MHD in non-inductive tokamak plasmas: simulations and comparison to experiments on Tore Supra

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Aknowlegments to

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Motivations

Non-inductive tokamak scenario: continuous energy production

- Unlimited plasma discharge
- Plasma current driven by
  - Self-generated bootstrap current (~15% on Tore Supra, more than 50% needed for ITER)
  - Auxilliary systems (~85% provided by Lower Hybrid (**LH**) waves on Tore Supra)

Non-inductive discharges tend to have hollow current profiles

- On Tore Supra: hollow current profile driven by LH deposition
- On ITER: hollow current profile driven by off-axis bootstrap

Magneto-Hydro-Dynamic stability of hollow current profiles

- At moderate $\beta=\mu_0 p/B^2$: (Double-)Tearing instabilities
- At high β: pressure driven modes (Neoclassical Tearing, Interchange, Ballooning, kink)

Motivations

- **Impact of MHD instabilities on Tore Supra non-inductive pulses**
  - **Loss of wave-driven fast electrons**
    - Less current drive
    - Risks for plasma facing components (water leaks…)
  - **Degradation of energy confinement**

- This has motivated **studies aiming at**
  - Understanding **what determines MHD-stable domain**
  - Understanding **how MHD modes impact energy confinement**
  - *Understanding **how Tore Supra case extrapolates to higher-β***

- **Tools**
  - **MHD diagnostics** (δT_e, δB)
  - **Numerical codes for linear and non-linear studies**

Outline

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- ❑ **Numerical tools**
- ❑ **Impact of the Double-Tearing Mode**
- ❑ **Fully developed non-linear regimes**
- ❑ **Issue on n=1 mode stability**
- ❑ **Conclusion and perspectives**

Outline

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Numerical tools

❑ **Tool for non-linear MHD**: XTOR [Lütjens '08] (full MHD, toroidal)

Toroidal rotation

$$\rho\,(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = \mathbf{J}\times\mathbf{B} - \nabla p + \boxed{\mu_\varphi\,(\mathbf{V}_{\mathbf{src}} - V_\varphi)} + \nu\nabla^2\mathbf{v}$$

$$\partial_t p + \mathbf{v}\cdot\nabla p = -\Gamma p\nabla\cdot\mathbf{v} + \nabla\cdot\chi_\perp\nabla p + \cdots$$

$$\cdots + \boxed{\mathbf{B}\cdot\nabla[\chi_{//}(\mathbf{B}\cdot\nabla p)/B^2]} + H$$

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E}$$

Parallel transport

$$\mathbf{E} + \mathbf{V}\times\mathbf{B} = \eta\,(\mathbf{J} - \mathbf{J}_{\mathbf{NI}})$$

$$\boxed{\mathbf{J}_{\mathbf{NI}} = \mathbf{J}_{\mathbf{cd}} + \mathbf{J}_{\mathbf{boot}}}$$

Non-inductive sources

$$\mathbf{J}_{\mathbf{cd}} = (\mathbf{J} - \mathbf{J}_{\mathbf{bs}})_{t=0}$$

❑ **Standard MHD model adapted for Tore Supra experiments**

- **Non-inductive current sources**
- **Anisotropic Heat transport**
- **Toroidal rotation source (as generated by ripple)**

Numerical tools

❑ Re-Scaling of dimensionless coefficients

- Lundquist number ($S=\tau_R/\tau_A$)~ 10^9 in experiments
- Hardly tractable in numerical simulations for resistive MHD modes
  - ➢ Fine radial discretisation ($\Delta r \sim S^{-2/5}$)
  - ➢ Long time evolution ($T \sim S^{3/5}$)

- Using lower S implies rescaling other parameters
  - ➢ **Option MHD / pressure dynamics**
    - ✓ Correct pressure dynamics during mode growth
    - ✓ $\tau_{MHD}/\tau_E=(\tau_{MHD}/\tau_E)^{exp}$: $S^{3/5}\chi_\perp=(S^{3/5}\chi_\perp)^{exp}$
  - ➢ **Option current / pressure dynamics**
    - ✓ Correct pressure dynamics during equilibrium evolution
    - ✓ $\tau_R/\tau_E=(\tau_R/\tau_E)^{exp}$: $S\chi_\perp=(S\chi_\perp)^{exp}$
- Typically, we use $S=5\times10^6$-5×10^7

❑ Other parameters

- Anisotropic diffusivity: $\chi_{//} / \chi_\perp = 10^8$
- Prandtl number **:** $S\nu=1$

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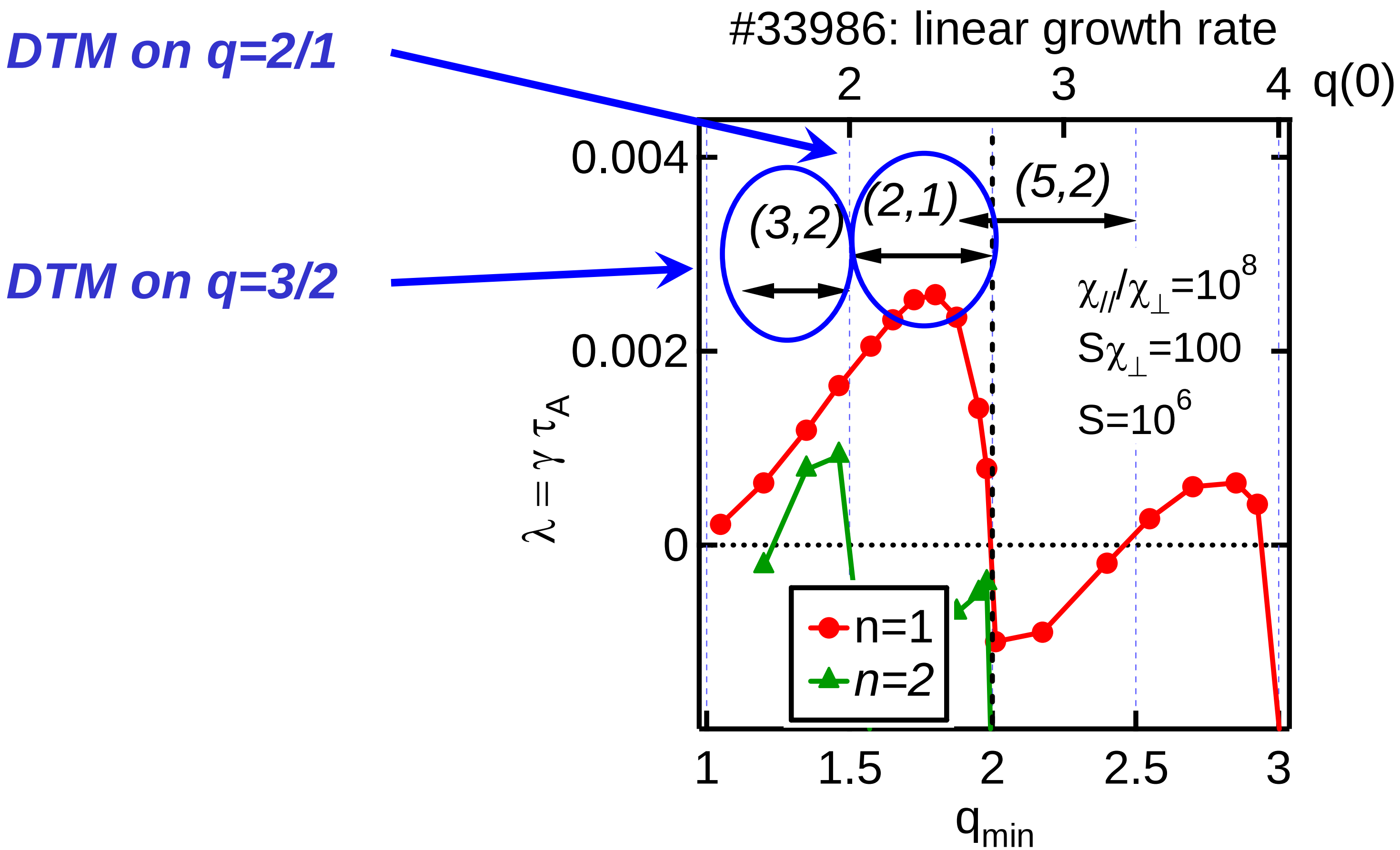
Double-Tearing Mode

