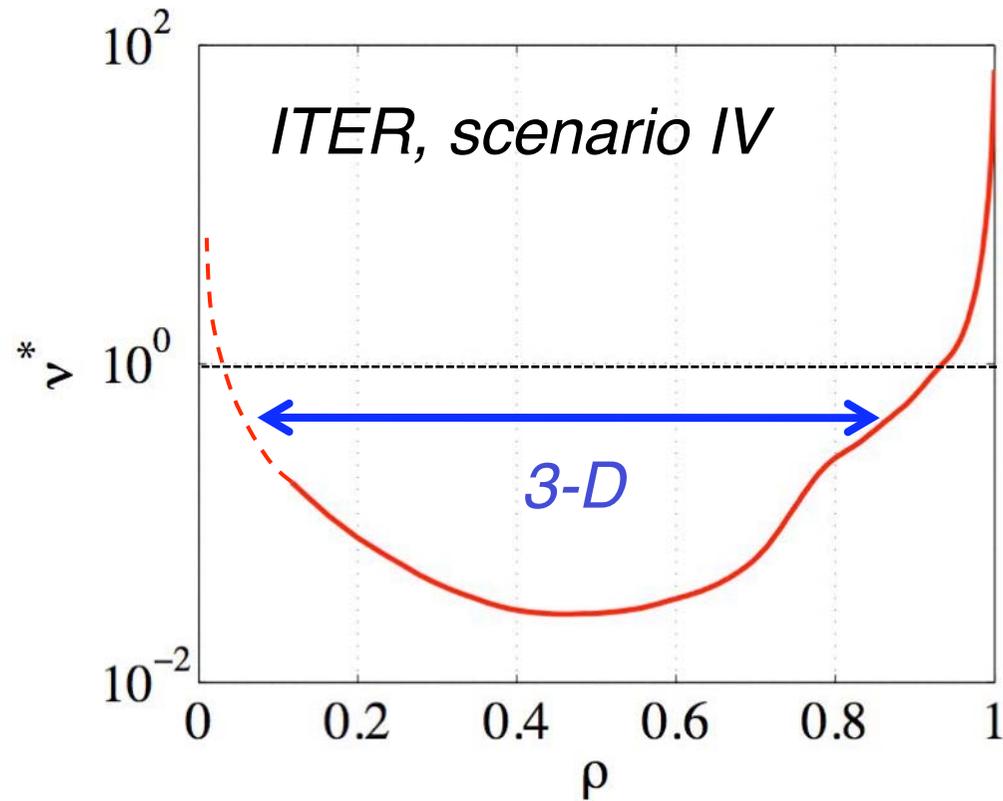


All the electron dynamics is projected at $B=B_{\min}$



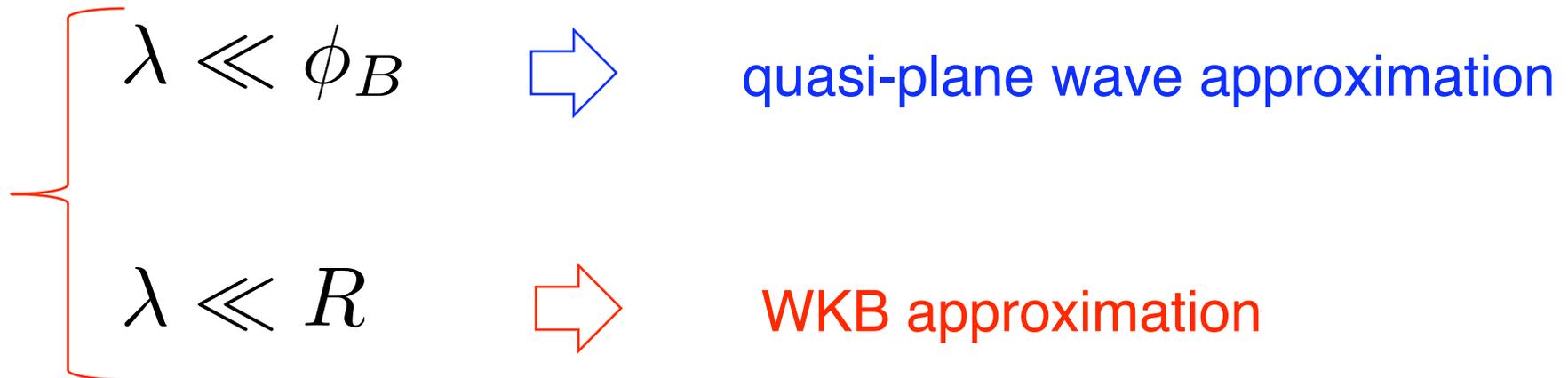
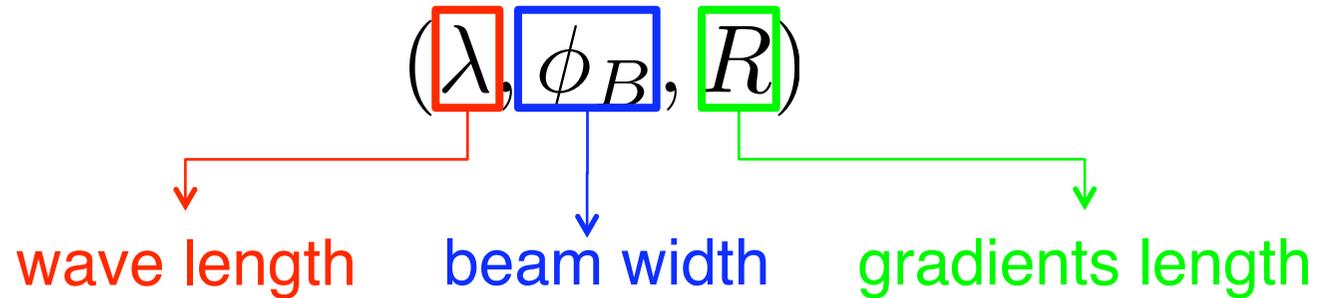
$$\xi_{0T} = \sqrt{1 - \frac{B_{\min}(\psi)}{B_{\max}(\psi)}}$$

Momentum space dynamics on the magnetic flux surface ψ



Domain of validity of the bounce-averaging procedure

rf wave dynamics



Eikonal $S(\mathbf{k}, \omega, \mathbf{x}, t)$

$$\mathbf{E}_1(\mathbf{x}, t) = \mathbf{E}_{\mathbf{k}, \omega}(\mathbf{x}, t) e^{i[\mathbf{k}_1(\mathbf{x}, t) \cdot \mathbf{x} - \omega(\mathbf{x}, t)t]}$$

Quasi-plane wave approximation

WKB approximation

Smooth wavefield envelope

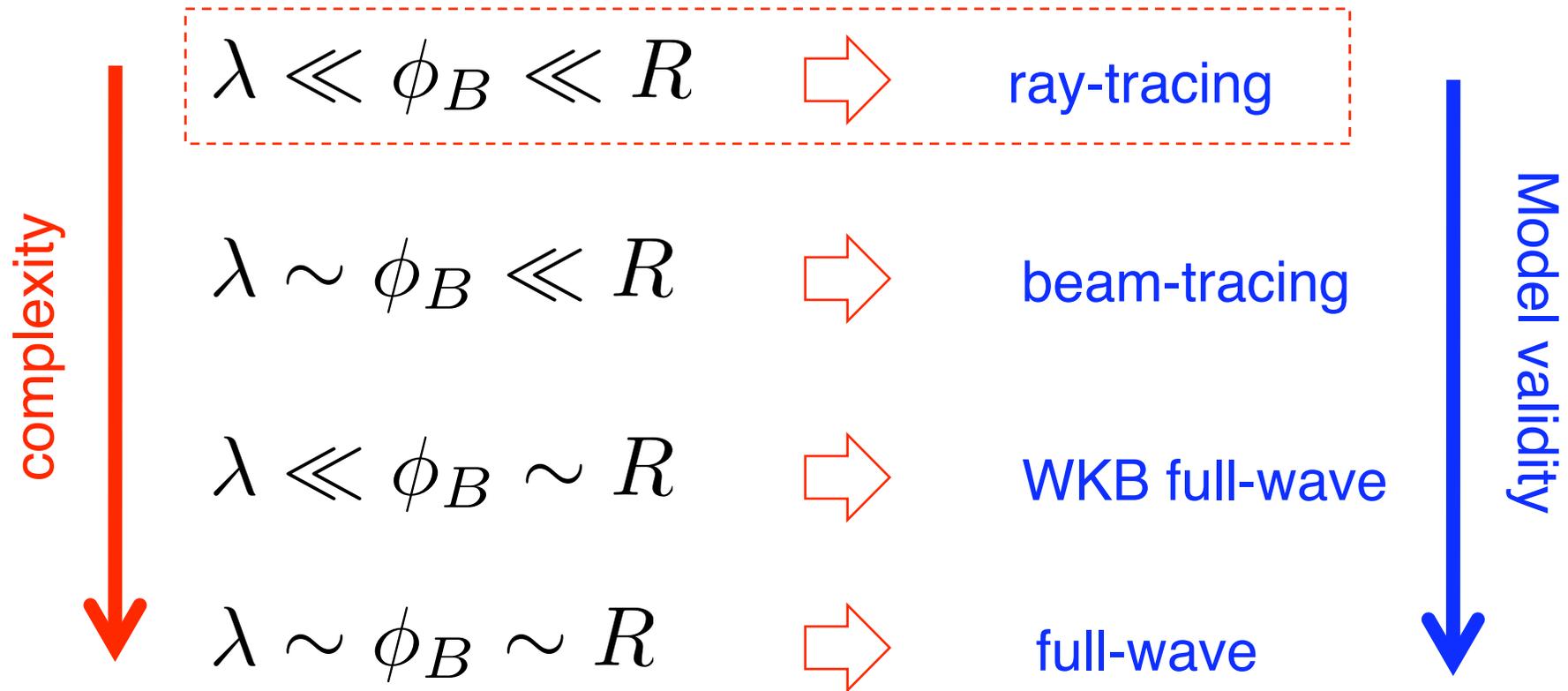
$$\left\{ \begin{array}{l} \lim_{t \rightarrow -\infty} \|\mathbf{E}_{\mathbf{k}, \omega}\|^2 = 0 \\ \lim_{\|\mathbf{x}\| \rightarrow \infty} \|\mathbf{E}_{\mathbf{k}, \omega}\|^2 = 0 \end{array} \right.$$

