Toroidal momentum transport

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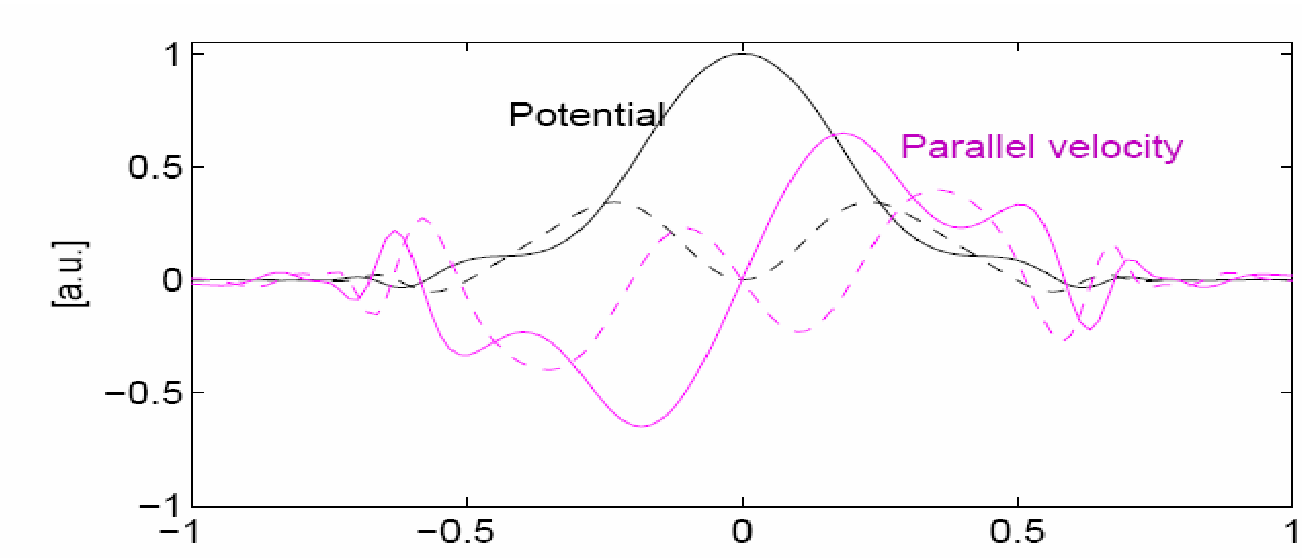
Symmetry

- The linear gyro-kinetic equation in circular geometry using the ballooning transform and without rotation or rotation gradient has as symmetry
- From this it follows that the momentum flux is zero in the ballooning transform



A.G. Peeters, PoP 2005

Momentum flux

- It then follows that the momentum flux has the form

Diffusion coefficient   angular rotation frequency   Pinch   Particle flux

$$\Gamma_\phi = -nm\chi_\phi R\nabla\Omega + nmV_\phi R\Omega + V_{ExB} + \rho_*(\nabla T, \nabla n) + m\Gamma R\Omega$$

GK study diffusion

Peeters PoP 2005

Waltz PoP 2007

Strintzi PoP 2008

Coriolis pinch effect

Peeters PRL 2007

TEP part of the pinch

Hahm PoP 2007

ExB shearing effect

Dominguez Phys Fluid 93

Garbet PoP 2002

Gurcan PoP 2007

Waltz PoP 2007

A.G. Peeters, PPCF 2006

Contents

- Basic mechanism of the Coriolis drift effect
- Fluid model
- Relation to the TEP formulation
- Influence of parallel dynamics
- Gyro-kinetic simulations: linear and nonlinear
- Self-consistent mode structure

Basic mechanism of the Coriolis drift effect

Transformation to the co-moving system

- Transforming the gyro-kinetic Lagrangian to the co-moving system of a rigid body toroidally rotating plasma gives the Hamiltonian

$$H = Ze < \phi > + \frac{1}{2} m {v_\parallel}^2 + \mu B - \frac{1}{2} m {\mathbf{u}_0}^2$$

Potential energy due to electro-static potential

Particle energy

Potential energy due to outward centrifugal force

- Where u_0 is the plasma velocity

$$\mathbf{u}_0 = \mathbf{\Omega} \times \mathbf{x} = R^2 \Omega \nabla \varphi, \quad \mathbf{\Omega} = (\nabla R \times \nabla \varphi) R \Omega,$$

Rotation vector

Toroidal angle

A. Brizard PoP 1995, A.G. Peeters submitted PoP 2008

Equation of motion

$$\frac{d\mathbf{X}}{dt} = v_{\parallel}\mathbf{b} + \frac{mv_{\parallel}^2}{ZeB_{\parallel}^*}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{Ze}\frac{\mathbf{b}\times\nabla B}{B_{\parallel}^*}$$

Parallel velocity Curvature drift Grad-B drift

$$+ \frac{\mathbf{b}\times\nabla\langle\phi\rangle}{B_{\parallel}^*} + 2\frac{mv_{\parallel}}{ZeB_{\parallel}^*}\mathbf{\Omega}_{\perp} - \frac{m\Omega^2 R}{ZeB_{\parallel}^*}\mathbf{b}\times\nabla R$$

ExB drift **Coriolis drift** **Centrifugal drift**

- New drift velocities: The Coriolis and Centrifugal drift

$$\mathbf{v} = \frac{\mathbf{F}\times\mathbf{B}}{ZeBB_{\parallel}^*}$$

Drift equation

$$\mathbf{F}_{co} = 2mv_{\parallel}\mathbf{b}\times\mathbf{\Omega}$$

Coriolis force

$$\mathbf{v}_{co} = 2\frac{mv_{\parallel}}{ZeB_{\parallel}^*}\mathbf{\Omega}_{\perp}$$

Coriolis drift

Equation of motion

$$\frac{d\mathbf{X}}{dt} = v_\parallel \mathbf{b} + \frac{m v_\parallel^2}{ZeB_\parallel^*}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{Ze}\frac{\mathbf{b}\times\nabla B}{B_\parallel^*}$$