❑ Basic linear MHD stability features

- q-scan by Rescaling of total current
- Tearing unstable when q_{min} below 2

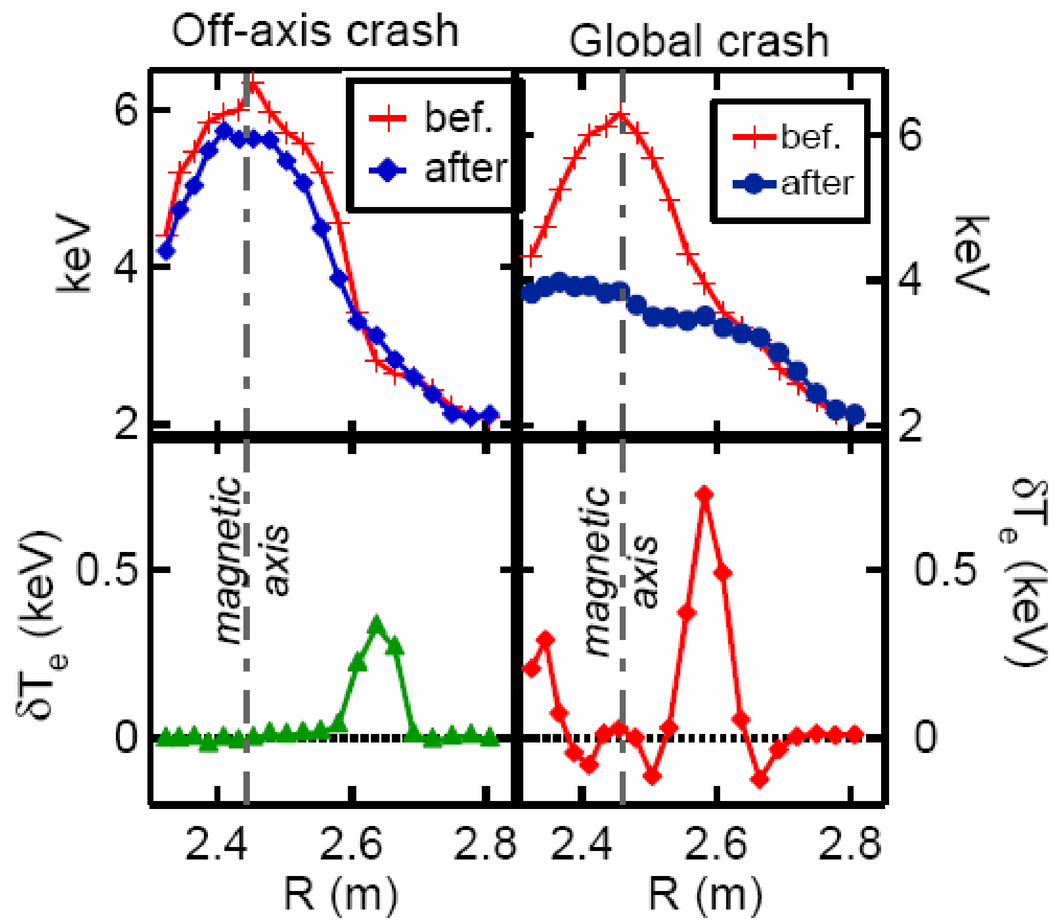


Double-Tearing Mode

❑ Impact of the Double-Tearing Mode (DTM) on confinement

- **Observation of different situations**
  - ➢ Off-axis crashes, complete crash (q=2 DTM)

What determines the impact of the DTM ?

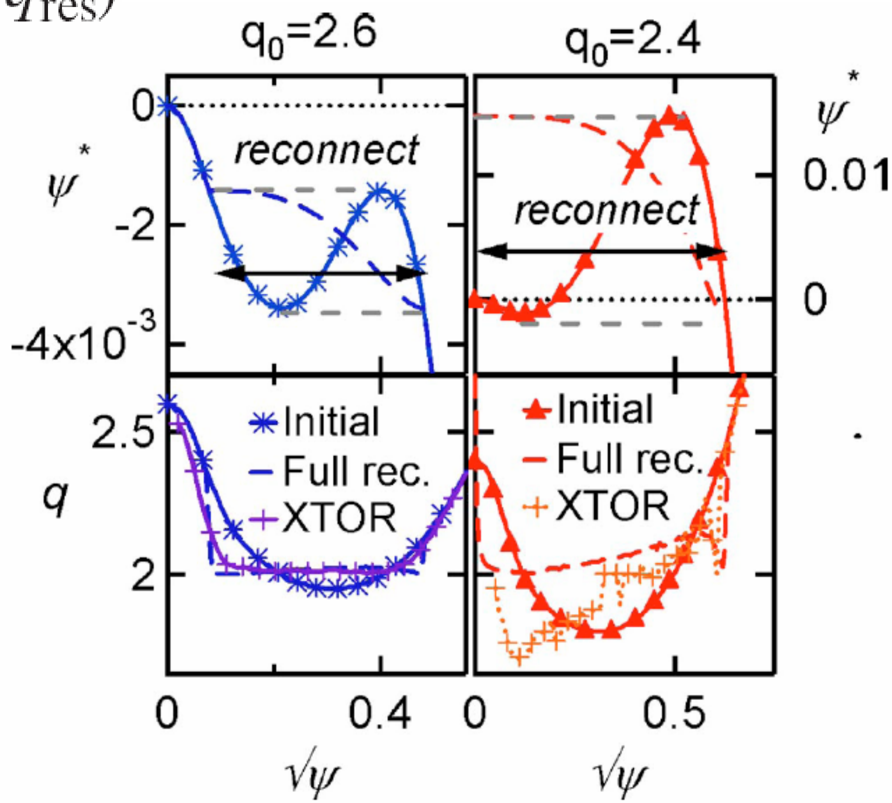


Double-Tearing Mode

❑ Unstable DTM experiences full reconnection

- Full reconnection model [Kadomtsev '75] extended to non-monotonic q-profiles [Carreras '79]
  - ➢ Helical flux reconnected in multiple-valued region

$$\psi^* = \int d\psi_N (1 - q/q_{\mathrm{res}})$$



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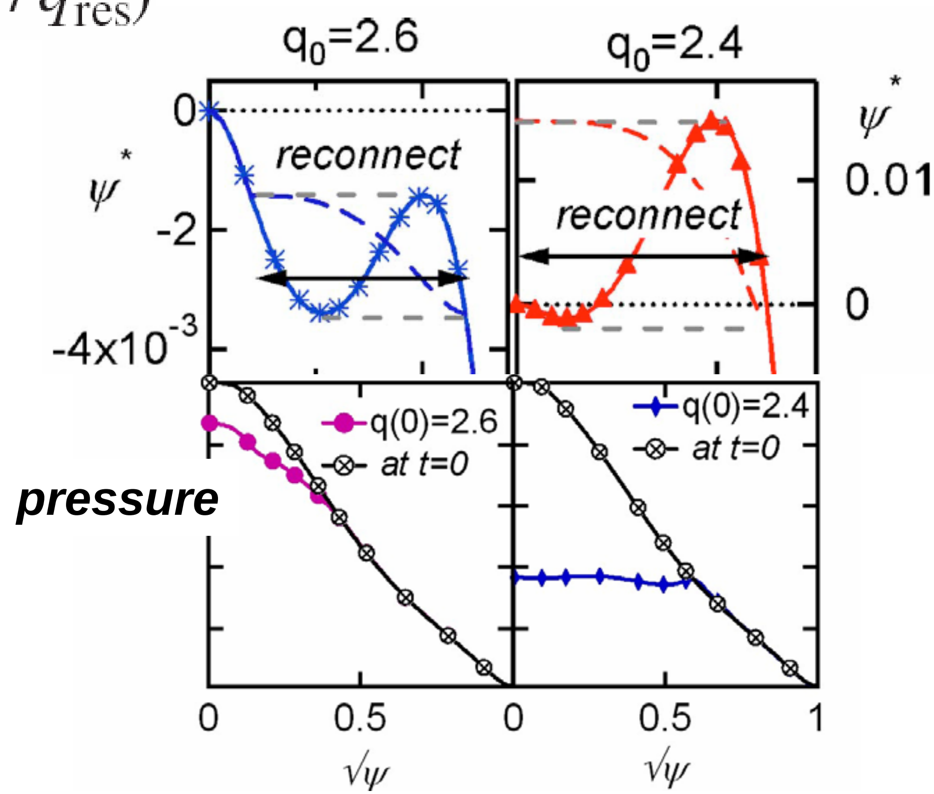
$$\psi^* = \int d\psi_N (1 - q/q_{res})$$

