From kinetic MHD in stellarators to a fully kinetic description of wave particle interaction

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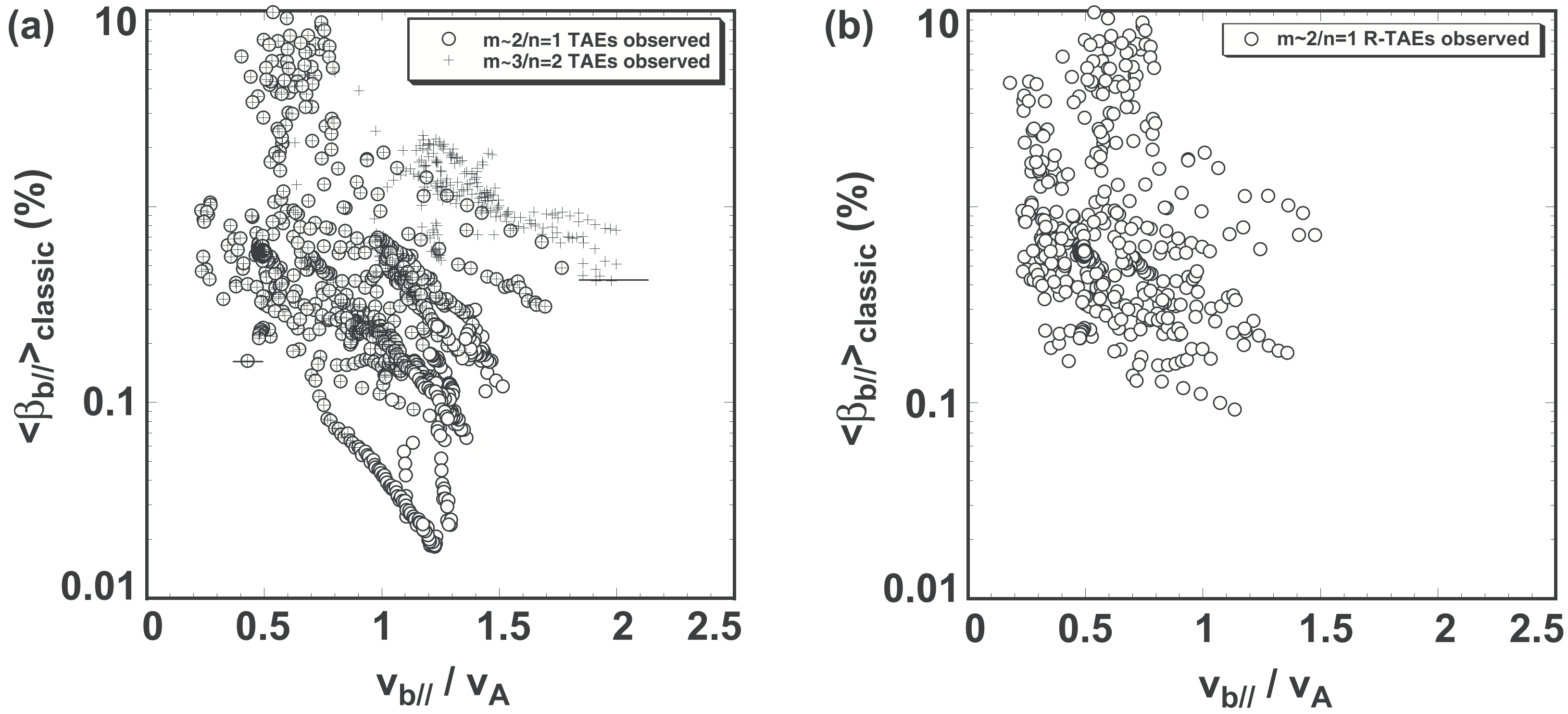
1. introductory remarks on wave particle-interaction
2. numerical 3D kinetic MHD model
3. application: stability boundaries for TAE in W7-AS/W7-X
4. limits and extentions of the model
5. gyro-kinetic approach for 2D systems

Introduction

kinetic effects may interact with ideal MHD modes:

- destabilization of MHD gap modes by resonant interaction of fast particles
- source of free energy: gradient of fast particles
- experimental observations:
  - W7-AS: A. Weller et al. (1998, 2000, 2003)
  - CHS/LHD: K. Toi et al. (2000, 2004)
  - HSX (D. Brower, J. Canik, et al. 2006)
- theoretical approaches for three dimensions:
  - analytical approach: Kolesnichenko et al. (2001, . . .)
  - gyrofluid model 2D (Don Spong)
  - M3D nonlinear two fluid - kinetic hybrid model (Strauss, Park et al., 2002, . . .)

experimental results from LHD



observation of TAE vs. R-TAE in LHD
Toi et al. (2002)

note: difference to MHD instabilities

- looking for particle interaction with the stable part of MHD spectrum
- source of free energy density or temperature gradient of fast particles
- kinetic description needed
- stable MHD spectrum:
  analogy to Schrödinger equation in a solid state