Parallel velocity Curvature drift Grad-B drift

$$+ \frac{\mathbf{b}\times\nabla\langle\phi\rangle}{B_\parallel^*} + 2\frac{m v_\parallel}{ZeB_\parallel^*}\mathbf{\Omega}_\perp - \frac{m\Omega^2 R}{ZeB_\parallel^*}\mathbf{b}\times\nabla R$$

ExB drift Coriolis drift **Centrifugal drift**

- Here the Centrifugal drift will be neglected

For centrifugal force effects see A.G. Peeters submitted PoP 2008

Equation of motion

$$\frac{d\mathbf{X}}{dt} = v_\parallel \mathbf{b} + \frac{m v_\parallel^2}{ZeB_\parallel^*}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{Ze}\frac{\mathbf{b}\times\nabla B}{B_\parallel^*}$$

Parallel velocity Curvature drift Grad-B drift

$$+ \frac{\mathbf{b}\times\nabla\langle\phi\rangle}{B_\parallel^*} + 2\frac{m v_\parallel}{ZeB_\parallel^*}\mathbf{\Omega}_\perp$$

ExB drift Coriolis drift

- NOTE : The toroidal rotation of the plasma enters the equations only through the Coriolis drift. Any flux proportional to the rotation velocity is due to the Coriolis drift effect

A.G. Peeters Phys. Rev. Lett 2007

Equation of motion

$$\frac{d\mathbf{X}}{dt} = v_\parallel \mathbf{b} + \frac{m v_\parallel^2}{ZeB_\parallel^*}\mathbf{b}\times(\mathbf{b}\cdot\nabla)\mathbf{b} + \frac{\mu}{Ze}\frac{\mathbf{b}\times\nabla B}{B_\parallel^*}$$

Parallel velocity   Curvature drift   Grad-B drift

$$+ \frac{\mathbf{b}\times\nabla\langle\phi\rangle}{B_\parallel^*} + 2\frac{m v_\parallel}{ZeB_\parallel^*}\mathbf{\Omega}_\perp$$

ExB drift   Coriolis drift

- NOTE :Unlike the curvature and grad-B drift the Coriolis drift is linear in the parallel velocity. While the former couple density and temperature fluctuations, the latter couples density and temperature fluctuations to parallel velocity fluctuations → Flux of parallel momentum is generated

A.G. Peeters Phys. Rev. Lett 2007

Fluid model
Main purpose is to highlight the physics mechanisms

Fluid model

- From the equations of motion the gyro-kinetic equation can be directly obtained
- The solution of this equation gives the radial flux of parallel velocity (toroidal momentum)

Insight is gained from a low field side gyro-fluid model

- Consistently derived from the gyro-kinetic equation
- Quantities evaluated at the low field side : parallel dynamics is introduced through a single wave vector
- Assume a single mode with a specified poloidal wave vector
- Gyro-fluid : build the moments of density, parallel velocity and temperature (no dissipative closures)

The fluid model

- This generates the following set of equations

Perturbed density (normalized to the background)

Perturbed Temperature (normalized to the background)

Perturbed parallel velocity (normalized to the thermal velocity)

$$\omega n + 4k_{\|} w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

Potential normalized with e/T

$$\omega w + 2(n+T)k_{\|} + 4w + 2(n+T)u = \left[u' - 2u - 2k_{\|}\right]\phi$$

Normalized density gradient

$$\omega T + \frac{8}{3}k_{\|} w + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi$$

Normalized Temp. Grad.

Mach number $u \equiv R\Omega / v_{th}$ **its gradient** $u' \equiv R^2 \nabla\Omega / v_{th}$

Relation to the gyro-kinetic equation

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

$$\omega T + \frac{8}{3}k_\parallel w + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi$$

$$\frac{\partial f}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla f + \mathbf{v}_d \cdot \nabla f + \mathbf{v}_{\rm Coriolis}\cdot\nabla f =$$

$$-\mathbf{v}_E\cdot\nabla f - \frac{eF_M}{T}\left[v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_{\rm Coriolis}\right]\cdot\nabla\phi$$

- The time derivative yields the frequency (normalized to the drift frequency)

Relation to the gyro-kinetic equation

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

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$$-\mathbf{v}_E\cdot\nabla f - \frac{eF_M}{T}\left[v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_{\mathrm{Coriolis}}\right]\cdot\nabla\phi$$

- Parallel dynamics couples density / temperature with the parallel velocity but in general the averaged parallel wave vector is zero.

Relation to the gyro-kinetic equation

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

$$\omega T + \frac{8}{3}k_\parallel w + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi$$

$$\frac{\partial f}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla f + \mathbf{v}_d \cdot \nabla f + \mathbf{v}_{\text{Coriolis}}\cdot\nabla f =$$

$$-\mathbf{v}_E\cdot\nabla f - \frac{eF_M}{T}\left[v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_{\text{Coriolis}}\right]\cdot\nabla\phi$$

- The drift due to the magnetic field in-homogeneity leads to a coupling of the density and temperature equations, but not with the perturbed parallel velocity.