- Core confinement
  - ➢ Small impact if off-axis
  - ➢ Else, global crash



Off-axis and global reconnections at q=2 consistent with full reconnection model and XTOR simulations

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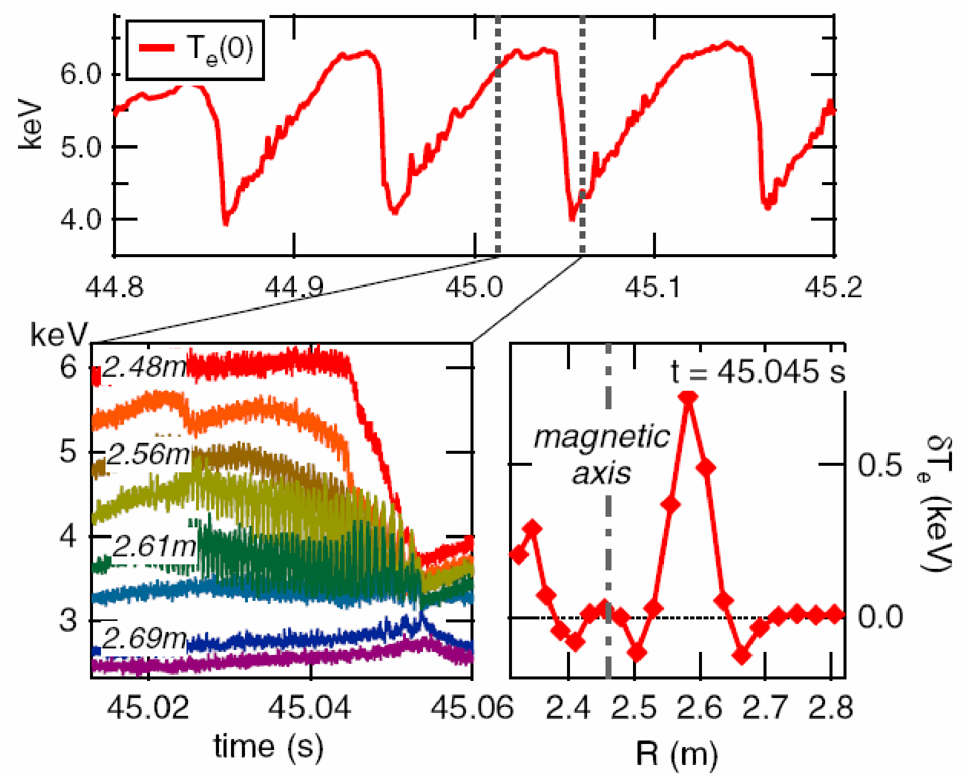
Non-linear regimes

- What is the dynamics of MHD-unstable plasma ?
  - **DTM can produce periodic relaxations**
    - The q=2 sawtooth regime [Chang '96]
    - *Example from Tore Supra*

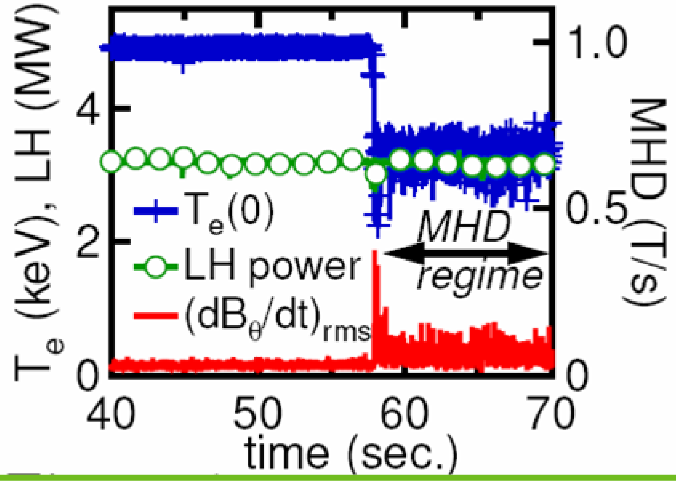
q=2 sawtooth regime with global crashs (consistent with full reconnection)



Non-linear regimes

- **What is the dynamics of MHD-unstable plasma ?**
  - **DTM can produce periodic relaxations**
    - The q=2 sawtooth regime [Chang '96]
  - **In other conditions, saturated regime with MHD activity**
    - Confinement degradation
    - Losses of fast electrons (LH-driven)

Transition to a regime with saturated MHD

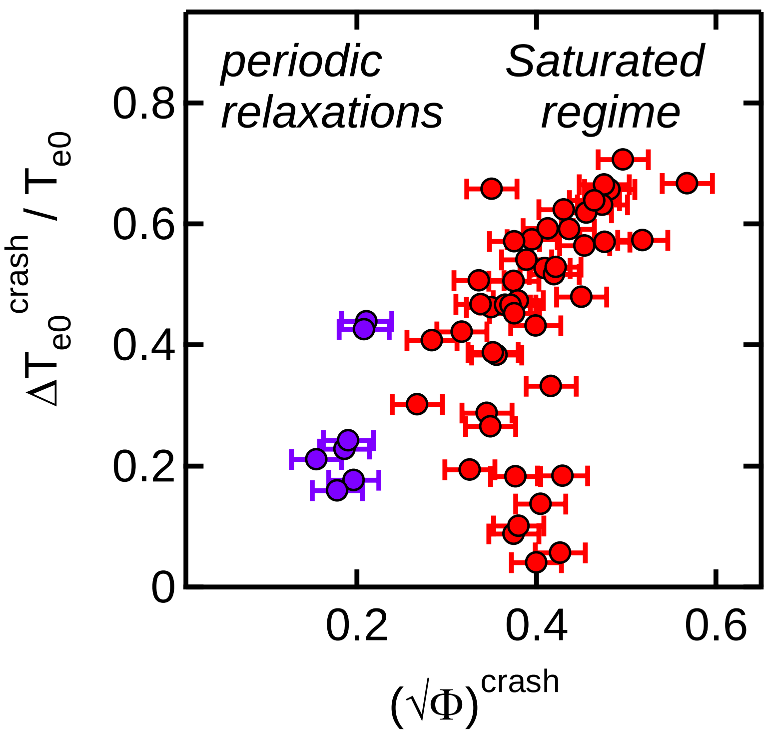


Non-linear regimes

❑ Discrimination between periodic relaxations and saturation

- **Experimental database for Tore Supra shows**
  - ➢ Periodic regime if crash radius small enough