Methods for uniform plasmas
can be applied locally:

- *Fourier space description*
- *group velocity*
- *local conductivity tensor*



rf wave dynamics



$$\lambda \ll \phi_B \ll R$$

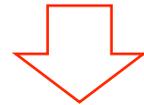
- Narrow spectral width transverse to the beam (\mathbf{v}_G)

$$\|\Delta \mathbf{k}_1 \times \mathbf{v}_G\| / \|\mathbf{k}_1 \times \mathbf{v}_G\| \sim \lambda / \phi_B$$

- weak damping approximation along the beam

$$|\nabla_{\mathbf{x}} \mathbf{E}_{\mathbf{k},\omega} \cdot \mathbf{v}_G| / |\mathbf{E}_{\mathbf{k},\omega} \cdot \mathbf{v}_G| \ll \|\mathbf{k}_1\|$$

$$|\partial \mathbf{E}_{\mathbf{k},\omega} / \partial t \cdot \mathbf{v}_G| / |\mathbf{E}_{\mathbf{k},\omega} \cdot \mathbf{v}_G| \ll \omega$$



Wave equation:

$$\mathbb{D}_{\mathbf{k},\omega} \cdot \mathbf{E}_{\mathbf{k},\omega} = i \nabla_{\mathbf{k}} \mathbb{D}_{\mathbf{k},\omega} : \nabla_{\mathbf{x}} \mathbf{E}_{\mathbf{k},\omega}$$

- dispersion tensor

$$\mathbb{D}_{\mathbf{k},\omega} = \mathbf{n}\mathbf{n} - n^2\mathbb{I} + \mathbb{K}_{\mathbf{k},\omega}(f_0)$$

- Permittivity and susceptibility tensors

$$\mathbb{K}_{\mathbf{k},\omega}(f_0) = \mathbb{I} + \mathbb{X}_{\mathbf{k},\omega}(f_0)$$

$$\mathbb{X}_{\mathbf{k},\omega}(f_0) = i\mathbb{S}_{\mathbf{k},\omega}(f_0) / (\varepsilon_0\omega)$$

- wave refractive index

$$\mathbf{n} = \frac{c}{\omega} \mathbf{k}_1 n_{\parallel} \hat{\mathbf{b}} + \mathbf{n}_{\perp}$$

- wave polarization vector

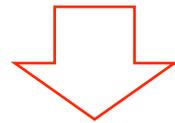
$$\mathbf{e}_{\mathbf{k},\omega} = \mathbf{E}_{\mathbf{k},\omega} / \|\mathbf{E}_{\mathbf{k},\omega}\|$$

Weak damping ordering

- Assuming ω and n_{\parallel} are real (rf wave propagates)
- weak damping approximation:

$$\delta \sim \left| \mathbb{D}_{\mathbf{k},\omega}^A(i,j) \right| / \left| \mathbb{D}_{\mathbf{k},\omega}^H(i,j) \right| \ll 1$$

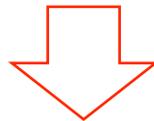
where $\mathbb{D}_{\mathbf{k},\omega} = \mathbb{D}_{\mathbf{k},\omega}^H + i\mathbb{D}_{\mathbf{k},\omega}^A$



Expansion in powers of $\delta \rightarrow \left\{ \begin{array}{l} n_{\perp} = n_{\perp r} + i n_{\perp i} \\ \mathbf{e}_{\mathbf{k},\omega} \\ \mathbb{D}_{\mathbf{k},\omega} \end{array} \right.$

- to order δ^0

$$\mathbb{D}_{\mathbf{k},\omega}^H (n_{\perp 0}) \cdot \mathbf{e}_{\mathbf{k},\omega,0} = 0$$



dispersion relation satisfied by propagative eigenmodes

$$\det \left(\mathbb{D}_{\mathbf{k},\omega}^H \right) = \mathcal{D}(n_{\perp 0}, n_{\parallel}, \omega) = 0$$

$$\hookrightarrow n_{\perp 0} = n_{\perp 0}(n_{\parallel}, \omega)$$

$$n_{\perp i0} = 0$$



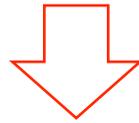
- to order δ^1

Equation of energy transfer between wave and electrons

$$\mathbf{v}_G \cdot \nabla_{\mathbf{x}} \|\mathbf{E}_1\| +$$

$$\frac{\mathbf{e}_{\mathbf{k},\omega,0}^* \cdot \mathbb{D}_{\mathbf{k},\omega}^A \cdot \mathbf{e}_{\mathbf{k},\omega,0}}{\partial(\mathbf{e}_{\mathbf{k},\omega,0}^* \cdot \mathbb{D}_{\mathbf{k},\omega}^H \cdot \mathbf{e}_{\mathbf{k},\omega,0}) / \partial\omega} \|\mathbf{E}_1\| = 0$$

Weak damping approximation



Separation propagation/absorption

Resonance condition

- non-resonant contribution

$\mathbb{D}_{\mathbf{k},\omega}^H \longrightarrow$ principal value of $S_{\mathbf{k},\omega}$

$$\mathbb{D}_{\mathbf{k},\omega}^H (f_0) \simeq \mathbb{D}_{\mathbf{k},\omega}^H (f_M)$$

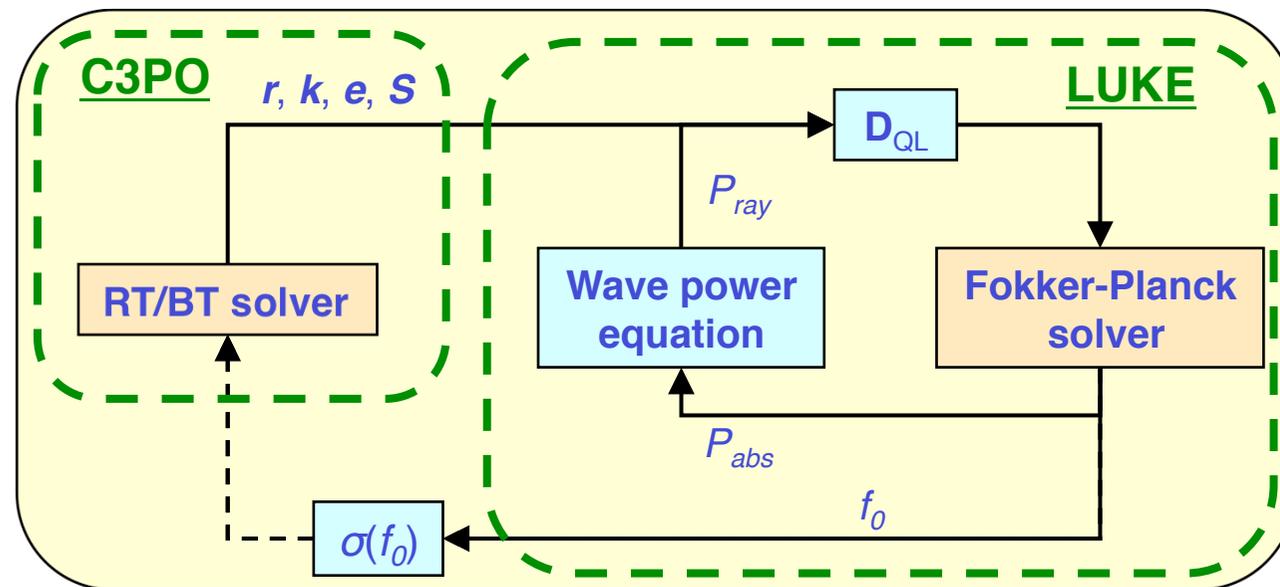
- resonant contribution $\gamma - n_{\parallel} p_{\parallel} - n\Omega/\omega = 0$

$\mathbb{D}_{\mathbf{k},\omega}^A \longrightarrow$ resonant part of $S_{\mathbf{k},\omega}$

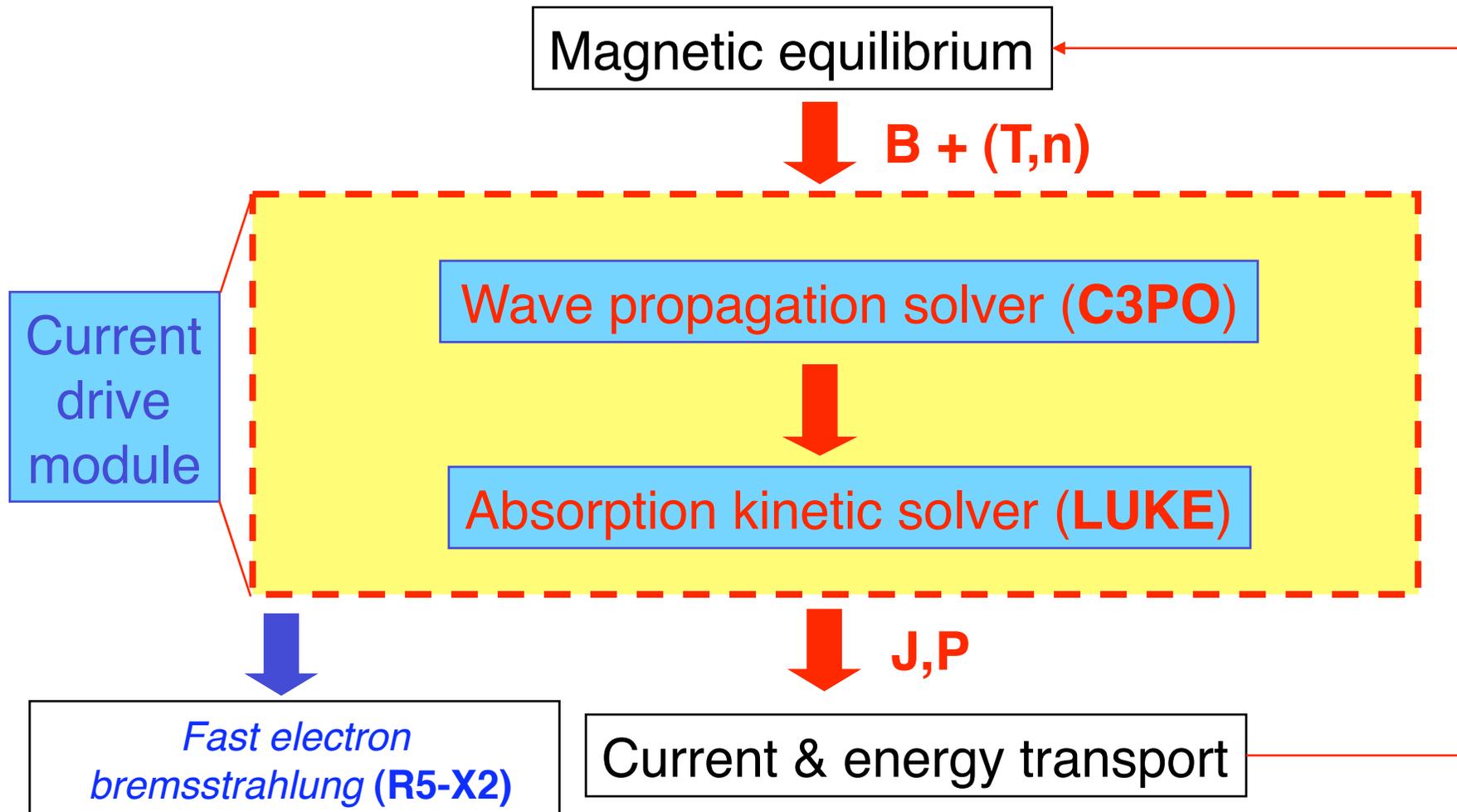
$$\mathbb{D}_{\mathbf{k},\omega}^A (f_0) \neq \mathbb{D}_{\mathbf{k},\omega}^A (f_M)$$