Relation to the gyro-kinetic equation

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

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$$\frac{\partial f}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla f + \mathbf{v}_d\cdot\nabla f + \mathbf{v}_{\text{Coriolis}}\cdot\nabla f =$$

$$-\mathbf{v}_E\cdot\nabla f - \frac{eF_M}{T}\left[v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_{\text{Coriolis}}\right]\cdot\nabla\phi$$

- All u terms (rotation) are due to the Coriolis drift effect
- NOTE: without Coriolis drift the perturbed velocity is proportional to u’ → Only a diffusive flux is obtained

Relation to the gyro-kinetic equation

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

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$$\frac{\partial f}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla f + \mathbf{v}_d \cdot \nabla f + \mathbf{v}_{\mathrm{Coriolis}} \cdot \nabla f =$$

$$-\mathbf{v}_E \cdot \nabla f - \frac{eF_M}{T}\left[v_\parallel \mathbf{b} + \mathbf{v}_d + \mathbf{v}_{\mathrm{Coriolis}}\right]\cdot\nabla\phi$$

- The ExB motion in the background gradient is responsible for all the gradient terms

Solution
Relation to turbulent equipartition and resonant particle effects

For simplicity I will neglect the parallel dynamics here

Turbulent equipartition

- It is well known from the work on particle transport that the terms on the right describe the ExB compression and convection

$$\omega n + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 4w + 2(n+T)u = [u' - 2u]\phi$$

$$\omega T + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi$$

- The equilibrium result can be read off by putting the term in the brackets to zero.
- This gives the ratio of pinch and diffusion

Equipartition of toroidal momentum

- Toroidal angular momentum $RnV_{\rm tor} \rightarrow R(nu + w)$

$$\omega n + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2 \right] \phi$$

$$\omega w + 4w + 2(n+T)u = [u' - 2u]\phi$$

- Multiply first Eq. with uR and second with R, and add

$$\omega[nuR + wR] + 4[nuR + wR] + 2TuR = R\left[u' + \frac{R}{L_N} u - 4u \right] \phi$$

- This leads to the TEP result

$$\Gamma_\phi \propto \chi_\phi u' + RV_\phi u$$

TEP $$\frac{RV_\phi}{\chi_\phi} = -4 + \frac{R}{L_N}$$

Equipartition of toroidal momentum

- Full solution assuming an adiabatic electron response

Full solution, adiabatic electrons $$\frac{RV_\phi}{\chi_\phi} = -4 - \frac{R}{L_N}$$

- But there are more terms in this expression

$$\omega[nuR+wR]+4[nuR+wR]+2TuR = R\left[u'+\frac{R}{L_N}u-4u\right]\phi$$

- This leads to the TEP result

$$\Gamma_\phi \propto \chi_\phi u' + RV_\phi u$$

TEP $$\frac{RV_\phi}{\chi_\phi} = -4 + \frac{R}{L_N}$$

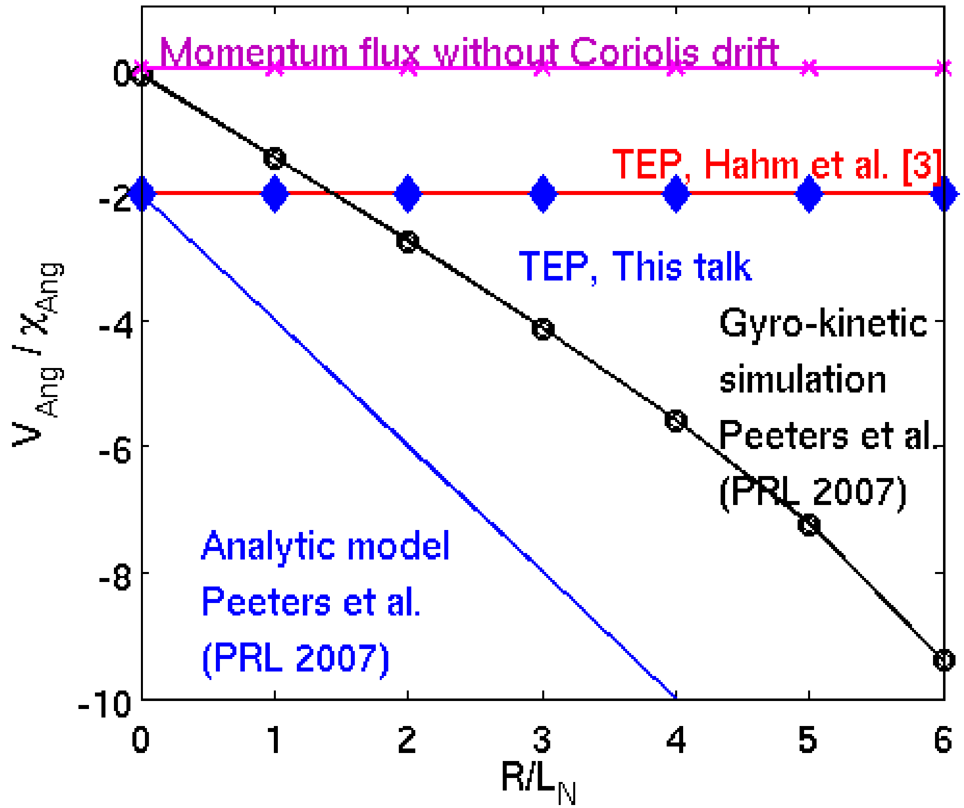
Comparison

$$\Gamma_{\mathrm{ang}} \propto \chi_{\mathrm{ang}} \frac{\mathrm{d}(Rnu)}{\mathrm{d}r} + V_{\mathrm{ang}} Rnu$$

$$\frac{\mathrm{d}(Rnu)}{\mathrm{d}r} = -n \left[u' + \frac{R}{L_N} u - 2u \right]$$

Write the flux for the angular toroidal rotation

- TEP result agrees with Hahm
- Additional terms important
- zero momentum flux w/o Coriolis drift
- Gyro-kinetics confirm the dependence on R/L_N



[3] Table 1, T.S. Hahm, Phys. of Plasmas (may 2008)

Influence of parallel dynamics
Linear gyro-kinetic calculations

Including the parallel dynamics yields

$$\Gamma_\phi = \chi_\phi u' + RV_\phi u + D_k \frac{k_{||} R}{k_z \rho}$$

- Where the transport coefficients are

$$\frac{RV_\phi}{\chi_\phi} = -4(1-u^2) + 12 k_{||N}^2 - \frac{R}{L_N}$$

$$\frac{D_k}{\chi_\phi} = -2(1-k_{||N}^2) + 6u^2 - \frac{1}{2}\frac{R}{L_N}$$

- Then a model for the parallel wave vector is needed. It is assumed here that the averaged value is set by the ExB shearing effect, while the average of the square is set by the connection length $k_{||}^2 \approx 1/q^2 R^2$