MHD

$\omega^2 W_{kin}(\xi^*, \xi) = W_{mag}(\xi^*, \xi)$

slab/cylinder dispersion relation:
$\omega^2 = k_{||}^2 v_A^2$

degeneracy
removed by symmetry breaking terms
$\Rightarrow$ gap

solid state

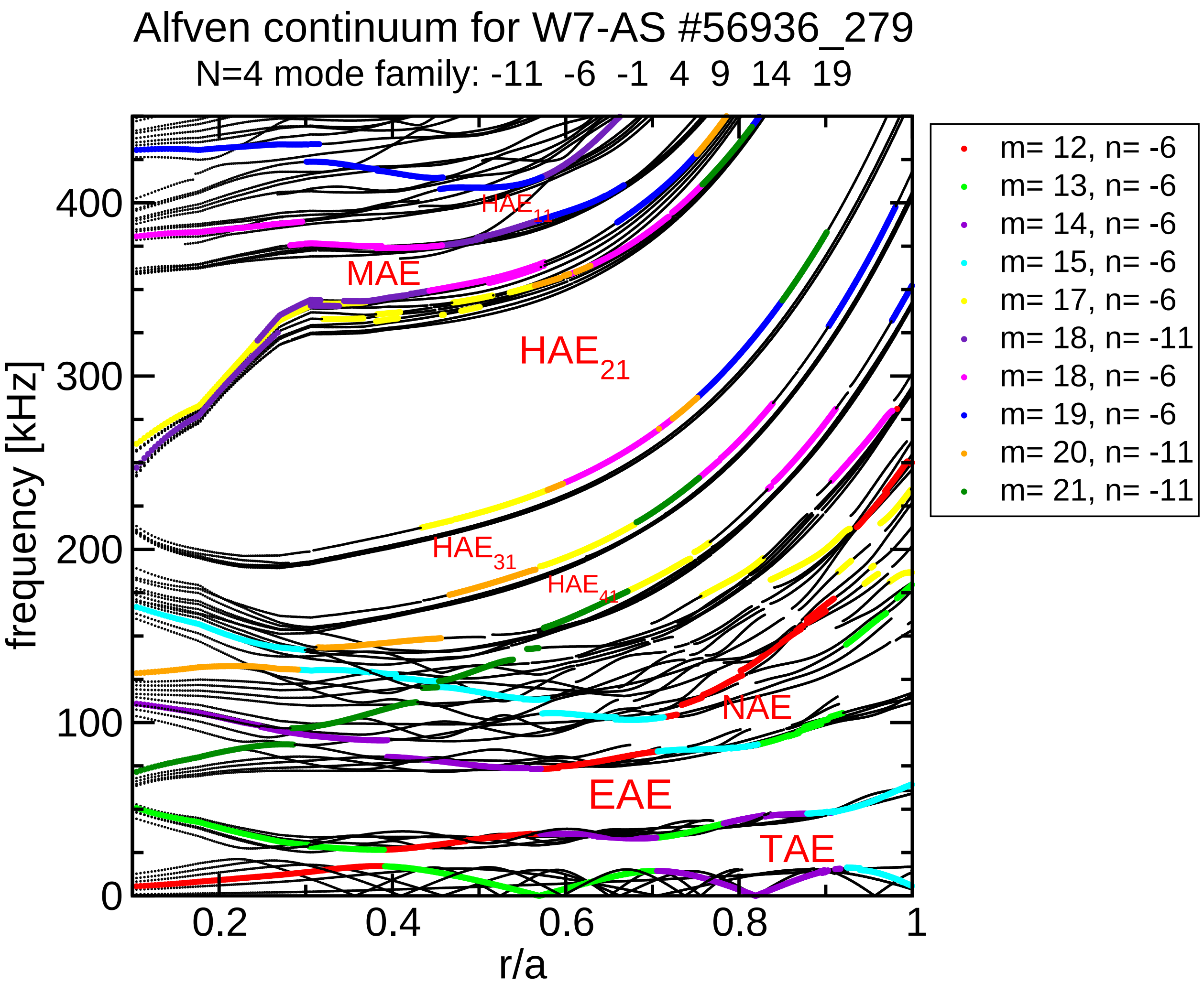
$E|\Psi|^2 = < \Psi^*|H|\Psi >$

free electron model:
$E = \hbar^2 k^2/(2m^*)$

degeneracy
removed by potential
$\Rightarrow$ gap

difference: MHD allows for global modes in the gap (breaking of symmetry)

A 3D ideal MHD continuum: W7-AS #56936



WENDELSTEIN 7-X

Kinetic MHD

1. restriction to MHD-like perturbations
   $\phi = 0$ no electrostatic potential
   $\vec{A}^{(1)} = \vec{\xi} \times \vec{B}$
   $\vec{B}^{(1)} = \vec{\nabla} \times \left(\vec{\xi} \times \vec{B}\right)$

2. derivation of an energy functional from the MHD moment equation

$$\vec{\nabla} \cdot \overleftrightarrow{P} = -\vec{B} \times \left(\vec{\nabla} \times \vec{B}\right)$$

3. replace $\overleftrightarrow{P}$ with a kinetic expression, i.e. an expression involving integrals of the distribution function

remark: this is equivalent to calculate growth/ damping rates considering the particle-wave energy transfer

Drift kinetic equation

Vlasov equation after transformation to guiding center variables and averaging over the gyro phase:
(e.g. Porcelli et al. 1994, Catto et al. 1980, Littlejohn 1983, cf. Hahm 1988)

$$\frac{\partial f}{\partial t} + \dot{\vec{R}} \cdot \frac{\partial f}{\partial \vec{R}} + \dot{v}_{\|} \frac{\partial f}{\partial v_{\|}} + \dot{y} \frac{\partial f}{\partial y} = 0$$