- the ray-tracing $\delta\mathcal{D}(n_{\perp 0}, n_{\parallel}, \omega) = 0$ is function of f_M .
- the wave amplitude, i.e. the quasilinear diffusion coefficient D_{ql} must be calculated self-consistently with the distribution f_0 .
- Global consistency: *power lost by the wave = power gained by electrons from quasilinear operator*



The current drive module



The ray-tracing C3PO

- Separation between plasma dispersion models and the metric associated to the magnetic equilibrium

$$\frac{\partial \mathbb{X}_{ij}^s}{\partial \mathbf{Y}} = \frac{\partial \mathbb{X}_{ij}^s}{\partial n_{\perp}} \frac{\partial n_{\perp}}{\partial \mathbf{Y}} + \frac{\partial \mathbb{X}_{ij}^s}{\partial n_{\parallel}} \frac{\partial n_{\parallel}}{\partial \mathbf{Y}} + \frac{\partial \mathbb{X}_{ij}^s}{\partial \beta_{T_s}} \frac{\partial \beta_{T_s}}{\partial \mathbf{Y}} + \frac{\partial \mathbb{X}_{ij}^s}{\partial \bar{\omega}_{ps}} \frac{\partial \bar{\omega}_{ps}}{\partial \mathbf{Y}} + \frac{\partial \mathbb{X}_{ij}^s}{\partial \bar{\Omega}_s} \frac{\partial \bar{\Omega}_s}{\partial \mathbf{Y}}$$

$$\mathbf{Y} = (\mathbf{X}, \mathbf{k}, t, \omega)$$

$$\beta_s = \sqrt{kT_s / m_s c^2}$$

$$\bar{\omega}_{ps} = \omega_{ps} / \omega \quad \bar{\Omega}_s = \Omega_s / \omega$$



- Curvilinear coordinate system: $(\rho(\psi), \theta, \phi)$
- Vectorization of the magnetic equilibrium: Fourier series + piecewise cubic interpolation using Hermite polynomials: **no interpolation performed at each time step**
- 6th order Runge-Kutta
- rays are calculated inside the separatrix. Specular reflexion enforced (if needed) at $\rho=1$.
- ray calculation stopped when the rf power is linearly damped
- cold, warm, hot and relativistic dielectric tensors
- written in C (MatLab mex-file)
- distributed computing capability (1ray/processor)

The Fokker-Planck solver LUKE

- Fully 3-D conservative formulation

$$\partial f^{(0)} / \partial t + \nabla \cdot \mathbf{S}^{(0)} = s_+^{(0)} - s_-^{(0)}$$

$$\nabla \cdot \mathbf{S}^{(0)} = \frac{B_0}{\tilde{q}\lambda} \frac{\partial}{\partial \psi} \left(\frac{\tilde{q}\lambda}{B_0} \|\nabla \psi\| S_\psi^{(0)} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 S_p^{(0)} \right) - \frac{1}{\lambda p} \frac{\partial}{\partial \xi_0} \left(\lambda \sqrt{1 - \xi_0^2} S_\xi^{(0)} \right)$$

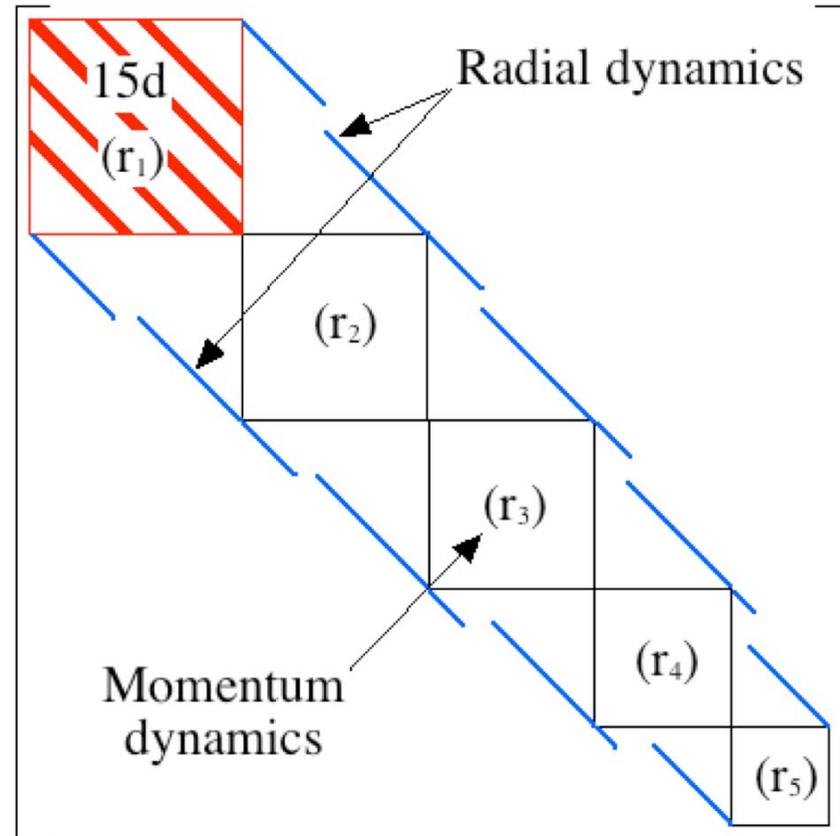
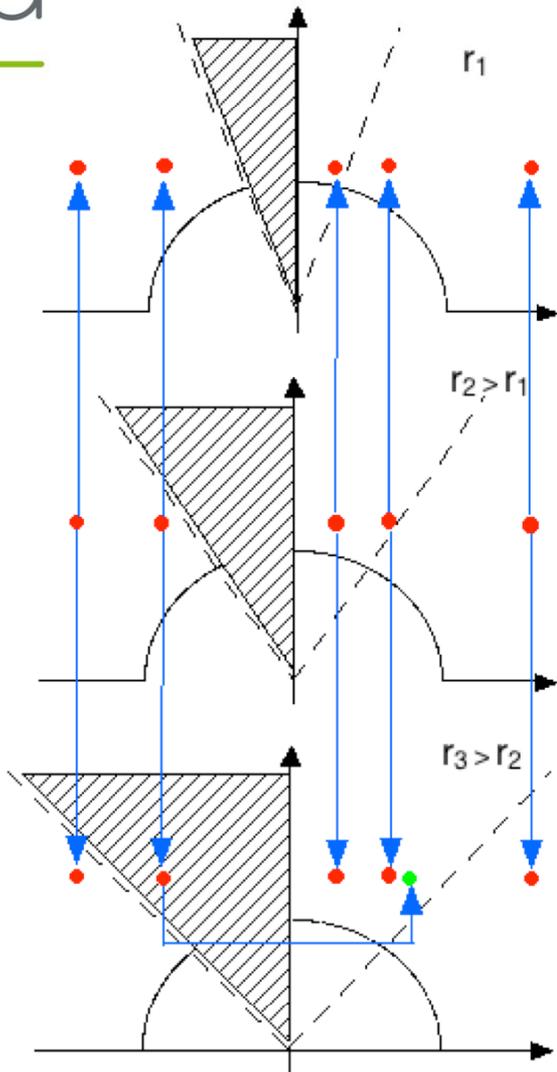
momentum space
↓

configuration space

$$\mathbf{S}^{(0)} = -\mathbb{D}^{(0)} \cdot \nabla f^{(0)} + \mathbf{F}^{(0)} f^{(0)}$$



- Non-uniform grids (f and fluxes)
- Fully implicit time scheme: *large time step* $\Delta t v_c \sim 10^4$
- Usual Chang & Cooper interpolation for p grid
- Linear interpolation for radial and pitch-angle grids
- Discrete cross-derivatives consistent with boundary conditions \rightarrow *high rf power densities*
- Generalized incomplete LU factorization technique for an arbitrary number of non-zero diagonals (highly sparse L and U matrices)
- written in MatLab
- Iterative inversion method (MatLab built-in or external linear solvers **MUMPS**, PETSc, SUPERLU)
- Distributed and parallel computing
- *Coupled with ray-/beam tracing and full-wave codes*



$10^6 \times 10^6$ entries



rf current drive simulations

- The RF wave is described by a set of rays
- The plasma is divided into incremental flux surfaces
- D_{ql} is calculated on each flux surface:
 - contribution of all rays
 - contribution of all passes of the same ray