Including the parallel dynamics yields

$$\Gamma_\phi = \chi_\phi u' + RV_\phi u + D_k \frac{k_\parallel R}{k_z \rho}$$

- Where the transport coefficients are

$$\frac{RV_\phi}{\chi_\phi} = -4(1-u^2) + \frac{3}{q^2 (k_z \rho)^2} - \frac{R}{L_N}$$

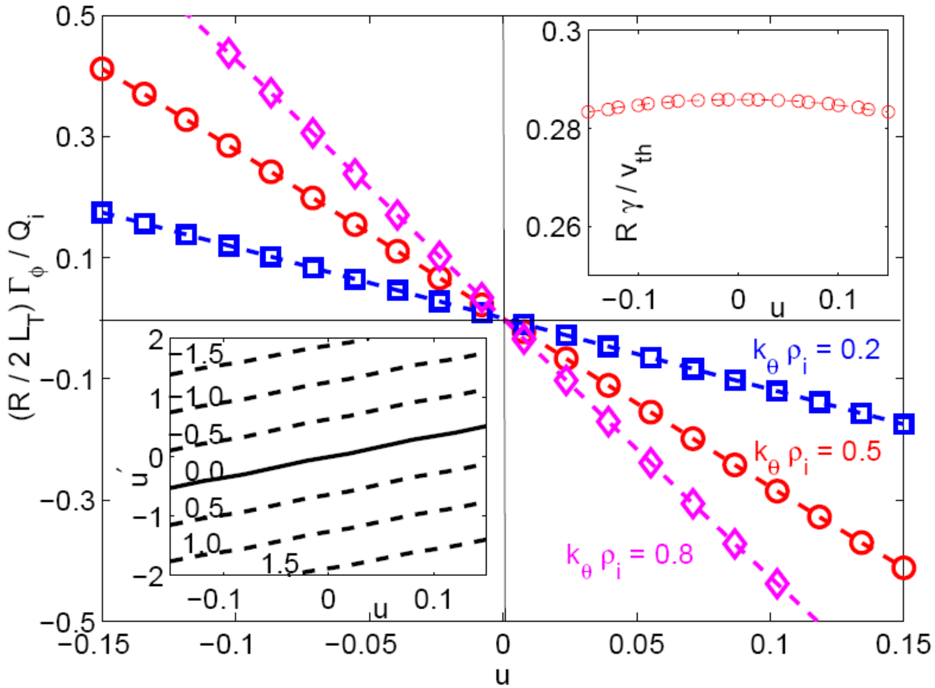
$$\frac{D_k}{\chi_\phi} = -2 + \frac{1}{2q^2 (k_z \rho)^2} + 6u^2 - \frac{1}{2}\frac{R}{L_N}$$

- The parallel dynamics as well as strong rotation weakens the inward pinch
- Similarly strong rotation and parallel dynamics weakens the ExB shear effect

Linear gyro-kinetic calculations

- Reproduce the pinch of toroidal momentum
- The pinch enhances the absolute value of core rotation
- Flux does depend on the poloidal wave number
- Small influence on growth rate

Toroidal momentum flux calculated by GKW for zero background gradient as a function of the toroidal velocity u (different values of the poloidal wave vector)



A.G. Peeters et al., Phys. Rev. Lett. **98**, 265003 (2007)

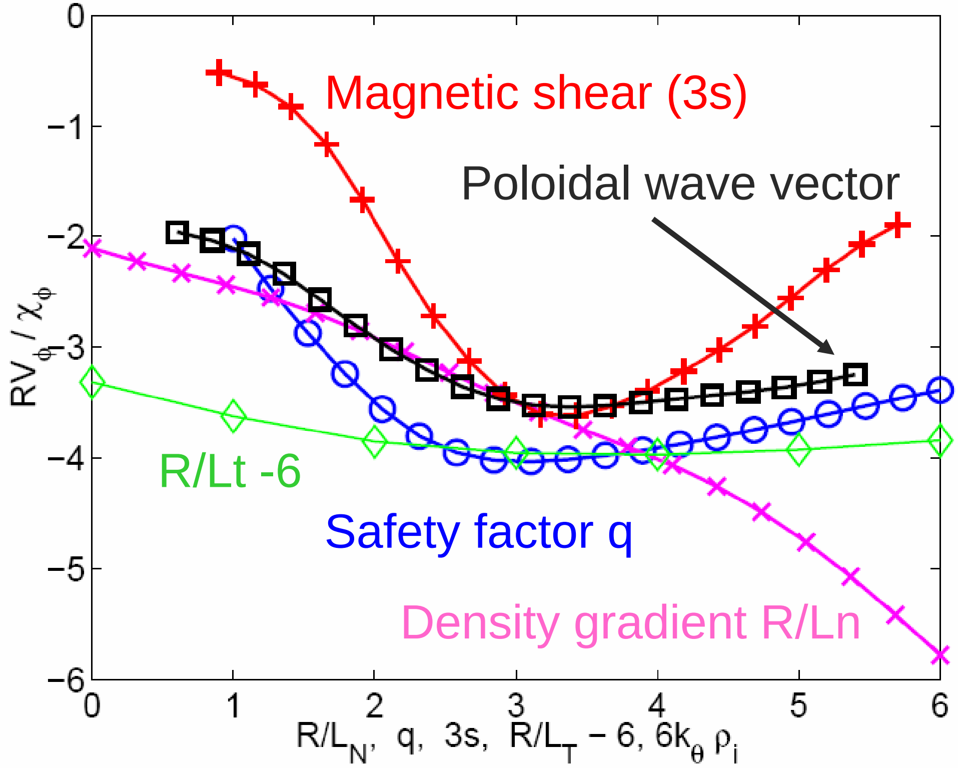
Pinch velocity normalised to diffusion coefficient

Fluid prediction

$$\frac{RV_\phi}{\chi_\phi} = -4(1-u^2) + \frac{3}{q^2(k_z\rho)^2} - \frac{R}{L_N}$$