Non-linear regimes and position of inversion radius after MHD mode



Non-linear regimes

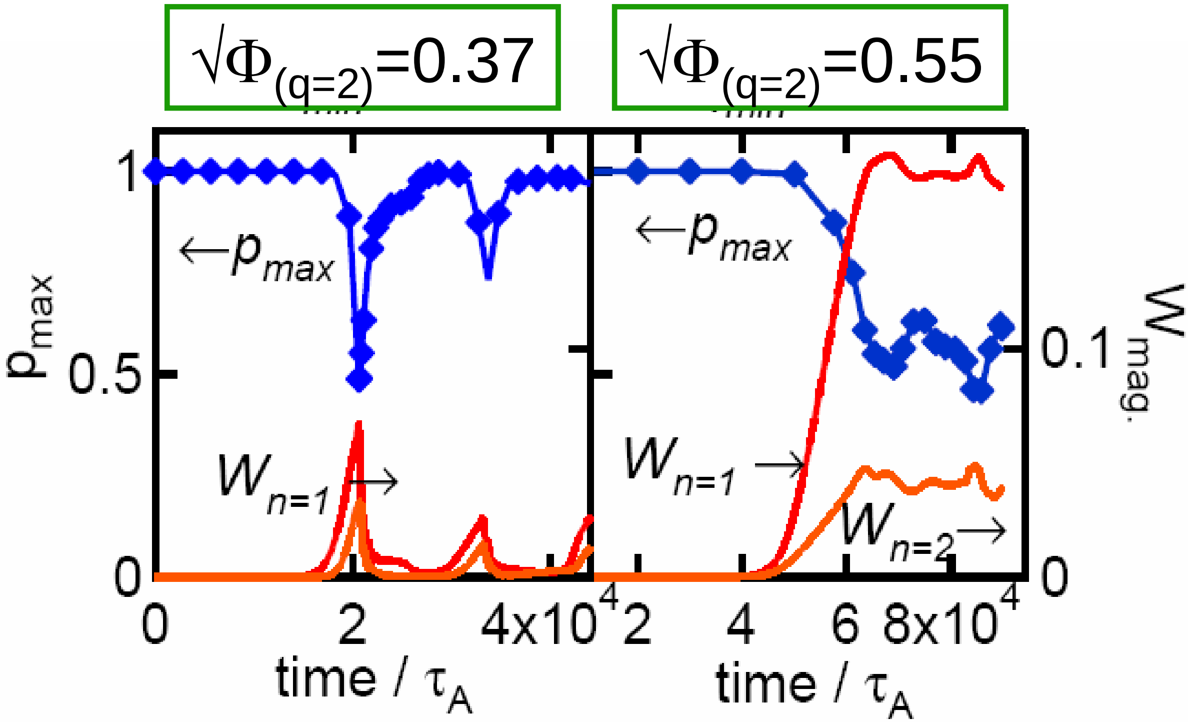
❑ Non-linear MHD simulations

- Here $\tau_R/\tau_E=(\tau_R/\tau_E)^{exp}$ to have consistent pressure / equilibrium dynamics
- **Total current rescaled to increase q=2 radius**
  - *At the same time, goes from Double to Single tearing*

Non-linear regimes for different q=2 positions

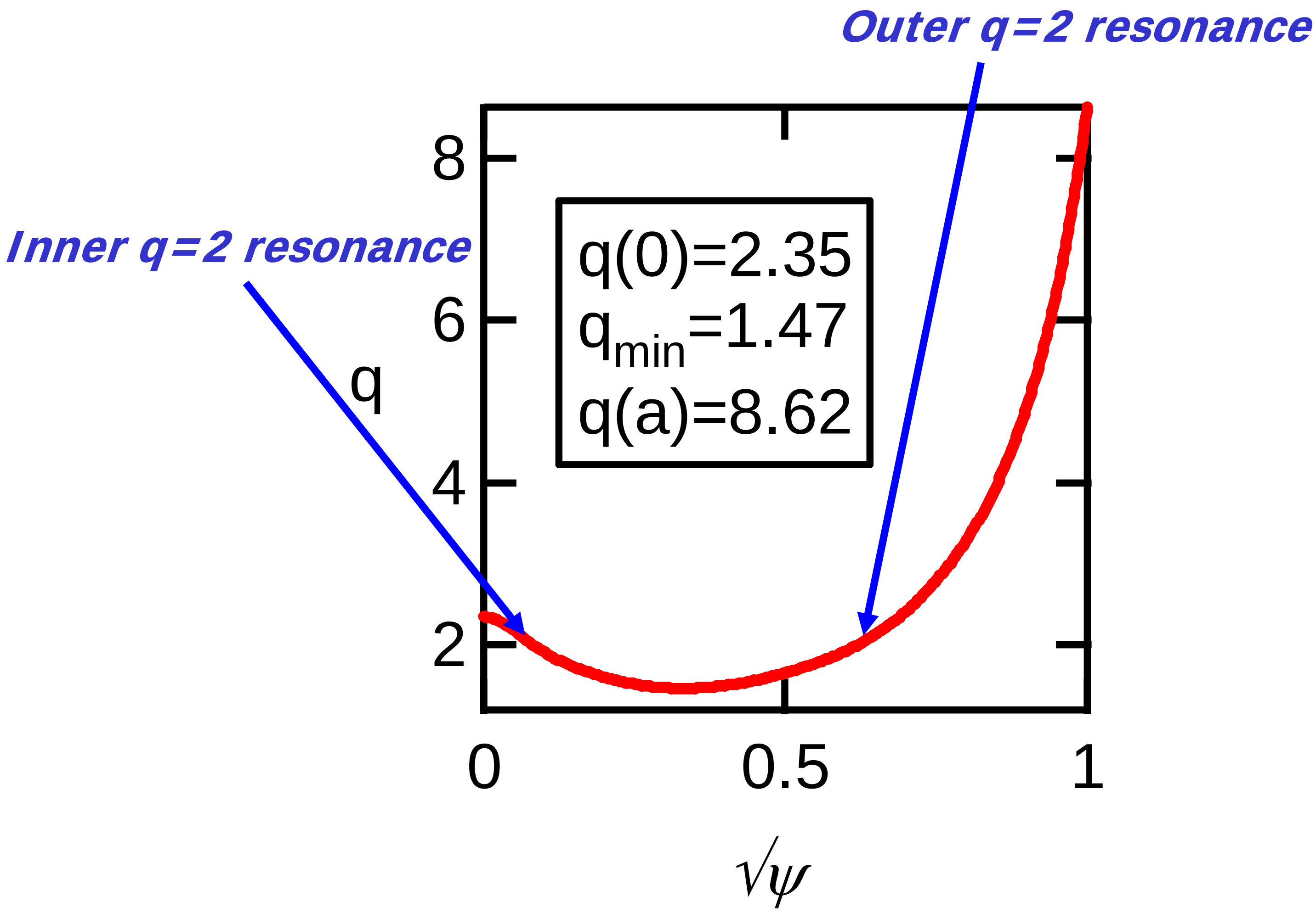
• periodic relaxations for DTM

• saturated state when q=2 more outside and STM



Non-linear regimes

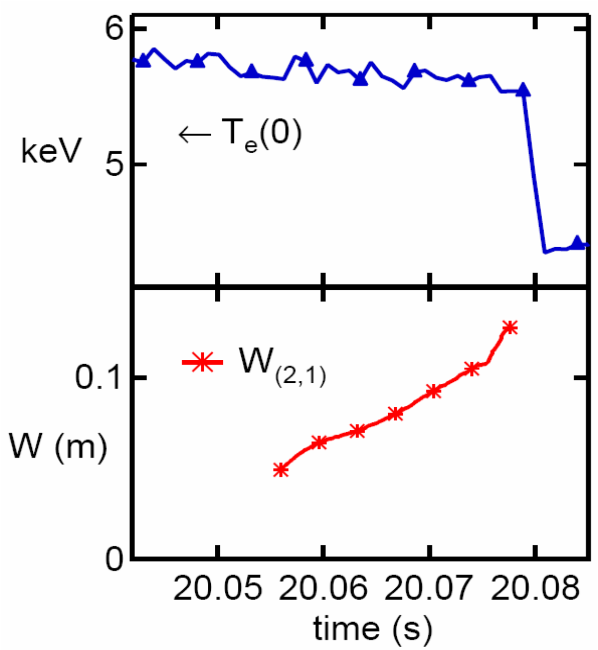
- **Fully non-inductive discharges (q_{min}~3/2):**
  - Magnetic equilibrium



Non-linear regimes

- **Fully non-inductive discharges (q_{min}~3/2): two regimes**
  - Successfull long discharges
    - No sign of n=1 activity
  - But in similar conditions
    - n=1 mode can grow and saturate at large size