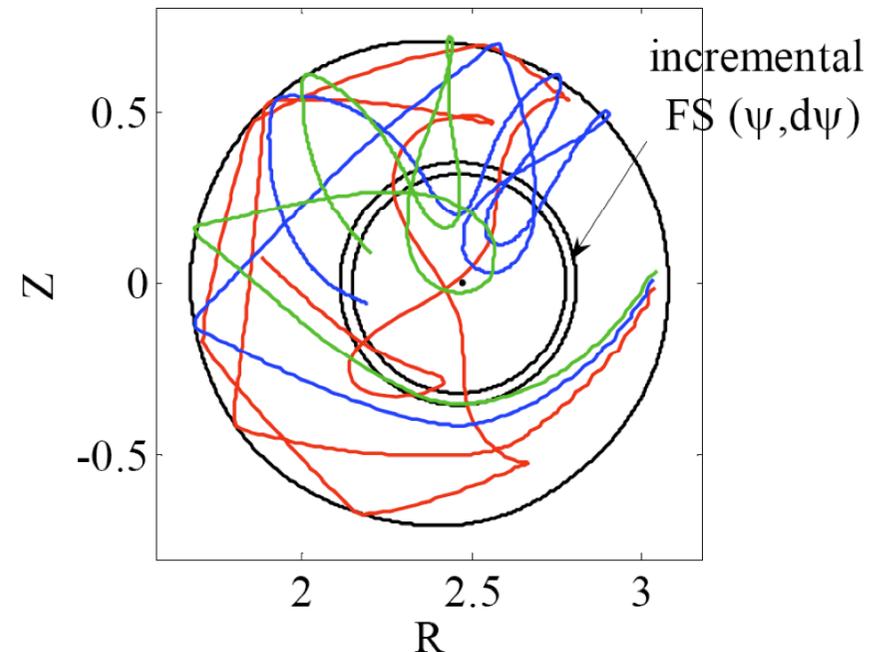
$$\mathbf{D}_{QL}(\psi, \vec{p}) = \sum_y \mathbf{D}_{QL}^y(P_y, \psi, \vec{p})$$

Ray power flow equation

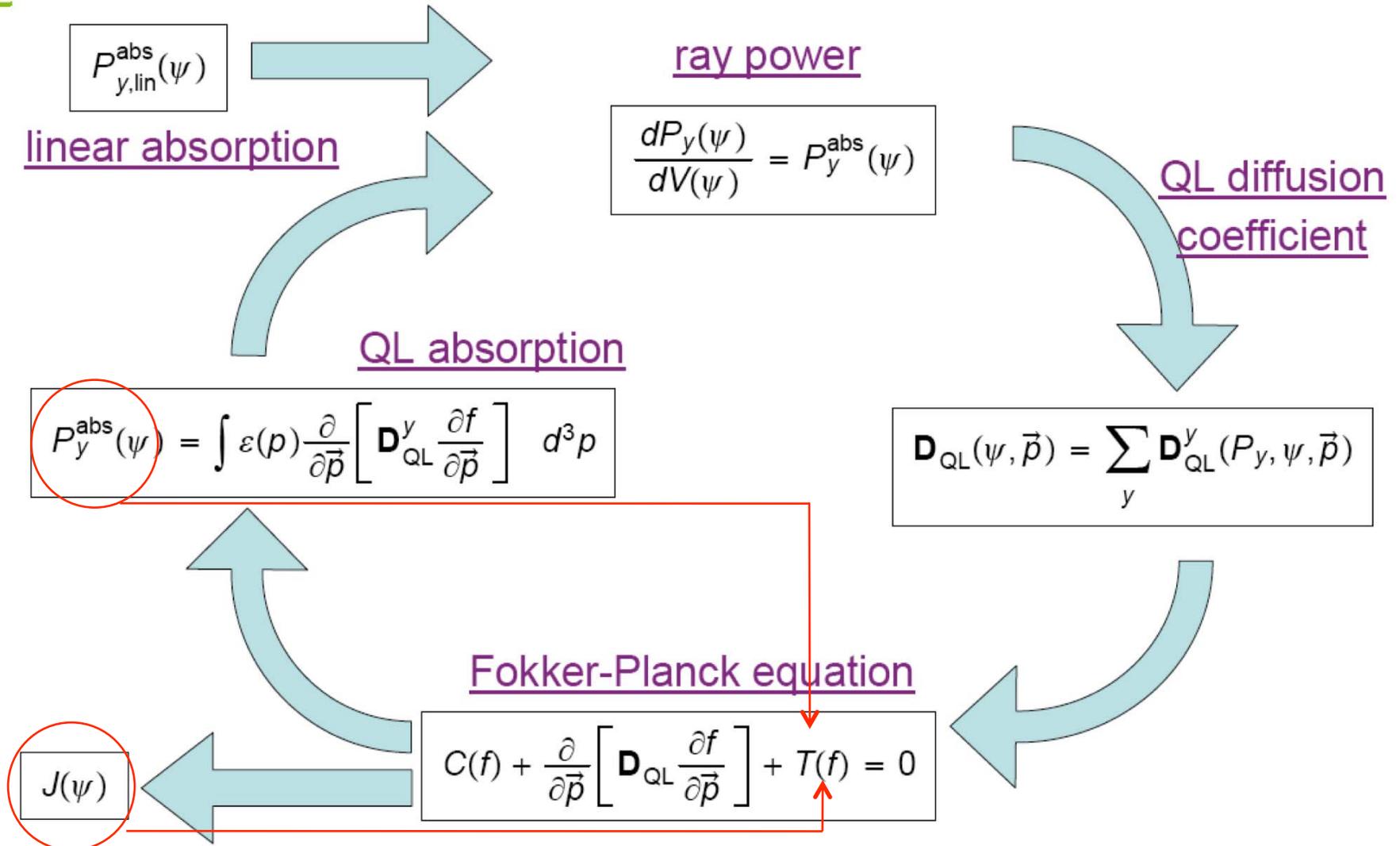
$$\frac{dP_y(\psi)}{dV(\psi)} = P_y^{\text{abs}}(\psi)$$

↓

Unknown



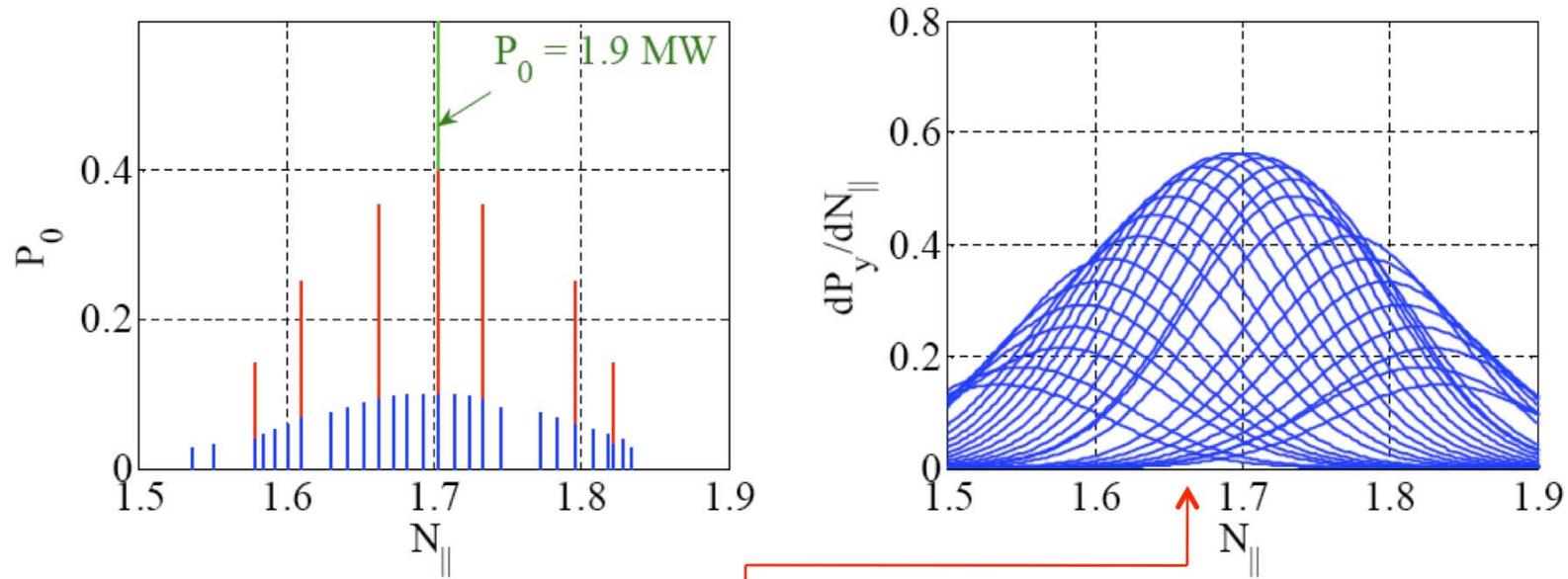
Self-consistent calculations





Few ray simulations

- The quasilinear convergence is carried out using the **power flow along each ray**, and not on the global power absorption profile.
- This technique requires to use a reduced number of **rays** (one per poloidal antenna row, one for the positive and negative lobes for the LH wave), *otherwise very weak or no convergence*.
- The small number of rays allows to reduce considerably the computational effort, and to interpret $\Delta n_{||}$ as a **real spectral width** (*consistency with Fourier theory*).