Gyro-kinetic simulations in toroidal geometry (based on GKW)

Momentum pinch normalised with the diffusion coefficient as a function of various parameters : Density gradient R/Ln (3), safety factor q (2), magnetic shear s (1), temperature gradient R/Lt (9) and poloidal wave vector k



A.G. Peeters et al., Phys. Rev. Lett. **98**, 265003 (2007)

Self consistent mode structure

Fluid equations

$$\omega n + 4k_\parallel w + 2(n+T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 2(n+T)k_\parallel + 4w + 2(n+T)u = \left[u' - 2u - 2k_\parallel\right]\phi$$

$$\omega T + \frac{8}{3}k_\parallel w + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi$$

- One notices that the parallel dynamics and the Coriolis drift appear in a symmetric way

A.G. Peeters submitted PoP 2008

Fluid equations

$$\omega n + 2(n+T) + 4\hat{u}w = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 4w + 2(n+T)\hat{u} = [u' - 2\hat{u}]$$

$$\omega T + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}\hat{u}w = \left[\frac{R}{L_T} - \frac{4}{3}\right]$$

- Combined, with

$$\hat{u} = u + k_{||N}$$

- Important difference between the two: u is the toroidal rotation which is a background quantity. The parallel wave vector however is determined by the mode structure which can adjust
- Most unstable mode often has $\hat{u} = 0$ or $k_{||N} = -u.$

A.G. Peeters submitted PoP 2008

Fluid equations

$$\omega n + 2(n+T) + 4\hat{u}w = \left[\frac{R}{L_N} - 2\right]\phi$$

$$\omega w + 4w + 2(n+T)\hat{u} = [u' - 2\hat{u}]$$

$$\omega T + \frac{14}{3}T + \frac{4}{3}n + \frac{8}{3}\hat{u}w = \left[\frac{R}{L_T} - \frac{4}{3}\right]$$

- Combined, with

$$\hat{u} = u + k_{||N} \qquad \hat{u} = 0$$

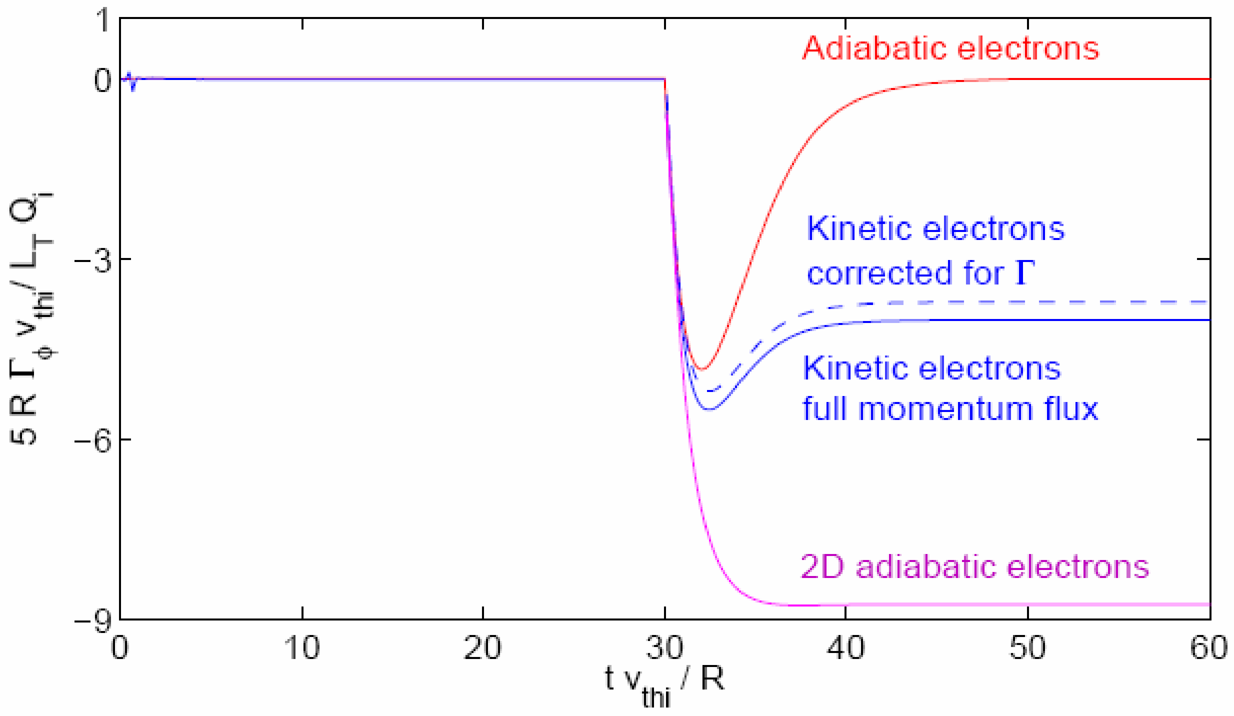
- This solution will make that the perturbed velocity is again proportional to the gradient of the toroidal rotation
- Transport is purely diffusive: NO PINCH

A.G. Peeters submitted PoP 2008

Gyro-kinetic simulations

- Fluid result partly related to the approximations. The transformation is not possible for both electrons and ions
- Result, however, carries over to the gyro-kinetic regime