Example of n=1 becoming unstable



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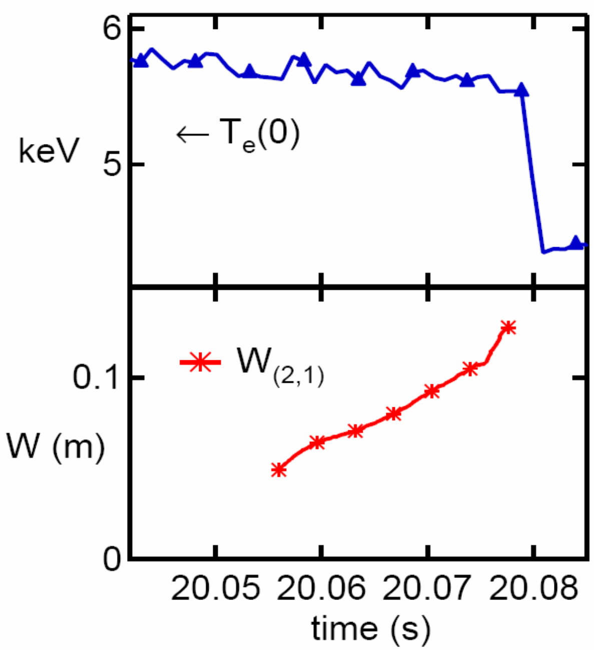
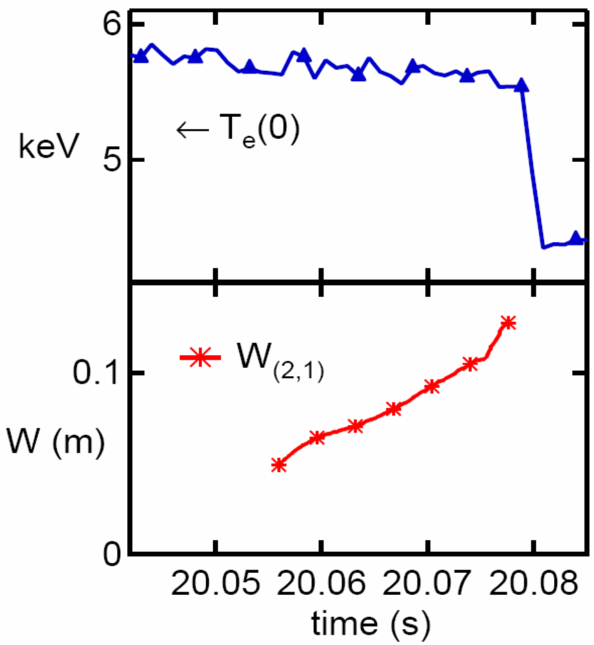


Can non-linear MHD model recover these two regimes ?

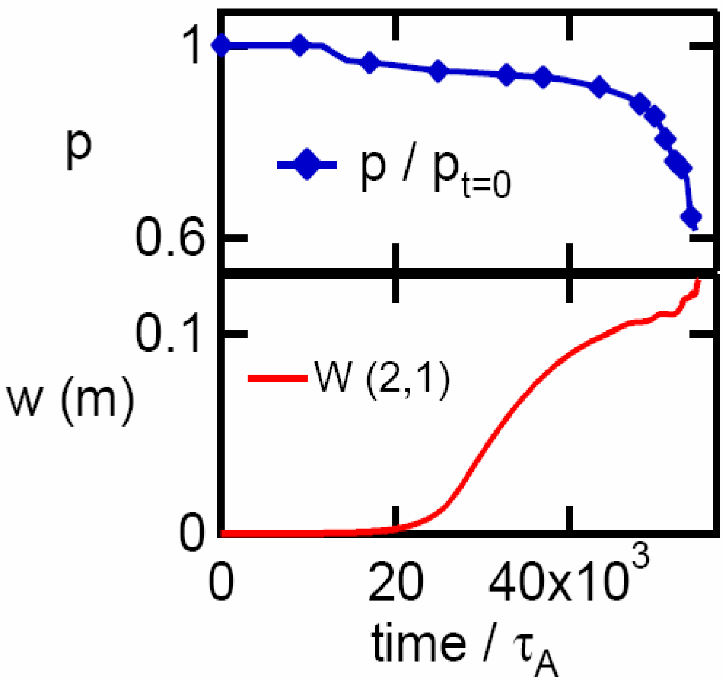
Non-linear regimes

- **Long fully non-inductive discharges: wrong n=1 prediction**
  - ***Non-linear MHD simulations including transport in agreement with global crash: n=1 mode found unstable***
  - ***Successfull discharges (no n=1 mode) cannot be recovered***

Example of n=1 becoming unstable



XTOR S=5x10^6



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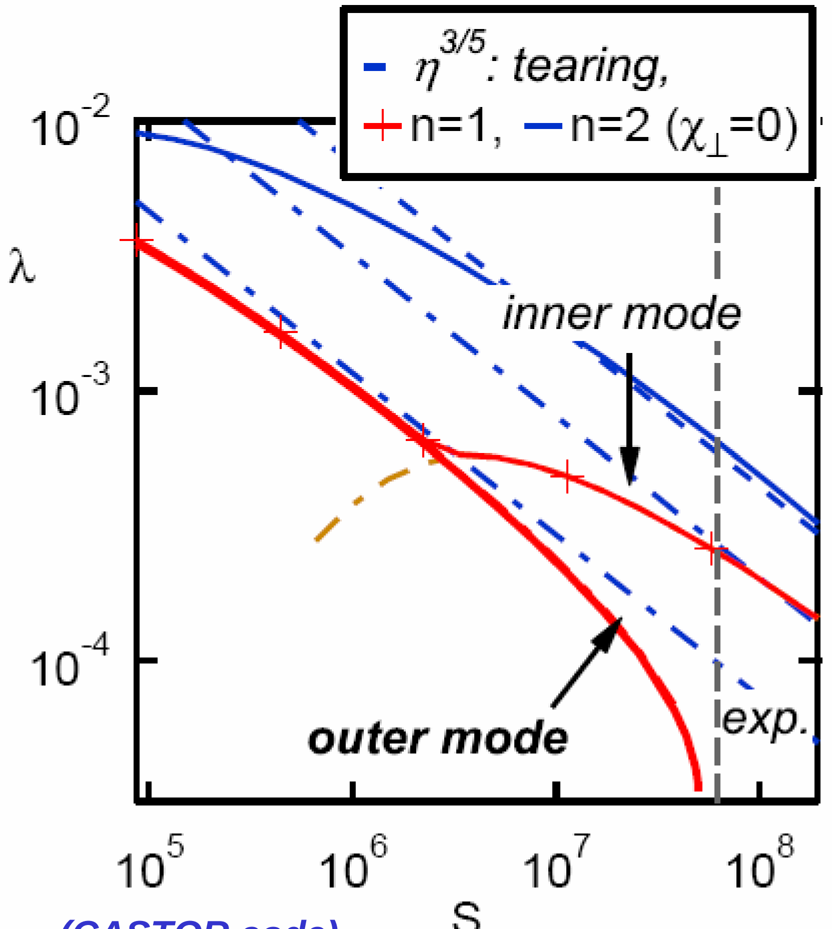
Issue on n=1 mode

- **Condition for recovering n=1 stability in standard MHD**
  - Curvature term at high S *[Glasser '75 (GGJ)]*
  - No transport

2 tearing branches (2 resonances)

Outer mode stable at S=S^exp^



(CASTOR code)

Issue on n=1 mode

❑ Condition for recovering n=1 stability in standard MHD

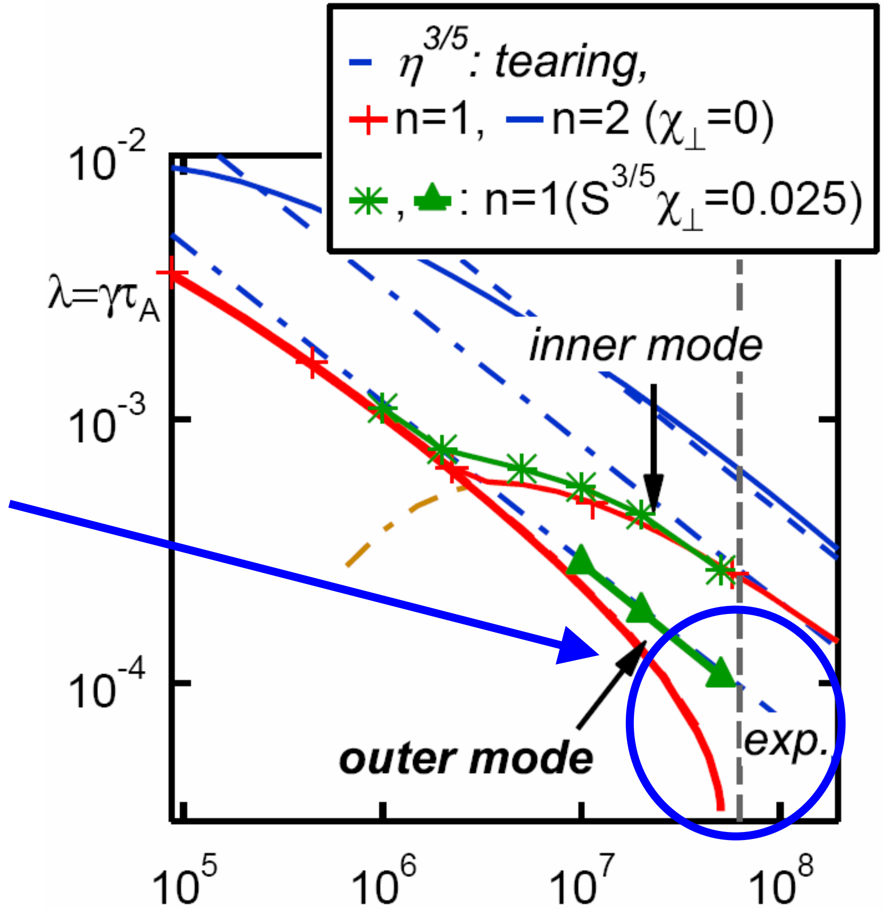
- Curvature term at high S *[Glasser '75 (GGJ)]*
- No transport