$y = \mu B$: perpendicular energy, $v_{\|}$: parallel velocity $\| \vec{B}$
$\vec{R}$: location of the guiding center

distribution function $f(\vec{R}, y, v_{\|}, t)$: distribution of guiding centers

correct up to first order in

$$\delta = \frac{\text{gyro radius}}{\text{system length}} \ll 1$$

Drift kinetic equation

Linearization: $f = F + f^{(1)}$

(equilibrium part: F + perturbation: $f^{(1)}$)

zero order:

$$\dot{\vec{R}}^{(0)} \cdot \left(\frac{\partial F(\epsilon, \mu, \vec{R})}{\partial \vec{R}} \right)_{\epsilon,\mu} = 0$$

- regard this equation as being approximatively solved
- for times scales with negligible drifts:

$$F = F(s, \epsilon, \mu, \sigma)$$

s: flux label; σ: sign of $v_{||}$

Drift kinetic equation

linearized 3D drift kinetic equation to first order:

- $f^{(1)}$ **splits into an adiabatic . . .**

$$f^{(1)} = \vec{A}^{(1)} \cdot \frac{\vec{b} \times \vec{\nabla} F}{B} + Ze\phi^{(1)} \left(\frac{\partial F}{\partial \epsilon}\right)_{\vec{R},\mu} - \frac{B^{(1)}}{B} \left(\frac{\partial F}{\partial \mu}\right)_{\vec{R},\mu} + h^{(1)}$$

- **. . . and non-adiabatic part:**

$$\frac{d}{dt} h^{(1)} = \left[\left(\frac{\partial F}{\partial \vec{R}}\right) \cdot \frac{\vec{b} \times \vec{\nabla}}{M\Omega} + \left(\frac{\partial F}{\partial \epsilon}\right) \frac{\partial}{\partial t} \right] L^{(1)}$$

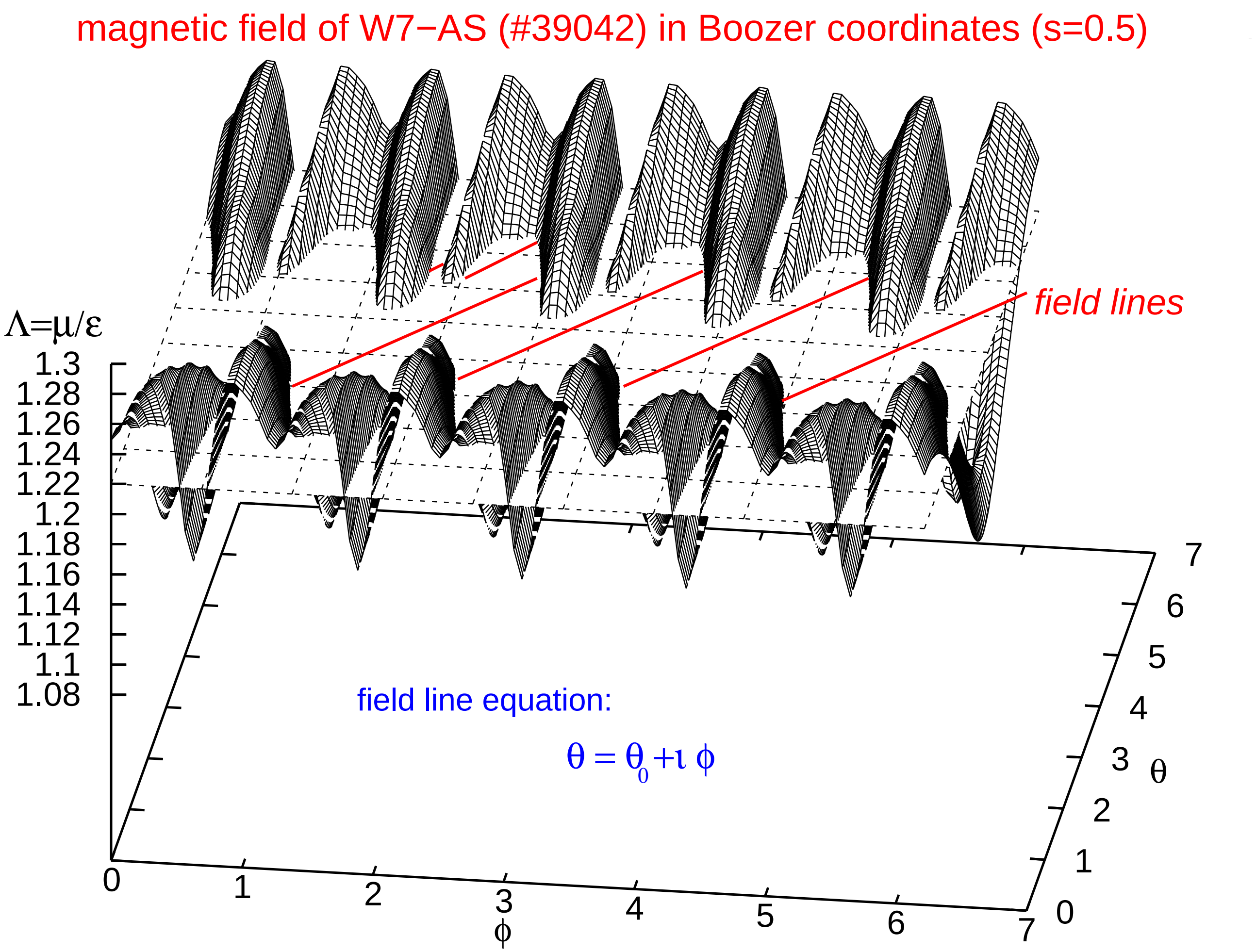
$L^{(1)}$: **perturbed Lagrangian** $L^{(1)} = [\dot{\vec{R}} \cdot \vec{A}^*]^{(1)} - \mu B^{(1)} - Ze\phi^{(1)}$