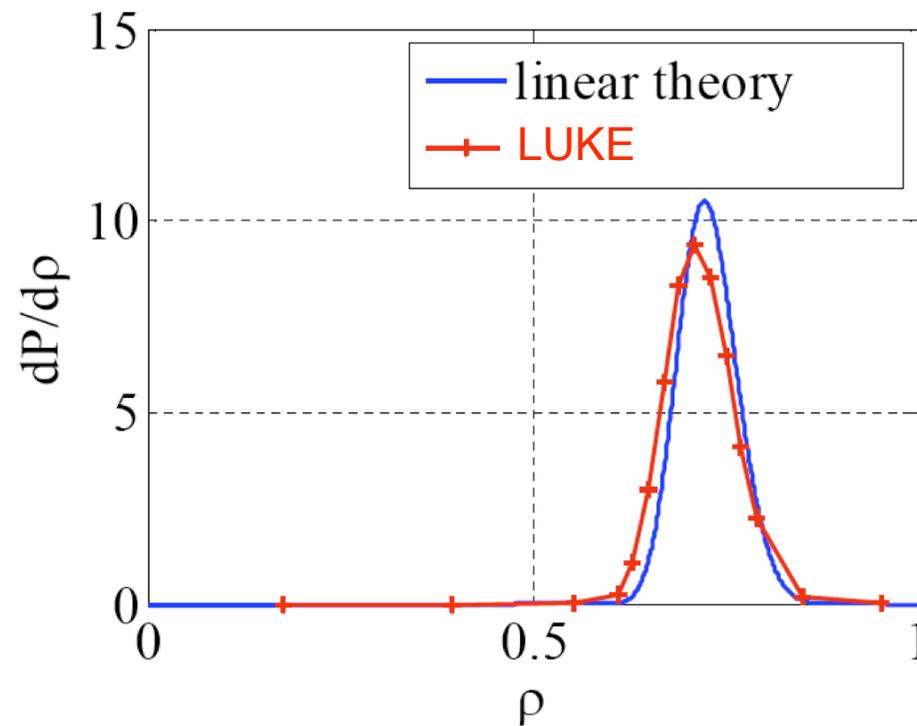


case	28 rays	7 rays	1 ray
ΔN_{\parallel} (each ray)	0.1	0.2	0.4

J. Decker, and Y. Peysson, in *33rd EPS Conference on Plasma Phys. and Contr. Fusion*, 2006.

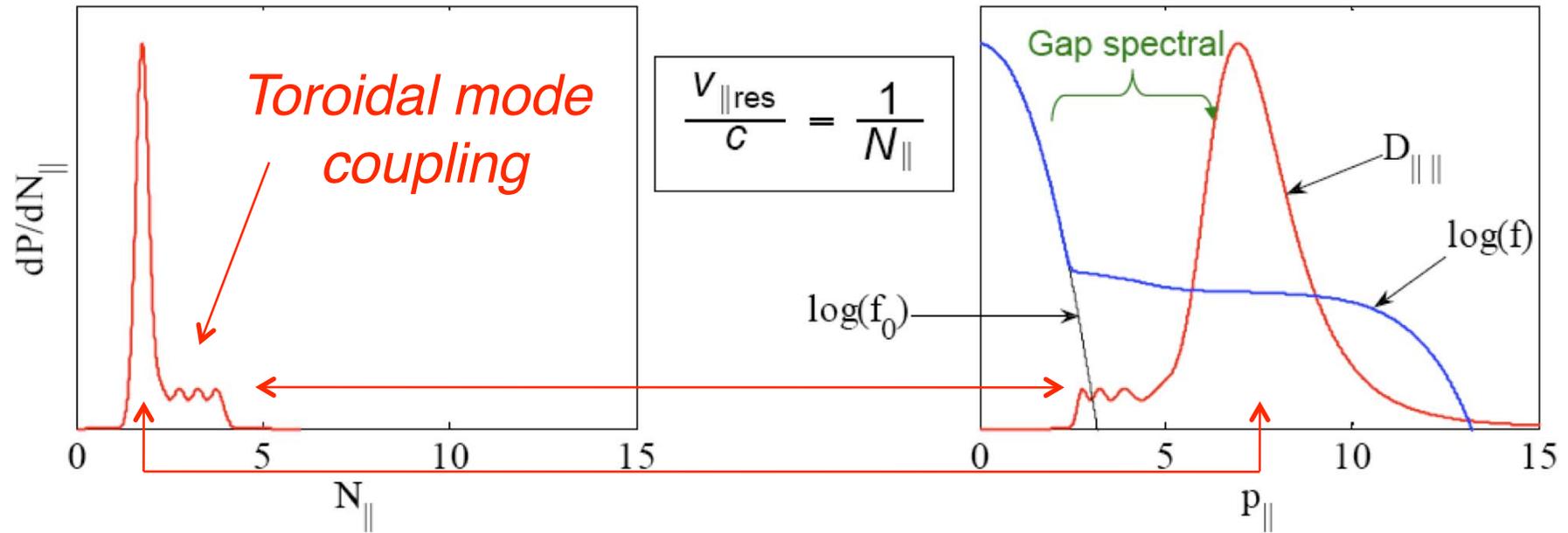
Linear limit

In the limit of low RF power level ($D \approx 0$), the result from the relativistic linear theory is well recovered (1A/1W)





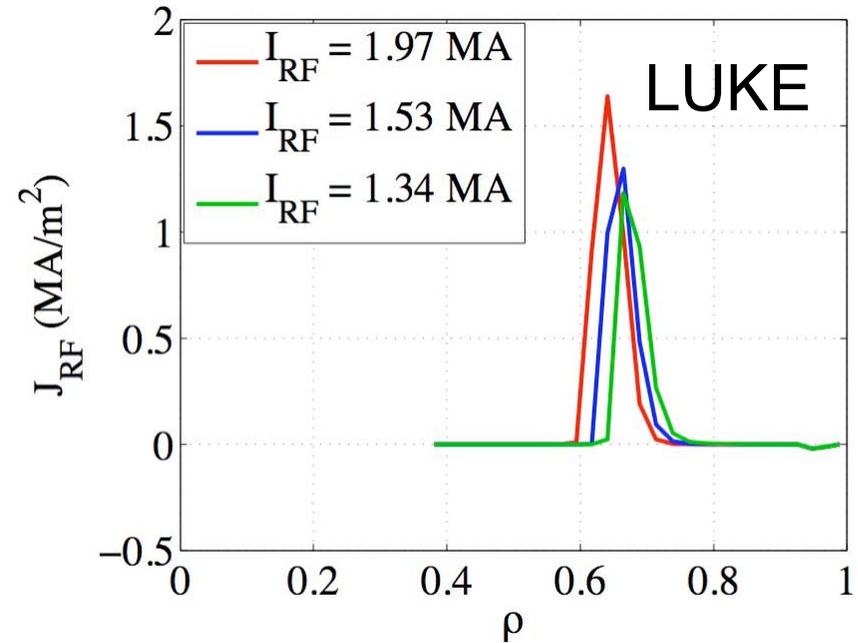
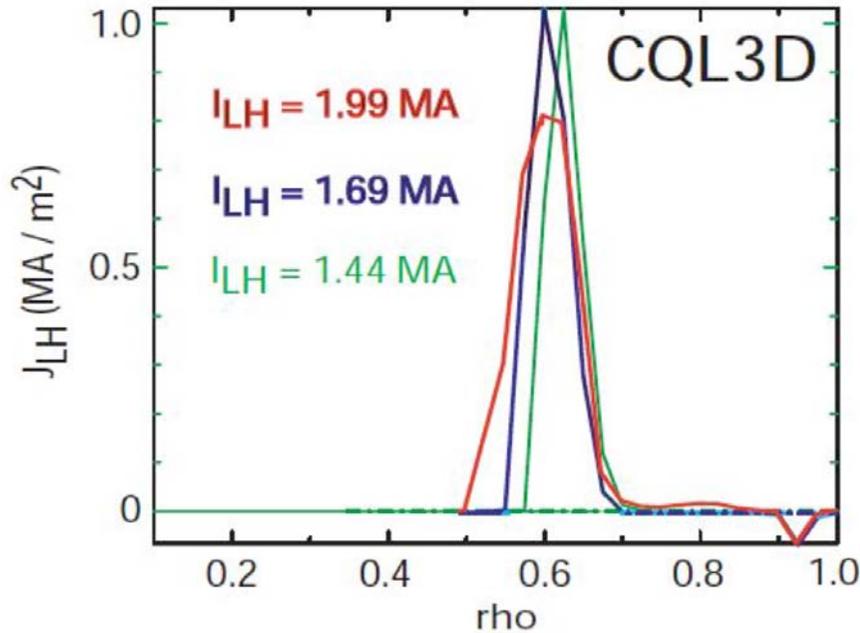
LH current drive simulations



The spectral gap is bridged by a small fraction of the LH power at high n_{\parallel} which pulls out a tail of fast electrons from the bulk which itself contributes to absorb the remaining part of the power at low n_{\parallel} .



LHCD in ITER (Scenario IV)



GENRAY - CQL3D: 80 rays

C3PO - LUKE: 3 rays

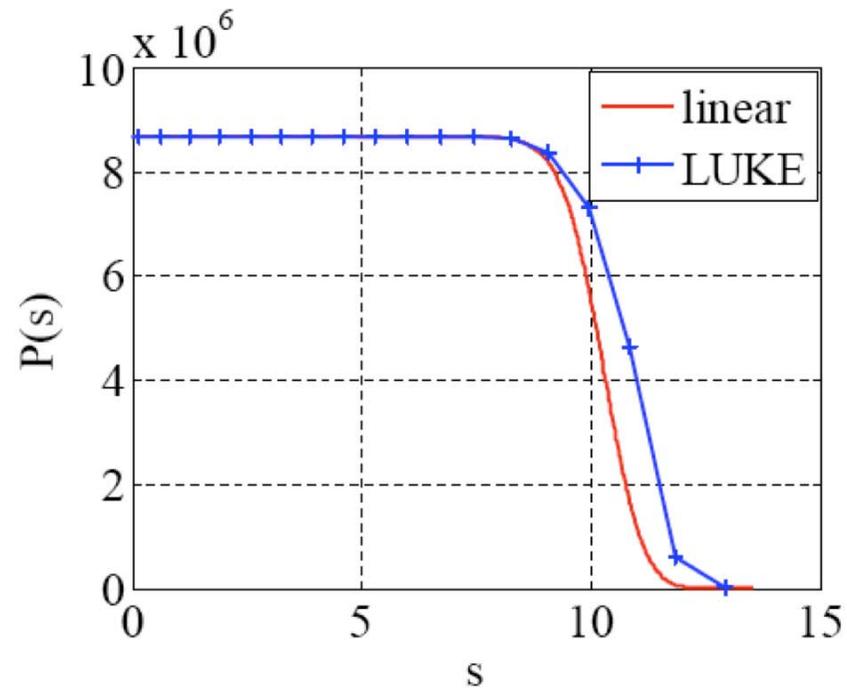
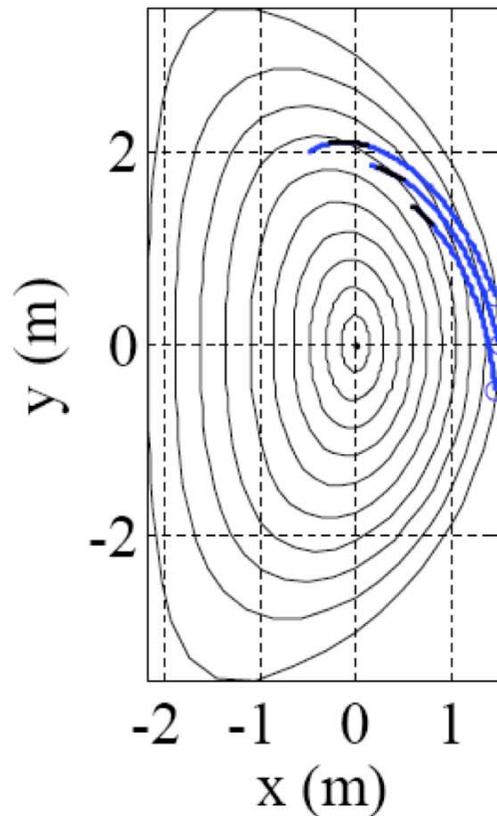
- $n_{||} = 1.9$
- $n_{||} = 2.0$
- $n_{||} = 2.1$



$$v_{||}/c \propto 1/n_{||}$$

P. T. Bonoli, et al., in *Proceedings of the 21st IAEA Conference*

LHCD in ITER (Scenario IV)