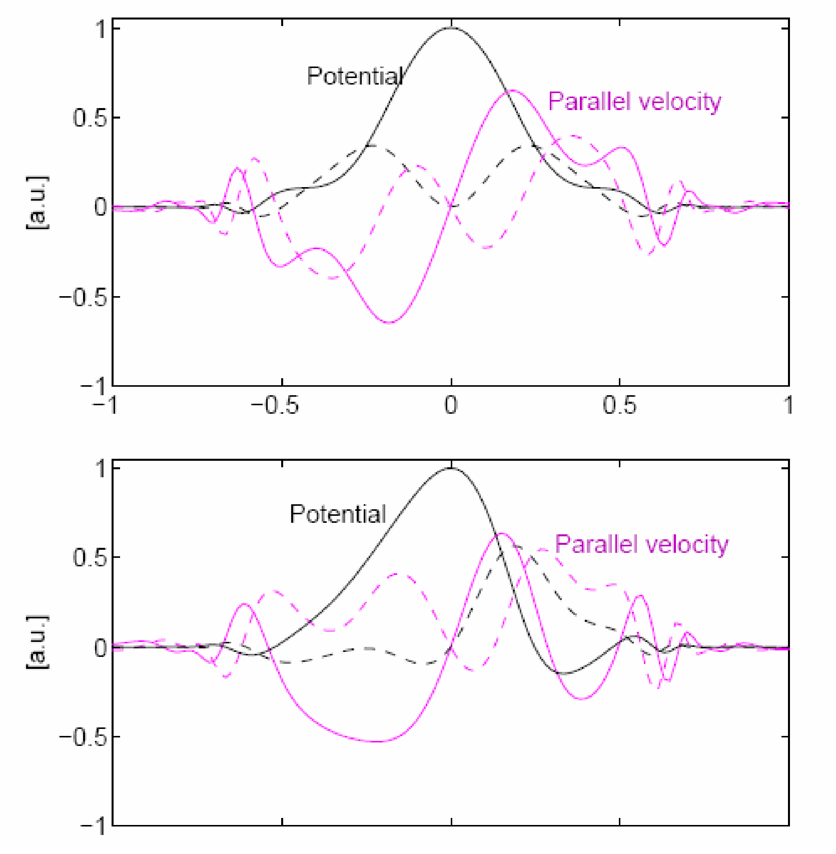
Normalized momentum flux as a function of time. Simulation is started with u = 0, after which a restart is made with u = 0.2.



Mode structure adjusts

- Mode structure for u = 0 is symmetric
- After the restart a pinch develops
- The mode structure adjusts to yield a zero pinch in the adiabatic electron case
- Parallel dynamics has a tendency to compensate the Coriolis drift effect
- Strong constraints on analytic modes. One can not average over a mode structure obtained without considering the toroidal rotation

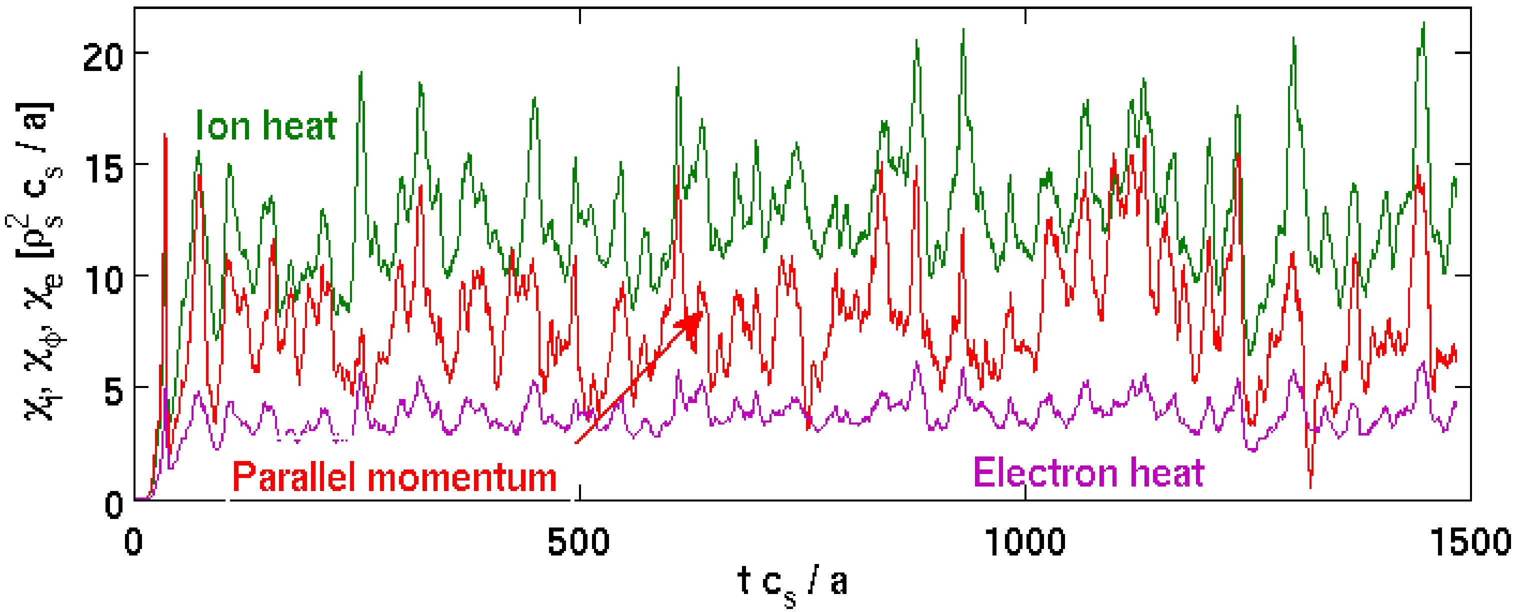
Top mode structure for u = 0.
Bottom mode structure for u = 0.2