integration along field lines

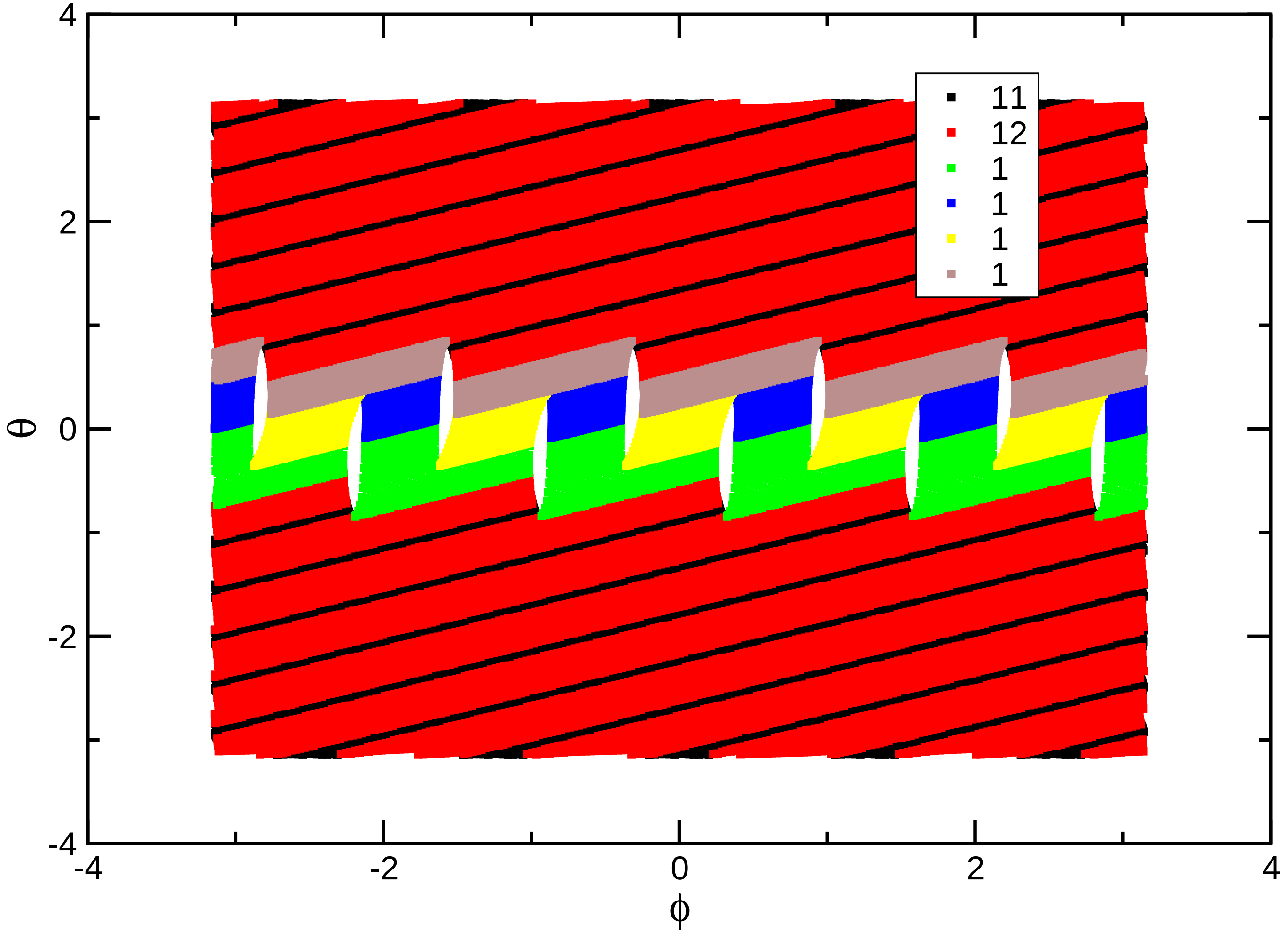
bounce averaged drifts within flux surface considered

no radial drifts

field line orbits (W7-AS)



field line orbits (W7-AS)