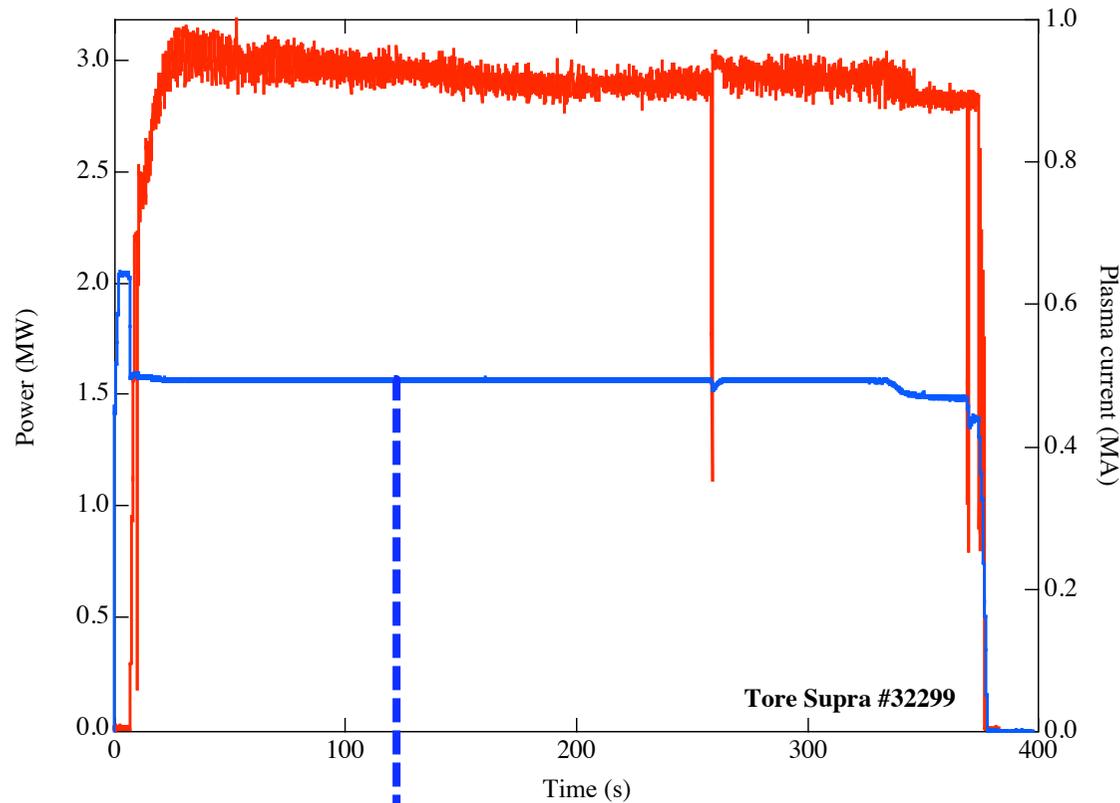


WKB approximation almost valid

Almost linear single pass absorption: results are independent of the number of rays !



LHCD in Tore Supra



- Full LHCD (6 min.)
- 2 antennas
- $n_{||0} = 1.7 \pm 0.2$
- $P_{lh} = 3 \text{ MW}$
- directivity: 0.6 & 0.7

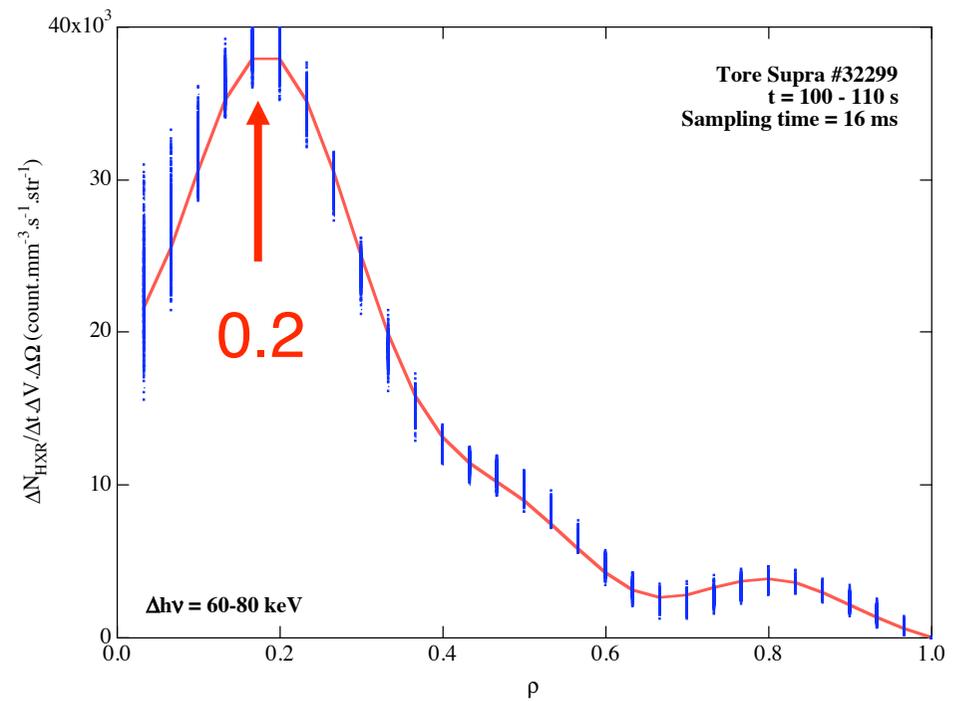
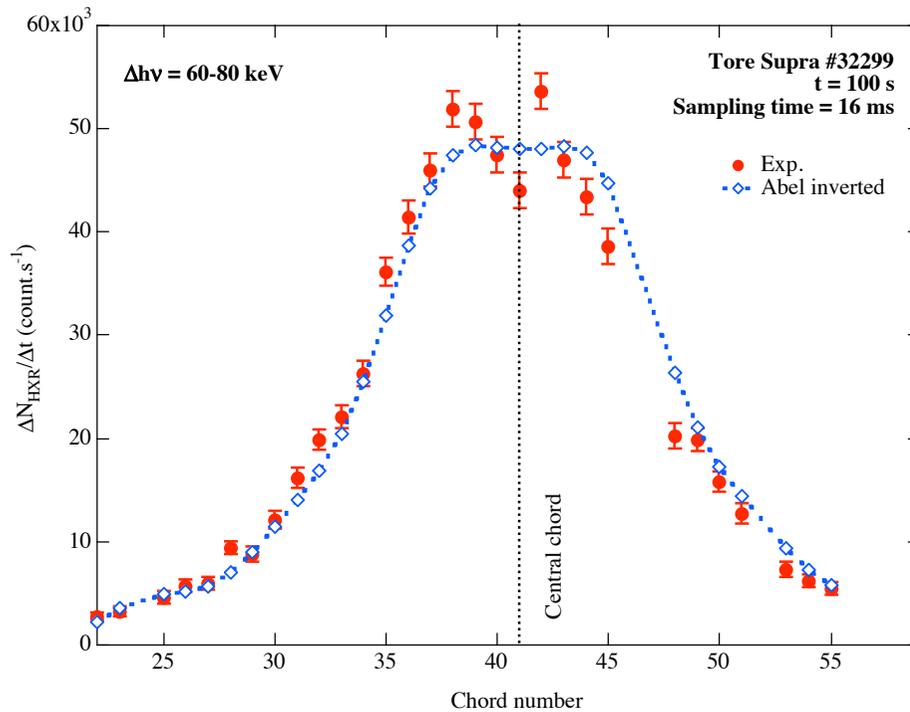
$$I_p \approx 500 \text{ kA}$$

D. van Houtte, et al., *Fusion Eng. and Design* 74, 651–658 (2005).

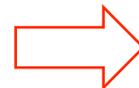


Fast electron bremsstrahlung (HXR)