Nonlinear simulations

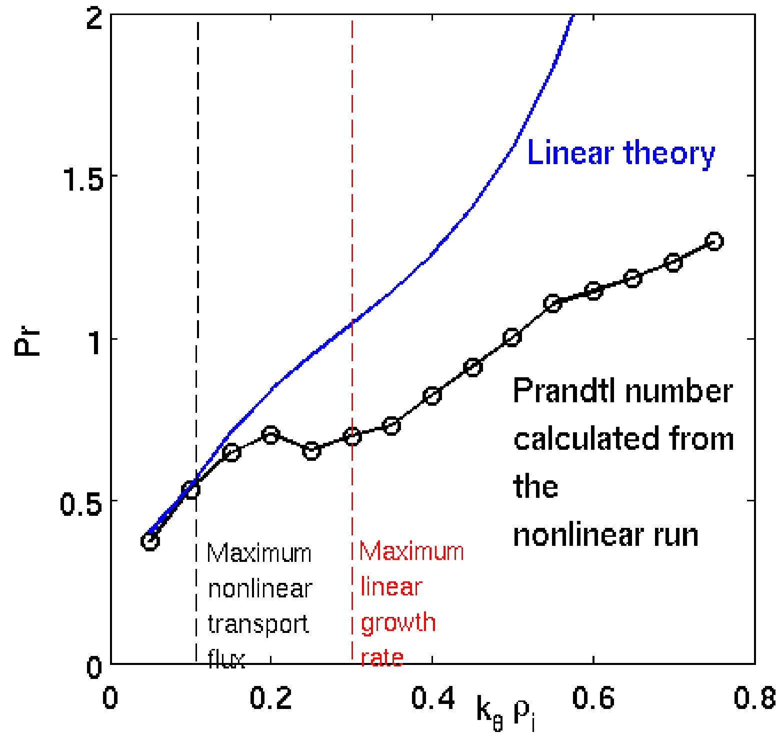
Nonlinear gyro-kinetic simulations

- Waltz standard case (q = 2, s = 1, ε = 0.16, R/L_T = 9, R/L_N = 3)
- Based on GKW
- Value is obtained (in gyro-Bohm units) χ_i = 11.3 ± 0.9 (GYRO 11) χ_ϕ = 8.2 ± 0.7 (GYRO ~8.8)
- Prandtl number Pr = 0.73 ± 0.07 (GYRO ~0.8)



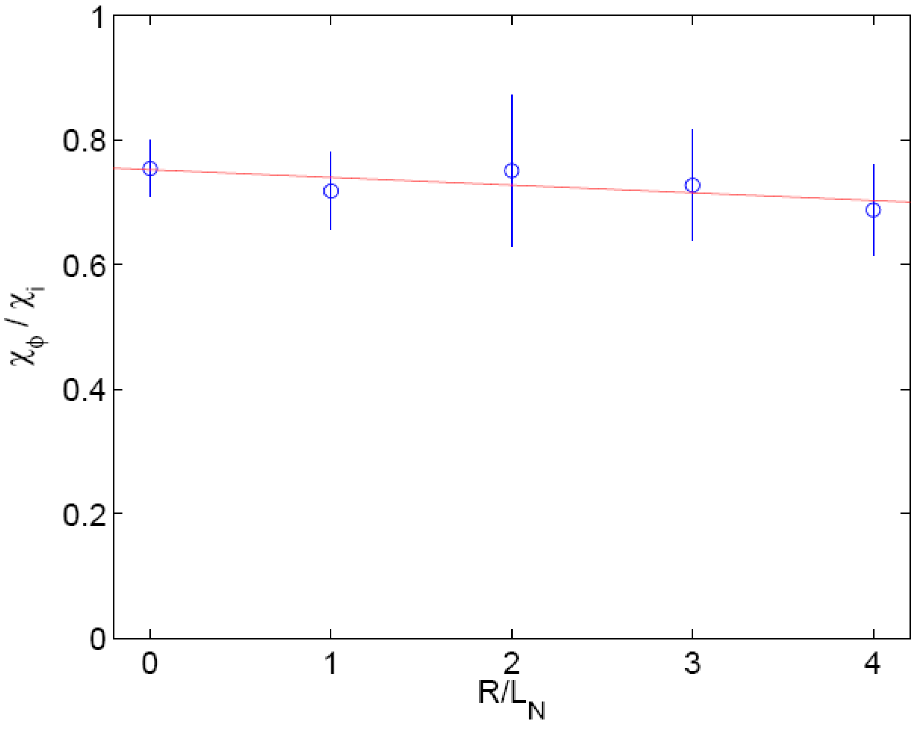
Diagonal part

- In linear theory the Prandtl number strongly increases with the poloidal wave vector
- Same trend in nonlinear theory, but far less strong
- Reasonable difference in the Prandtl number is obtained at poloidal wave number for which the growth rate is maximum 1 → 0.7
- Reasonable agreement for the value for which the nonlinear transport is maximum