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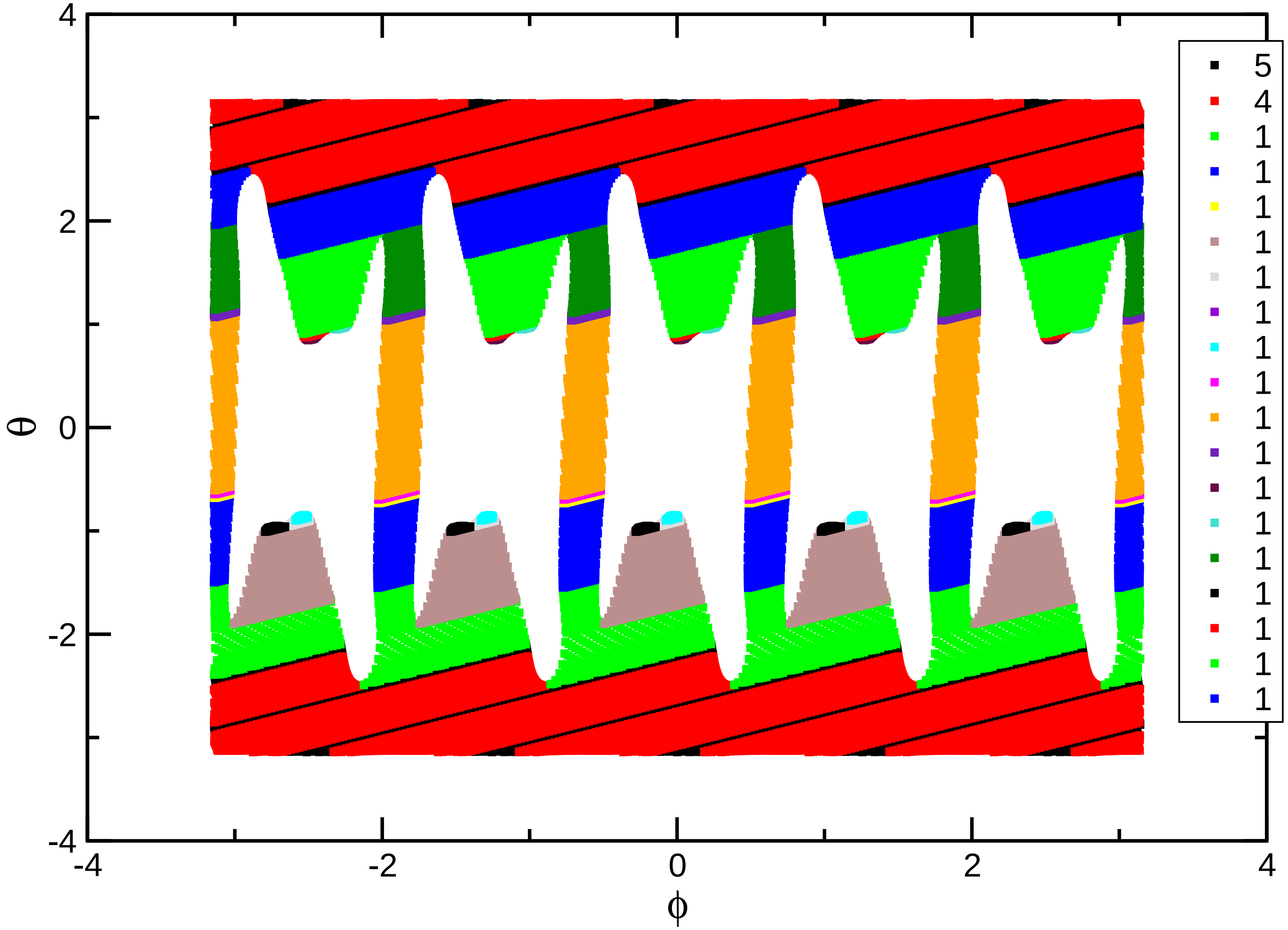

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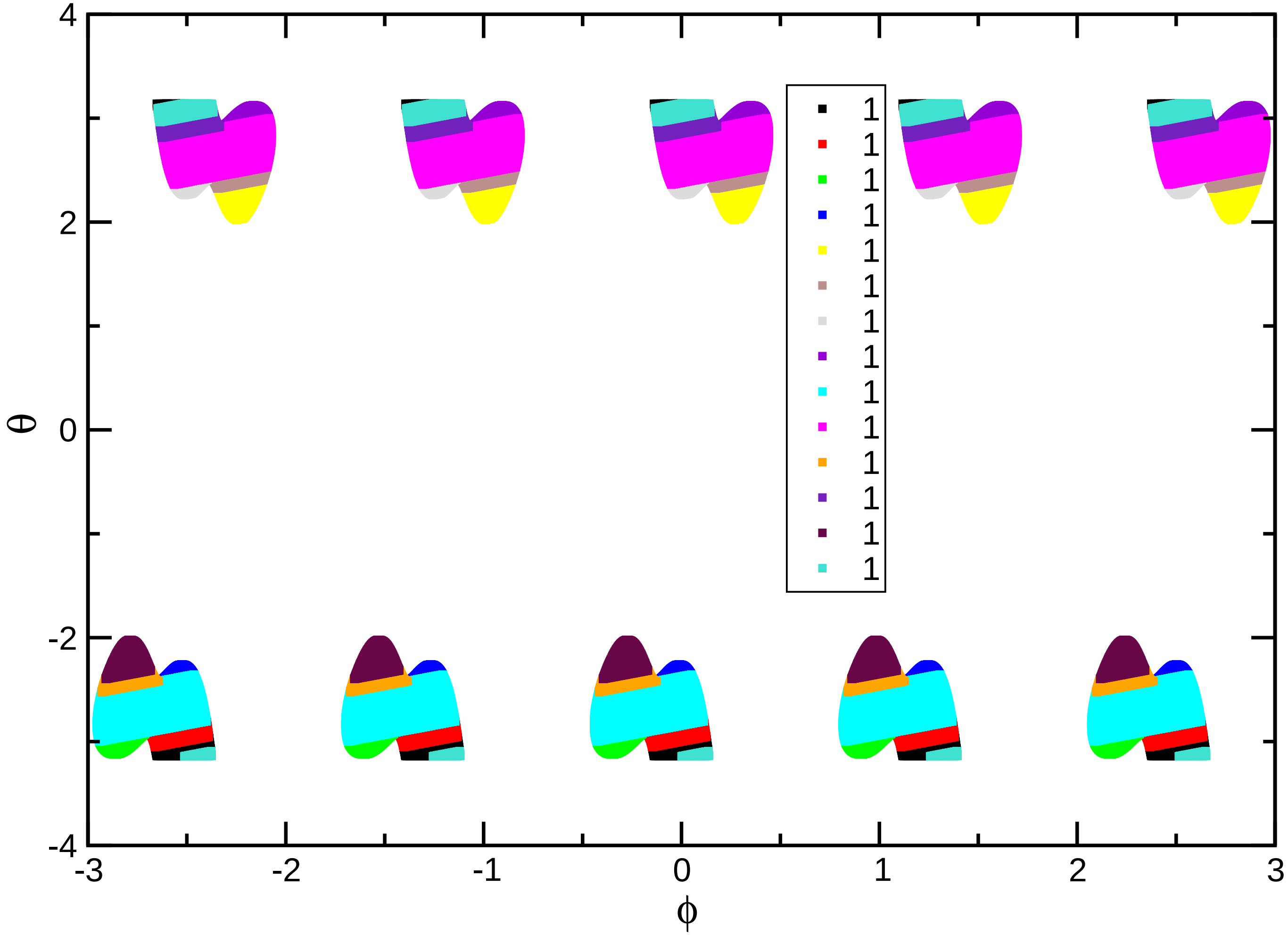