$\Delta k = 60-80$ keV



Line-integrated profile



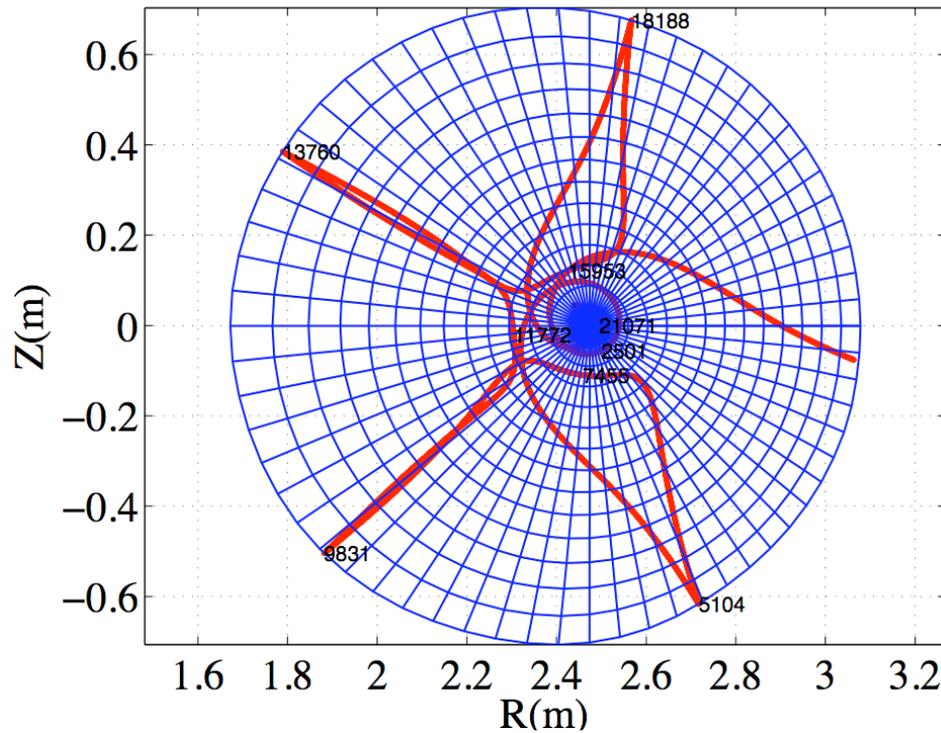
Abel inverted profile

Y. Peysson, et al., in vol. 4 of *Advances in Plasma Physics Research*, Nova Science Publisher, Inc. New York, 2003.

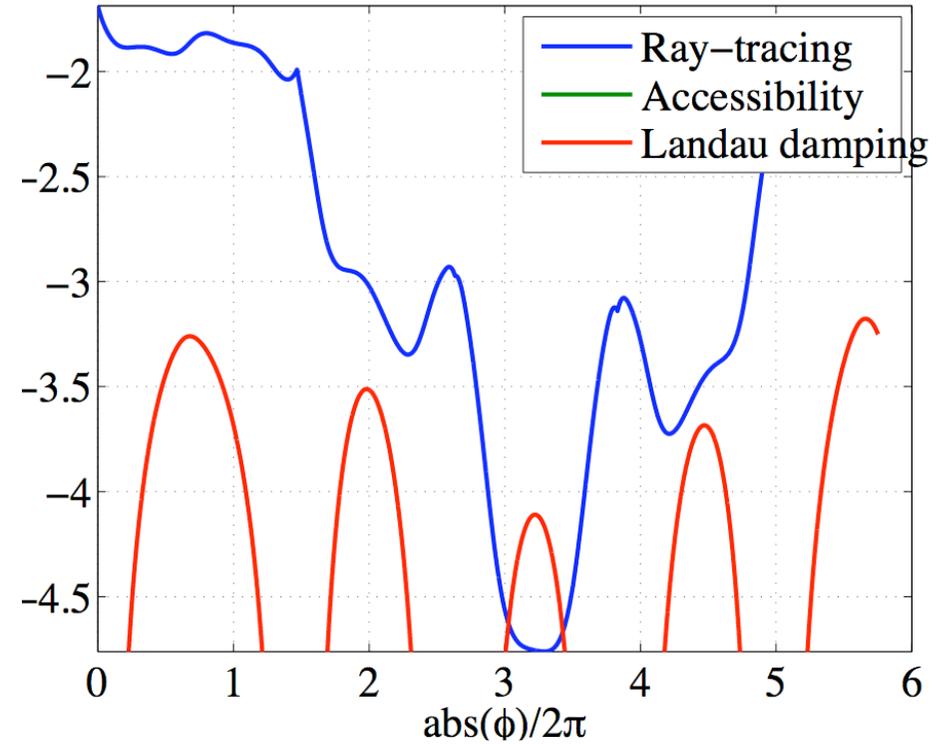
Theory of Fusion Plasmas, Varenna, Italy, 25-29 August 2008

Ray-tracing (C3PO)

Ray trajectory



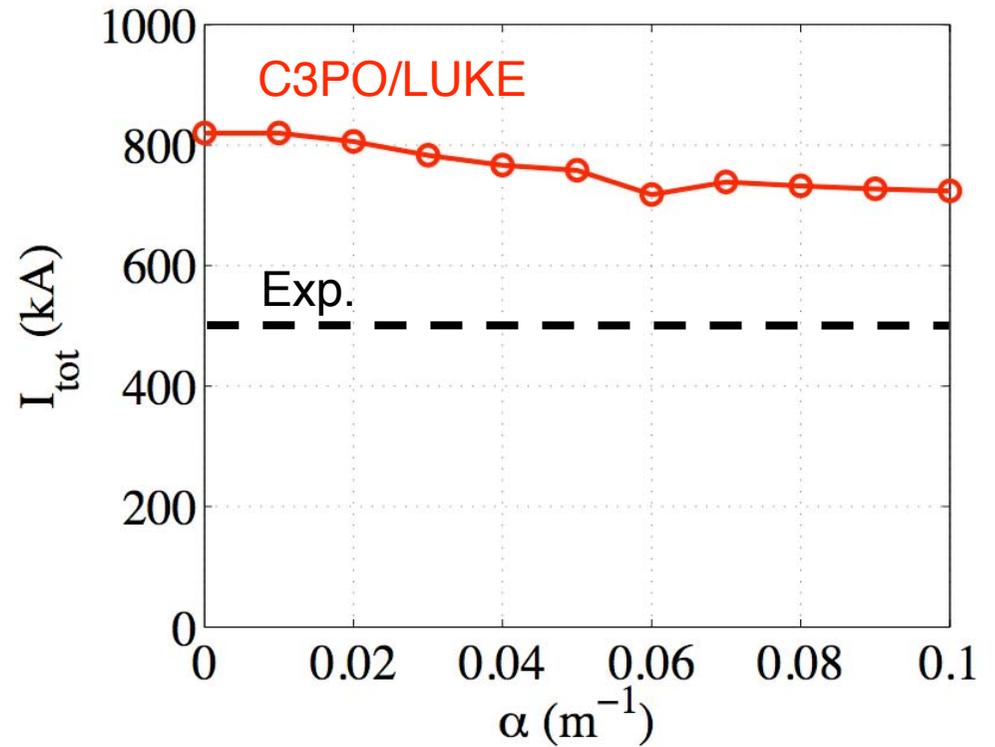
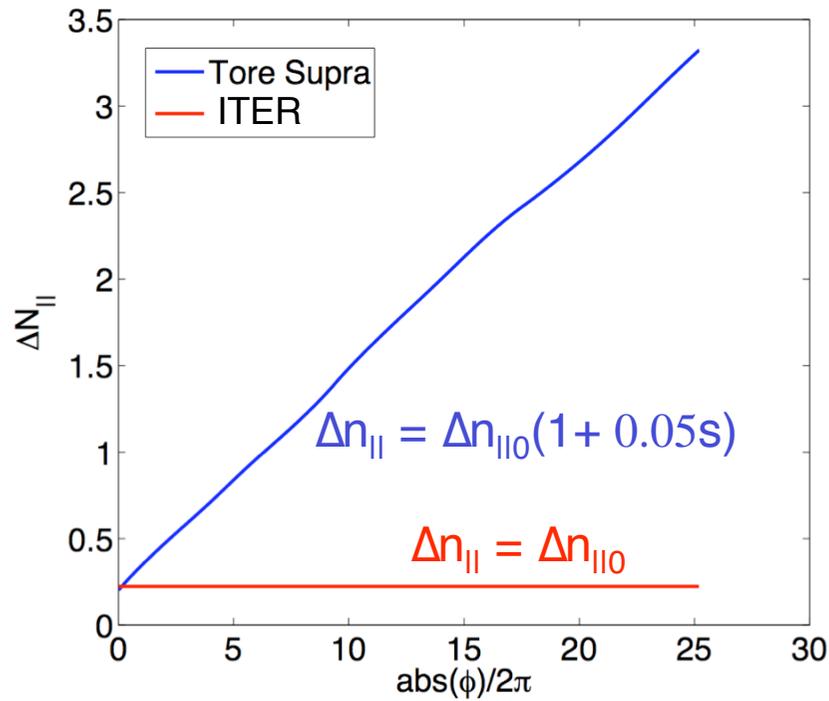
Parallel wave refractive index



WKB approximation not valid !

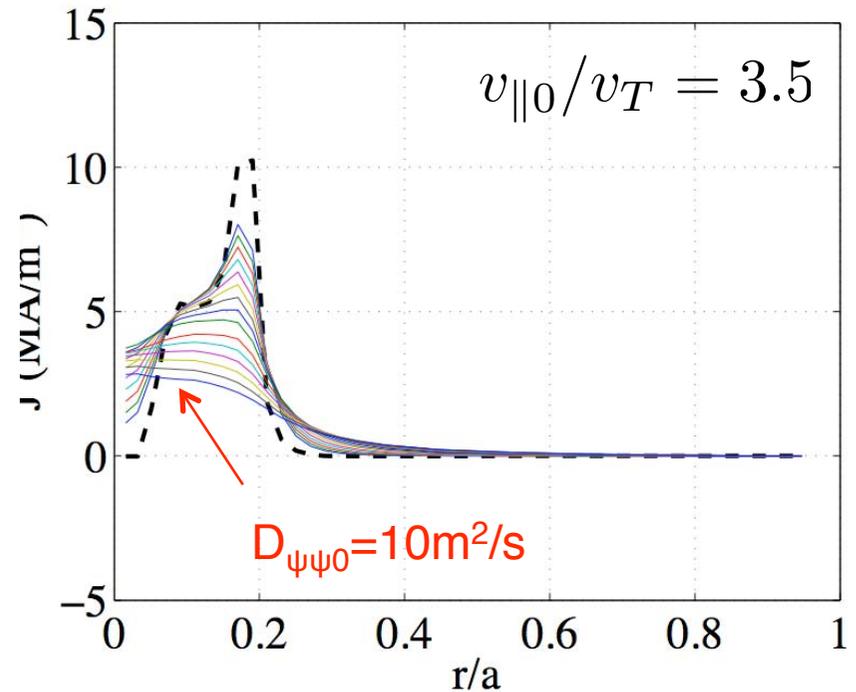
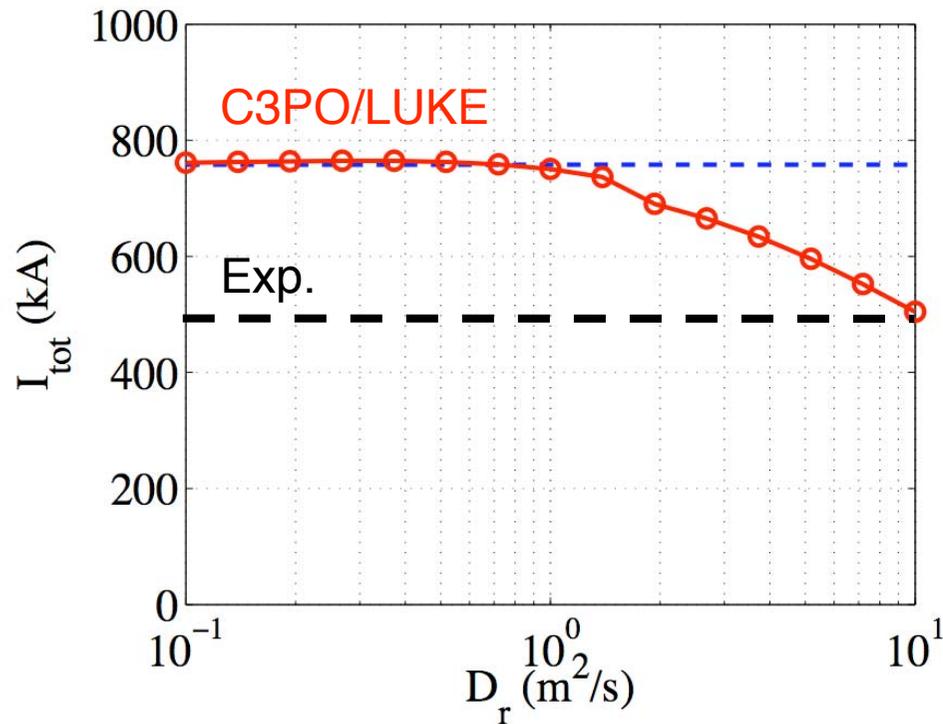
$$n_{||0} = -1.7$$