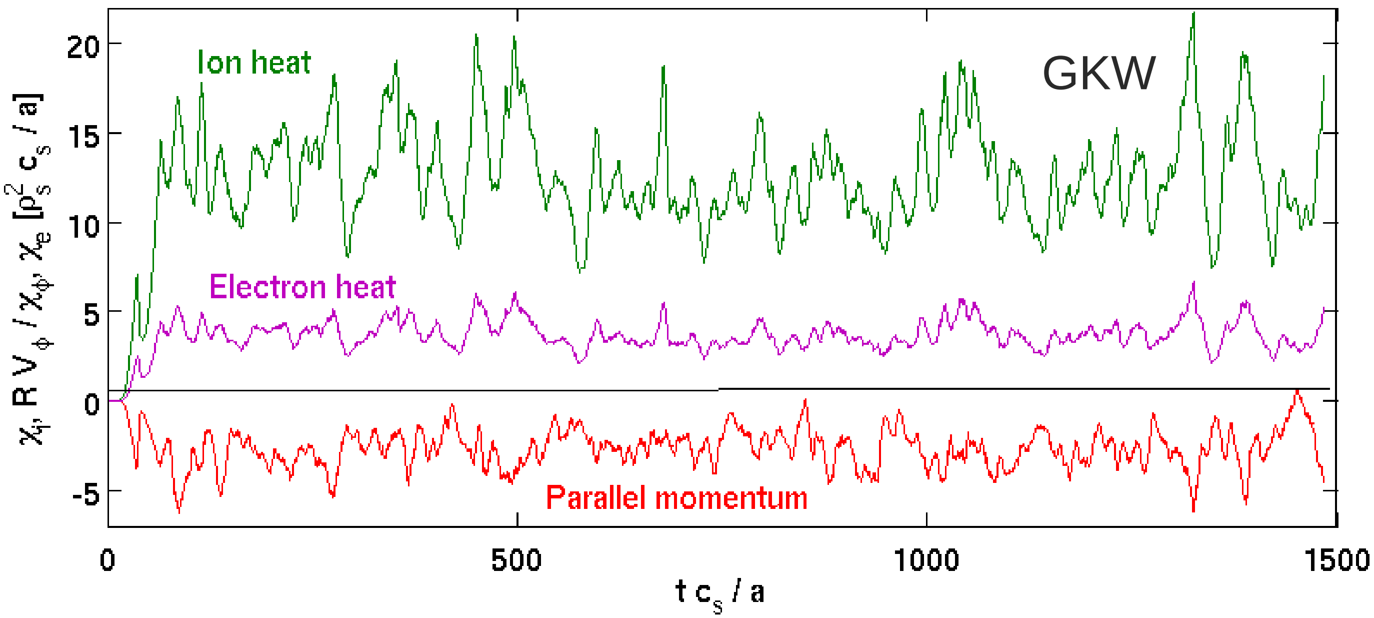
Diagonal part

- Prandtl number shows little parameter variation
- Is larger only close to the threshold of the ITG



Nonlinear simulations

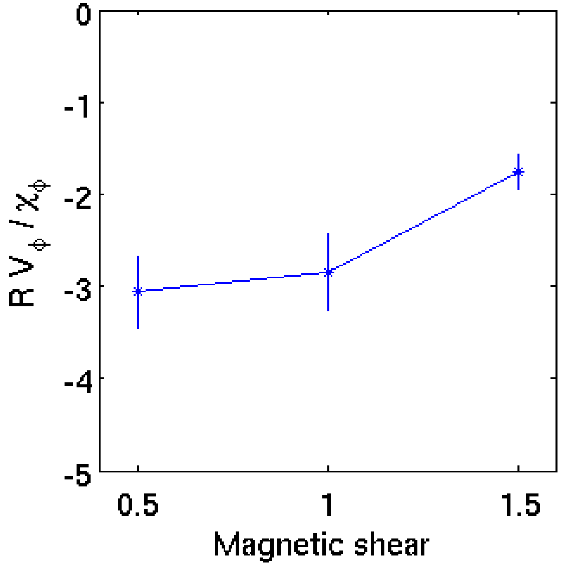
- First non-linear simulations of this effect obtained by R.E. Waltz using the GYRO code [6]
- Here confirmed by GKW, in red RV_ϕ / χ_ϕ (diffusion separate run)
- Waltz standard case (q = 2, s = 1, ε = 0.16, R/L_T = 9, R/L_N = 3)

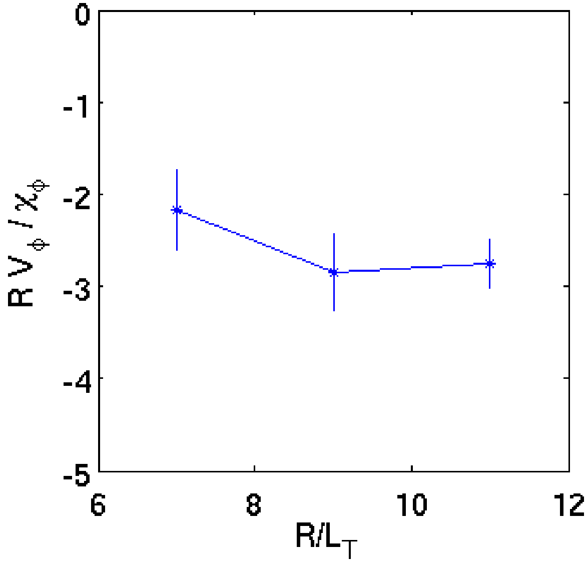


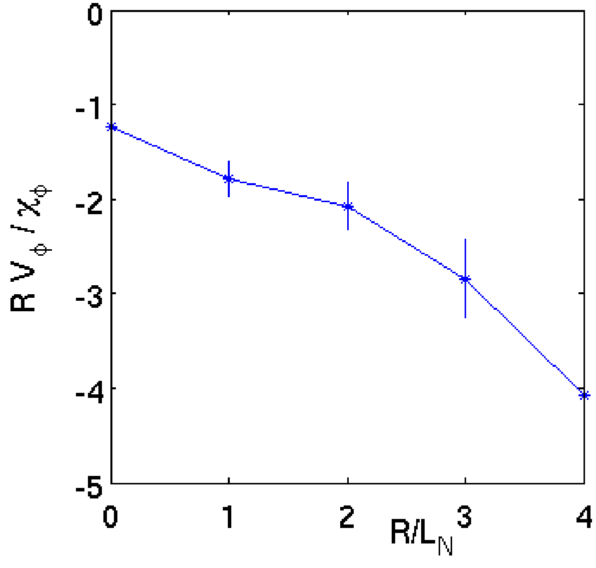
[6] R.E. Waltz, et al., Phys. Plasmas (2007)

Nonlinear simulations

- Magnitude similar to the linear theory (though somewhat smaller)
- Dependence on plasma parameters follows roughly (but not exactly) linear theory
- Dependence on temperature gradient is stronger, but mainly through a change in the diagonal term
- Main dependence appears to be the density gradient







Conclusions

- Consistent formulation of the gyro-kinetic equations in the co-moving frame of a toroidally rotating plasma reveals the existence of a Coriolis drift
- This drift depends linearly on the velocity and leads to the generation of parallel velocity fluctuations and eventually to a pinch of parallel momentum
- A fluid model has been derived in order to highlight the main physics mechanisms, and has been shown to include the effect of Turbulent equipartition.
- Gyro-kinetic simulations provide the full solution and reproduce the pinch effect
- Parallel dynamics is important. It can compensate the Coriolis drift and weaken the pinch.
- Nonlinear simulations in turn confirm the linear theory

That’s all folks