Kinetic energy integral

there is an energy integral considering kinetic effects

$$\delta W_{\mathrm{kin}} = \omega^2 \frac{1}{2} \int d^3\vec{x} \left| \vec{\xi}_\perp \right|^2 \rho_M = \delta W_{\mathrm{mag}} + \sum_{s=\mathrm{i,e,fast}} \delta W_s(\omega)$$

(Kruskal/Oberman 1958 ... Antonsen/Lee 1984)

$$\delta W_{\mathrm{mag}} = \frac{1}{2} \int d^3\vec{x} \left\{ \left| B_\perp^{(1)} \right|^2 + \left| B_\parallel^{(1)} \right|^2 + \vec{j}_\parallel \cdot \left(\vec{\xi}_\perp \times \vec{B}_\perp^{(1)} \right) - \frac{B_\parallel^{(1)}}{B} \vec{\xi}_\perp^* \cdot \vec{\nabla} p + \left(\vec{\nabla} \cdot \vec{\xi}_\perp^* \right) \left(\vec{\xi}_\perp \cdot \vec{\nabla} p \right) \right\}$$

the non-adiabatic contributions from the hot and thermal component replace the MHD fluid compression term
the contributions from the thermal plasma ($\delta W_{\mathrm{i,e}}$) and the fast particles δW_{fast}) depend on the perturbed particle Lagrangian $L^{(1)}$

(A. Könies, PoP 2000)

Kinetic contribution

particle- wave- energy- exchange by resonant interaction

$$\delta W_s = \frac{\pi}{M_s^2}\left\{\sum_\sigma\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\sum_{\substack{n,m\\ n',m'}}\sum_{p=-\infty}^{\infty} e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\times$$
$$\times\left(\frac{\partial F_s}{\partial\epsilon}\right)_\mu \frac{\omega-2\pi(\frac{n}{N_p}J-mI)\omega^*}{m\left\langle\omega_d^\vartheta\right\rangle+\frac{n}{N_p}\left\langle\omega_d^\varphi\right\rangle+\left\{\begin{matrix}\sigma(p+nq)\\ p\end{matrix}\right\}\omega_{\left\{\begin{smallmatrix}t\\ b\end{smallmatrix}\right\}}-\omega} L_{m'n'}^{(1)*}\mathcal{M}_{pn}^{m'n'*}L_{mn}^{(1)}\mathcal{M}_{pn}^{mn}$$

definition of $\mathcal{M}_{pn}^{m'n'}$:
for passing particles:

$$\mathcal{M}_{pn}^{m'n'} = \left\langle e^{i[2\pi(m'+n'q)\vartheta''-(p+nq)\omega_t t'']}\right\rangle_{\vartheta''}$$

for reflected particles:

$$\mathcal{M}_{pn}^{m'n'} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}\cos(p\omega_b t'')\right\rangle_{\vartheta''}$$

$\langle\ldots\rangle$ denotes the transit or bounce average

perturbed particle Lagrangian:

$$L^{(1)} = -(Mv_\parallel^2-\mu B)\vec{\xi}_\perp\cdot\vec{\kappa}+\mu B\vec{\nabla}\cdot\vec{\xi}_\perp$$

Realization of kinetic MHD in CAS3D-K

CAS3D-K: perturbative stability code based on a hybrid MHD-drift kinetic model

- 3-dimensional
- general mode structure and equilibrium
- particle drifts are approximated as bounce averaged drifts
- zero radial orbit width
- perturbative growth/damping rates from:

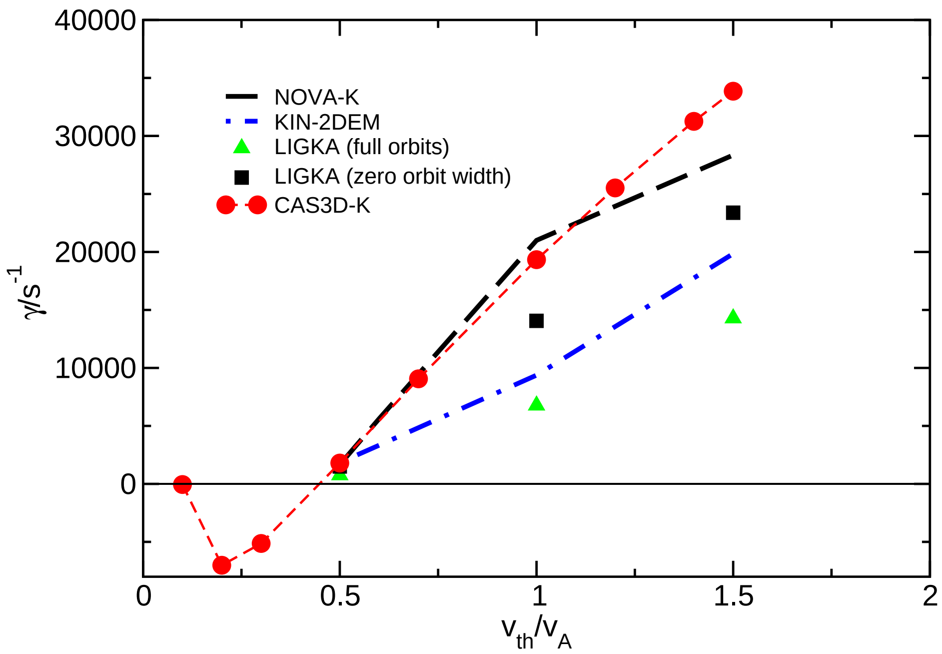
$$\Delta\omega_s + i\gamma_s \approx \frac{1}{2}\frac{\delta W_{\mathrm{s}}(\omega_0)}{\delta W_{\mathrm{mag}}}\omega_0$$

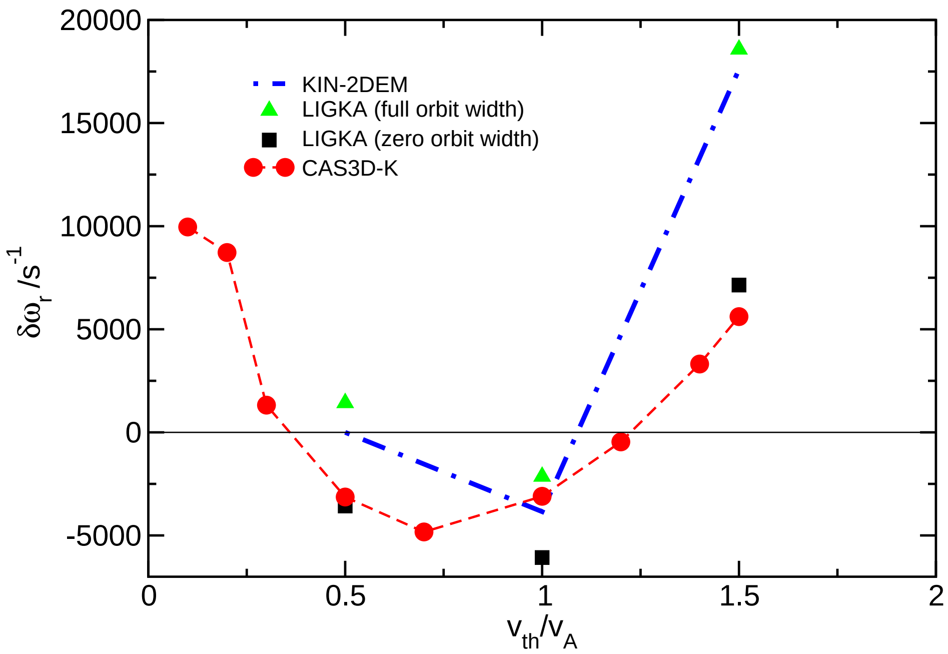
using the MHD eigenfunctions and the MHD frequency ω_0

- δW_{mag} from the ideal MHD stability code $CAS3D$ (C. Nührenberg, 1996, 1998, 2000, . . .)