Spectral width model





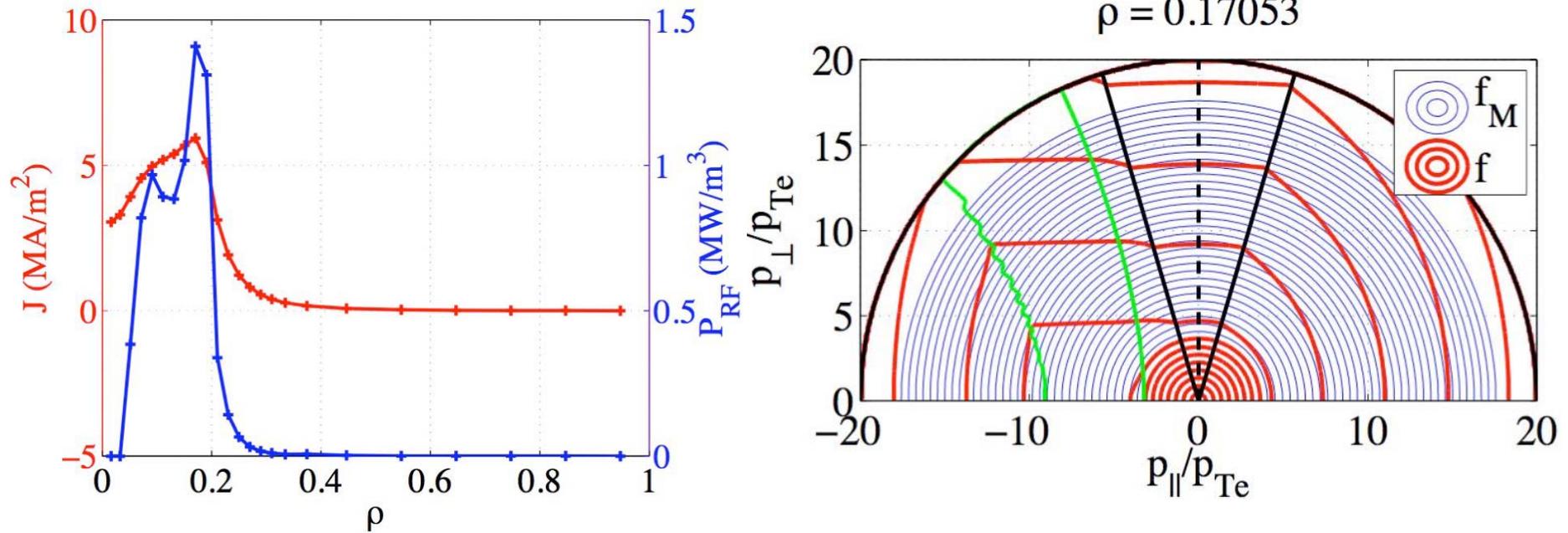
Fast electron transport model



$$D_{\psi\psi} = D_{\psi\psi 0} (|v_{\parallel}| - v_{\parallel 0}) H(|v_{\parallel}| - v_{\parallel 0})$$

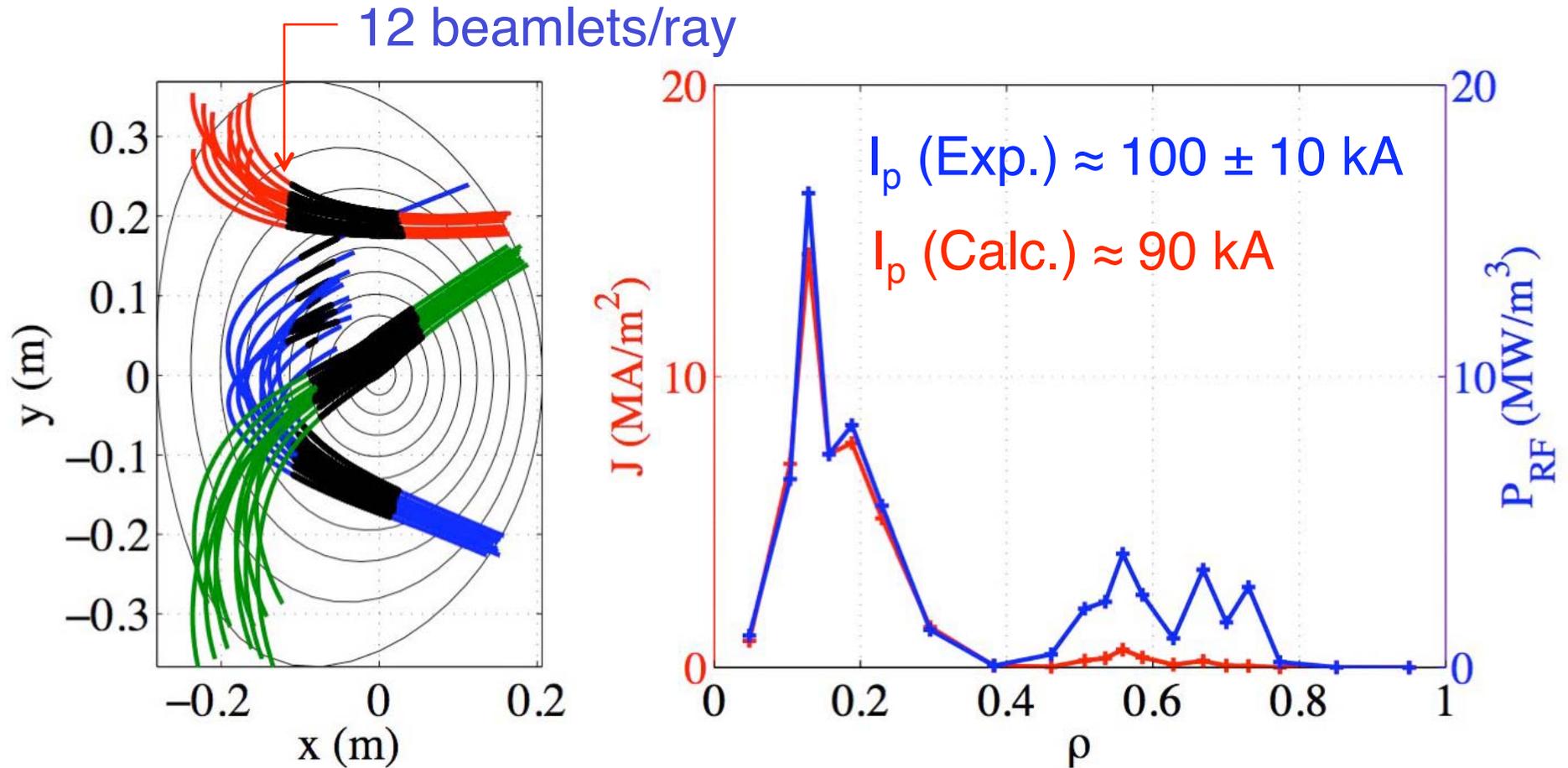


LHCD in Tore Supra



- predicted LH current too large by 40%
- large fast electron radial transport *inconsistent* with exp.
- weak effect of spectral broadening

ECCD in TCV



O. Sauter, et al., *Phys. Rev. Lett.* **84**, 3322–3325 (2000).



Conclusions & prospects

- A new current drive module for integrated tokamak modeling has been developed.
- Based on a strict implementation of the rf wave theory: *few ray approach*
- Fast electron radial dynamics consistent with quasilinear self-consistency: both P_{rf} and J_{rf} depend on the radial transport.
- Advanced algorithms for fast and robust calculations
- **LHCD**: linear theory holds for ITER and WKB approximation almost valid. Partial agreement with experiment on Tore Supra with toroidal spectral upshift and anomalous fast electron radial transport.
- **ECCD**: good agreement for full current drive in TCV without anomalous fast electron radial transport.



- **LHCD**: More physics must be incorporated in simulations within the WKB approximation for filling the spectral gap: *fluctuations*
- **LHCD**: C3PO ray-tracing → WKB full-wave for LH (ELECTRE-T)
- **LUKE**: bounce → orbit averaging + toroidal quasilinear operator (wave-induced radial transport, consistent description of the rf and bootstrap currents, ion physics)
- **LUKE**: electron back current calculations from non-Maxwellian ion distribution
- advanced tools: LUKE 3-D (ψ, p, ξ) → (θ, p, ξ), LUKE 4-D for edge current drive physics
- ...



References

J. Decker, and Y. Peysson, report EUR-CEA-FC-1736, Euratom-CEA (2004).
Y. Peysson, and J. Decker, Report EUR-CEA-FC-1739, Euratom-CEA (2008).

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LUKE: 2 articles submitted to Comp. Phys. Comm.