- **But transport is expected to play a role (low growth rate)**

n=1 then unstable at $S=S^{exp}$

- *consistent with theory expectations [Lütjens '01]*

Additional physics should be considered



(XTOR code)

Issue on n=1 mode

❑ Candidate mechanisms for n=1 stabilisation

- Toroidal rotation (ripple-driven in Tore Supra)
- Small island physics (outside standard MHD description)

❑ Rotation effect

- Rotation in Tore Supra consistent with theory prediction for ripple drive *[Trier '08]*: Mach number is low (M~0.05)

Toroidal rotation

$$\rho\left(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \mathbf{J}\times\mathbf{B} - \nabla p + \boxed{\mu_\varphi\left(\mathbf{V}_{\mathbf{src}} - V_\varphi\right)} + \nu\nabla^2\mathbf{v}$$

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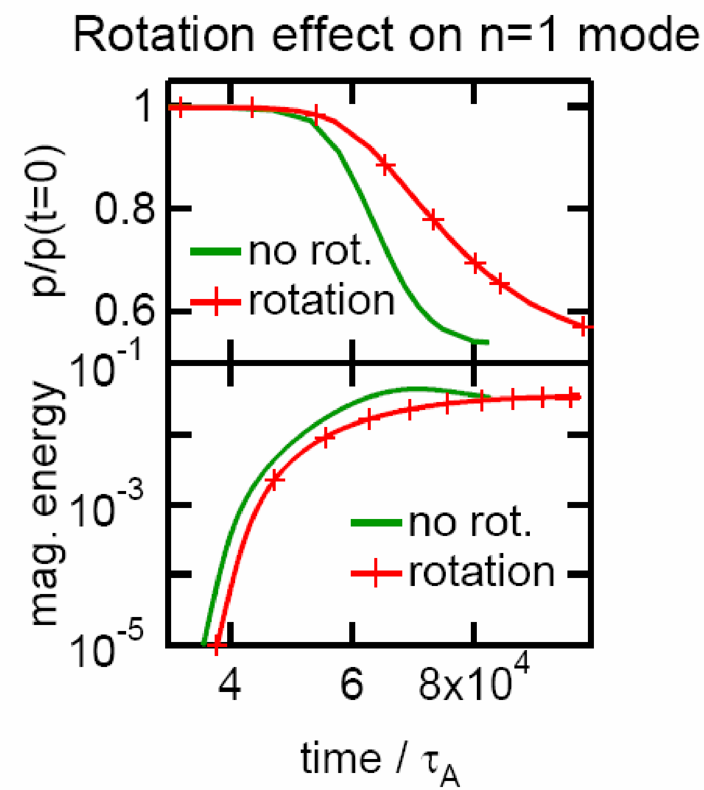
❑ Rotation effect

No effect on (2,1) tearing saturation

The pressure crash still occurs



Issue on n=1 mode

❑ Diamagnetic rotation: possible candidate

- Not negligible a priori: $\omega^*_e / \gamma \sim 10$
- Could potentially
  - ➢ Limit island growth in the non-linear regime *[Scott '87]*
  - ➢ Create multiple saturated states *[Ottaviani '04]*

- **Investigations**
  - ➢ 4-field reduced MHD, cylindrical geometry *[Hazeltine '87]*
  - ➢ Preliminary study with XTOR

Issue on n=1 mode

❑ 4-field Reduced MHD [Hazeltine '87]

- Poloidal flux (ψ), electrostatic potential (ϕ), electron pressure (p), parallel ion velocity (v)
- Isothermal model

$$
\begin{aligned}
\partial_t \psi &= \eta \nabla_\perp^2 \psi - \nabla_\parallel (\phi + \delta p) + e \\
\partial_t W &= \nu \nabla_\perp^2 W + [W, \phi + \delta \tau p] - \nabla_\parallel J + \delta \tau [\nabla_\perp p; \nabla_\perp \phi] \\
\partial_t p &= \kappa_\perp \nabla_\perp^2 p + [p, \phi] - \beta \nabla_\parallel (v - 2\delta J) + S_p \\
\partial_t v &= v_\parallel \nabla_\perp v + [v, \phi] - \frac{1+\tau}{2} \nabla_\parallel p + \delta \tau \beta \frac{1+\tau}{2} [p, v]
\end{aligned}
$$

$$W = -\nabla_\perp^2 \phi \qquad J = -\nabla_\perp^2 \psi$$

$$\nabla_\parallel \equiv \varepsilon \partial_\varphi - [\psi, .]$$

$$\omega_e^* = -(m/x)\delta(dp/dx) \text{ with } x = r/a$$

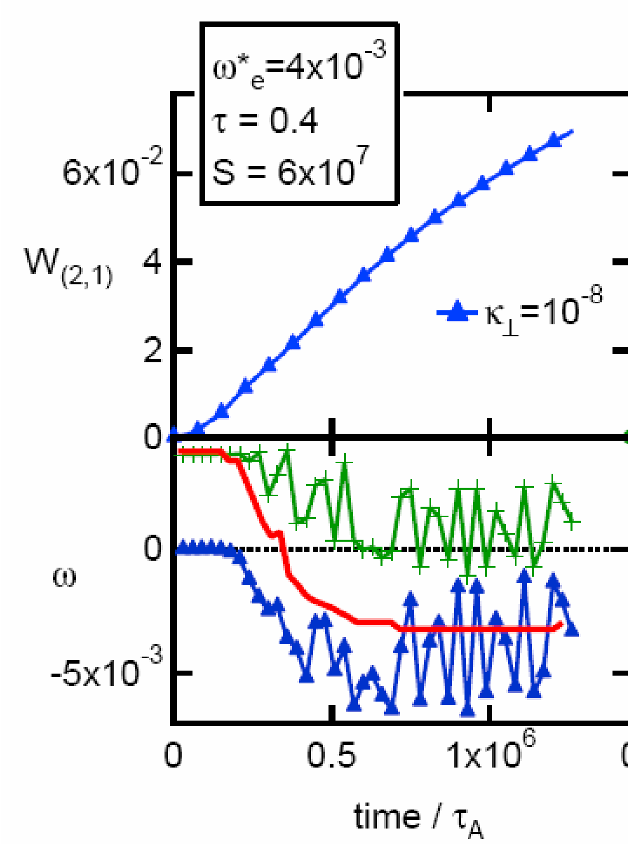
Issue on n=1 mode

❑ 4-field Reduced MHD [Hazeltine '87]

- Non-linear simulations with n=0,1
- Experimental resistivity $\eta=5\times10^{-9}$, $\nu=\nu_{//}=\eta$
- $\omega^*_e=4\times10^{-3}$ (higher than experimental $\omega^*_e=10^{-3}$)

Two non-linear regimes

- ***higher $\kappa_\perp$:***
  - ***large island***
  - ***$\omega \sim \omega_{ExB}$***
  - ***high ω_{ExB} generation***



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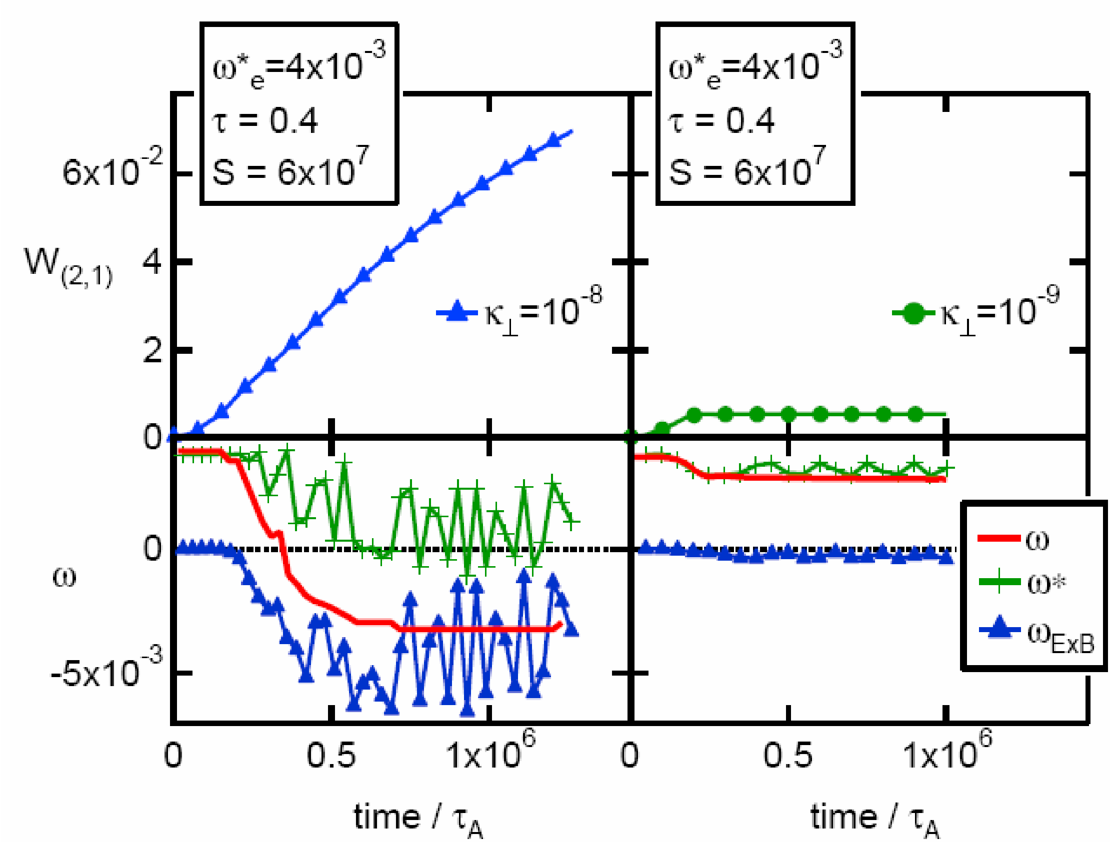
Two non-linear regimes

- ***higher $\kappa_\perp$:***
  - ***large island***
  - ***$\omega\sim\omega_{ExB}$***
  - ***high ω_{ExB} generation***
- ***lower $\kappa_\perp$:***
  - ***small saturated island,***
  - ***$\omega\sim\omega^*$***
  - ***low ω_{ExB} generation***



Issue on n=1 mode

❑ 4-field Reduced MHD [Hazeltine '87]

- Non-linear simulations with n=0,1
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- ***lower $\kappa_\perp$:***
    - ***small saturated island,***
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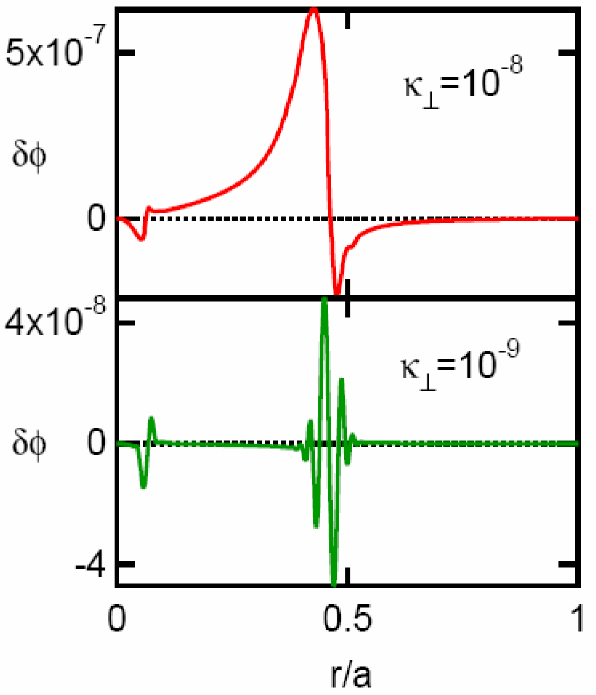
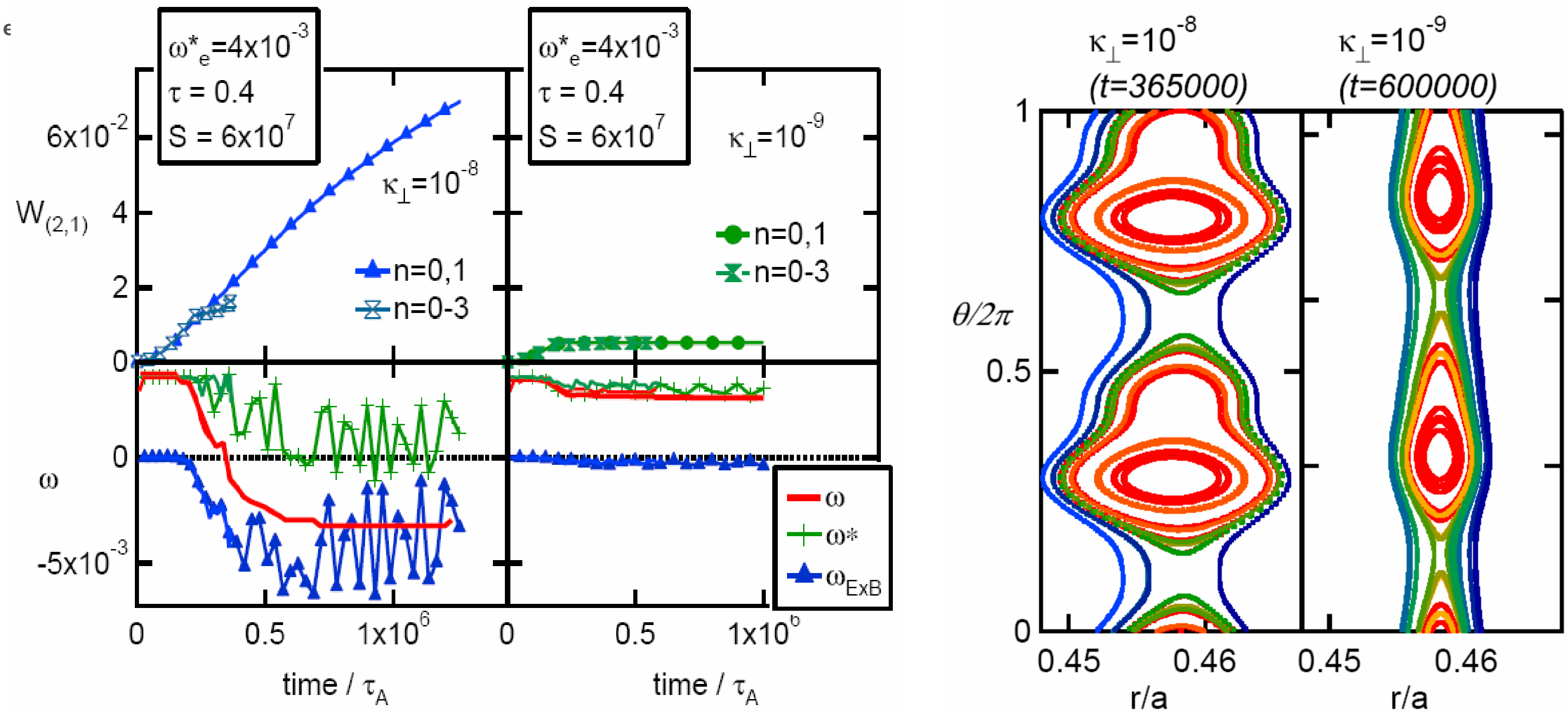
Distinct radial structures



Issue on n=1 mode

- **4-field Reduced MHD [Hazeltine '87]**
  - **Non-linear simulations (n=0,…3)**:
    - same results
    - Strong island distortion
    - Computation of the large island case crashes early



Issue on n=1 mode

❑ 4-field Reduced MHD [Hazeltine '87]

- **Two different non-linear regimes**
  - ➢ Controlled by $\kappa_\perp$ and ω^*_e
  - ➢ At low $\kappa_\perp$, pressure perturbation dominated by convection, otherwise dominated by transport
  - ➢ Qualitatively satisfaying for explaining experimental facts

- **Transition at $\kappa_\perp \in [10^{-9}, 4\mathrm{x}10^{-9}]$ for $\omega^*_e \sim 4 \mathrm{\ x\ } \omega^{*\,exp}_e$**
  - ➢ $\kappa_\perp$ consistent with collisional process ($\kappa_\perp = \eta(1+\tau)/2 = 3.5\mathrm{x}10^{-9}$)
  - ➢ But in experiment, $\kappa_\perp \sim 10^{-7}$: ***large island regime***

- Finally, the question of transport remains central even when diamagnetic effects are considered

Issue on n=1 mode

Diamagnetic rotation: toroidal effects

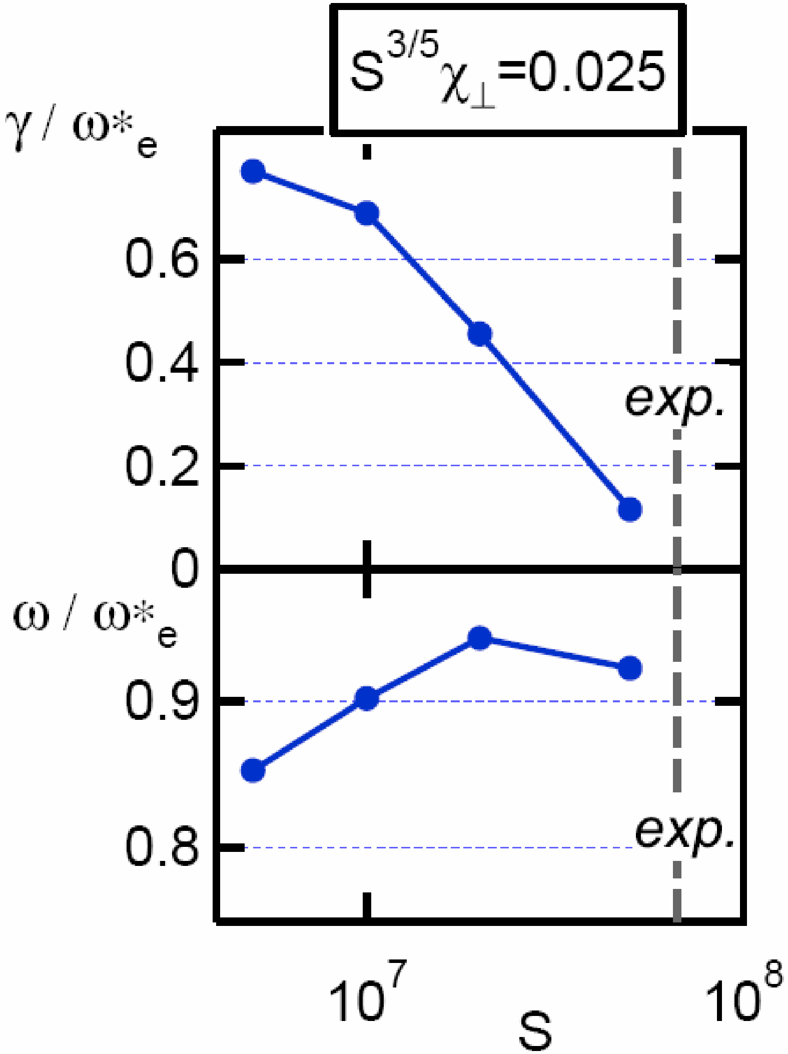
- Can toroidal geometry add significant stabilisation to n=1 mode ?
- Preliminary study with XTOR
- New version of the code: fully implicit time advance
- Electron diamagnetic effect in Ohm's law:

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} \quad = \quad \eta \left(\mathbf{J} - \mathbf{J_{NI}} \right) - \boxed{\frac{1}{1+\tau} \frac{\nabla_{\parallel} p}{en}}$$

Issue on n=1 mode

❑ Diamagnetic rotation: toroidal effects

- Linear study with transport
  - ➢ $S^{3/5}\chi_\perp = (S^{3/5}\chi_\perp)^{exp}$
  - ➢ At experimental S
    - ✓ $\gamma << \omega^*_e$
    - ✓ $\omega \sim 0.95\ \omega^*_e$



We expect significant effect of diamagnetic rotation in the full MHD model including transport

Issue on n=1 mode

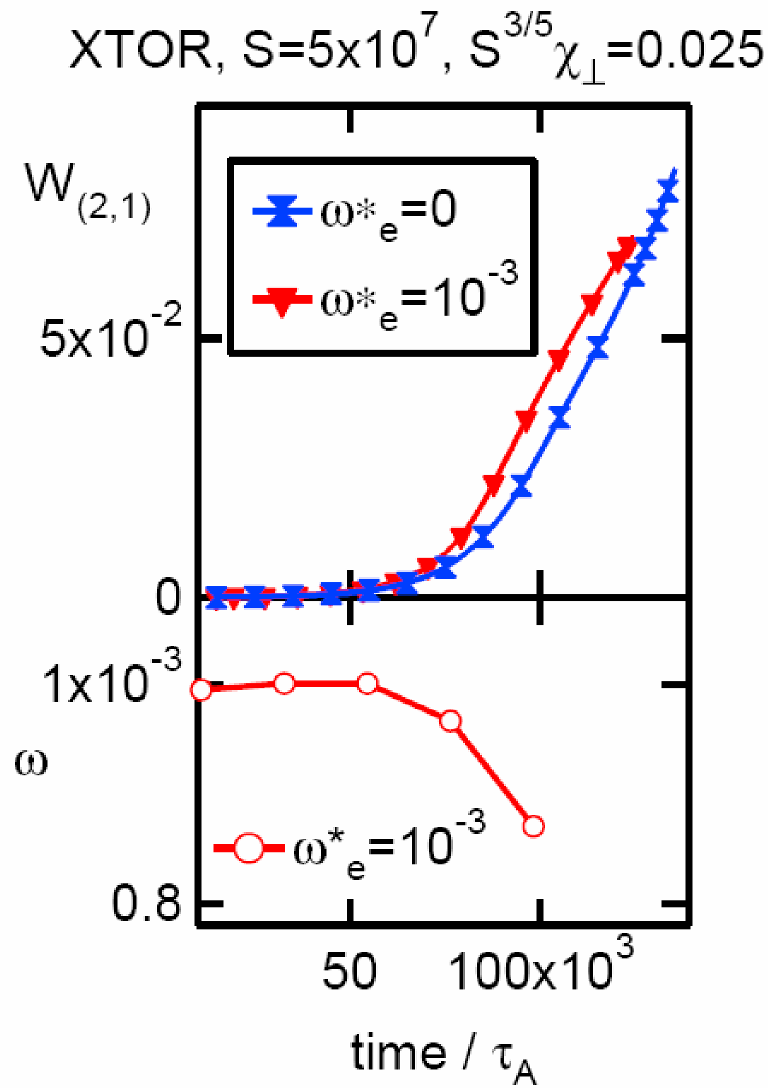
❑ Diamagnetic rotation: toroidal effects

- Non-linear regime with n=0,1
  - $S=5x10^7$, $\gamma/\omega^*_e \sim 0.16$

No significant difference compared to $\omega^*_e=0$ case

Consistent with reduced-MHD 4-field model:

• small island regime not accessible in presence of realistic transport coefficients



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Conclusion & Perspectives

❑ Dynamics of MHD modes in non-inductive discharges

- **Double-Tearing: full reconnection**
- **Saturation / Periodic relaxations: q=2 surface position**

❑ Stability of n=1 mode in very long discharges

- **Curvature stabilization at high S without transport**
- **But fails in more realistic conditions (finite transport)**
- **Two-fluid effect allows small island regime**
  - ➢ **But at collisional transport coefficient level only**

❑ Future work:

- more comprehensive MHD model needed ?
- Other rationale for transport coefficient implementation ?