(3,-2)/(2,-2) TAE Benchmark with LIGKA

circular tokamak $A = 4$, Maxwellian distribution of fast hydrogen ions



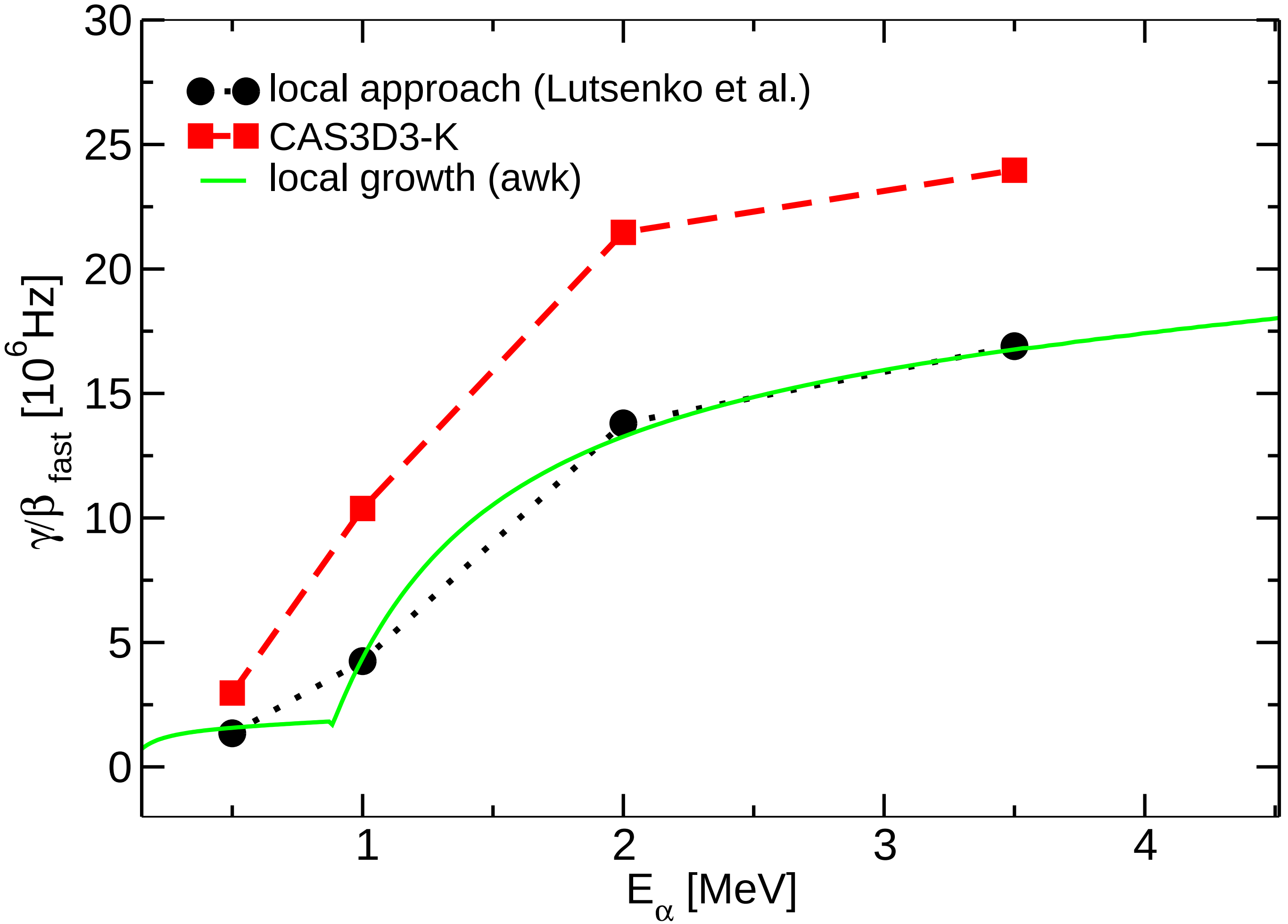


LIGKA gyro-kinetic eigenvalue code (Ph. Lauber et al., J Comp. Phys. 2007)

3D benchmark with analytical theory

growth rate in a heliotron configuration (N_P=20)

slowing down distribution of α- particles



3D analytical theory - What can we learn?

(see Kolesnichenko et al. 2002)

- proportionality to equilibrium quantities

$$\frac{\gamma}{\omega_0} \propto A^2 \sum_{m'n'} |\epsilon^{\kappa}_{m'n'}|^2 \approx A^2 \sum_{m'n'} |\epsilon^{B}_{m'n'}|^2$$

- coupling is approximately given by the structure of B
  $\Rightarrow$ investigate spectrum of B

- note, that for a TAE in a large aspect ratio tokamak: $\frac{\gamma}{\omega_0}$ is independent of the equilibrium

- the resonance condition $\omega - k_{||} v_{th} = 0$ determines

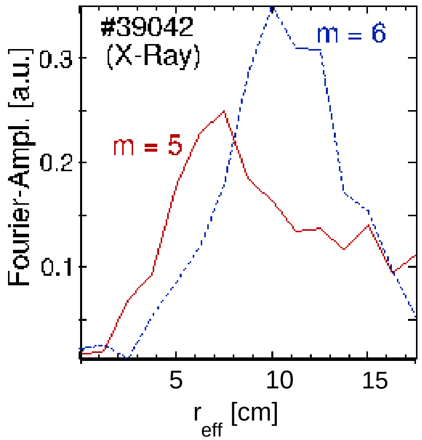
$$v^{\text{res}}_{m'n'} = v_A \left| 1 \pm \frac{m'\iota^* + n' N_p}{m\iota^* + n} \right|^{-1}$$

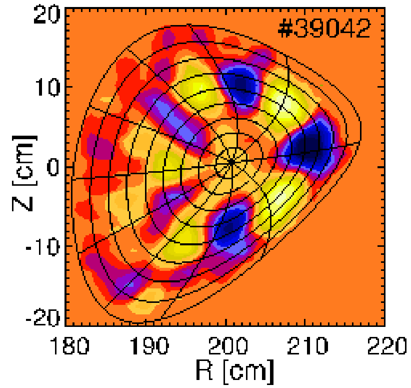
i.e. well known resonances at $v_0 = v_A$ and $v_0 = v_A/3$ for a Tokamak TAE

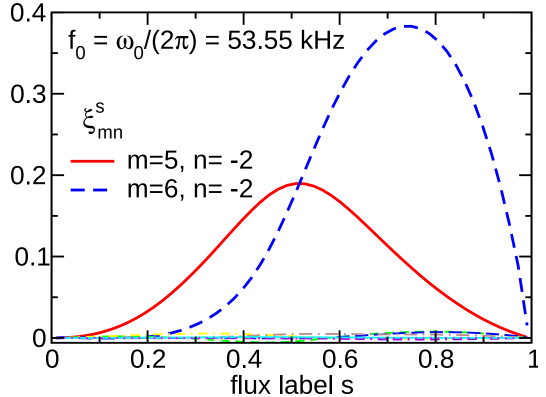
TAEs in W7-AS (#39042) and W7-X

W7-AS

A. Weller et al., Phys. Plasmas, **8**, 931 (2001):





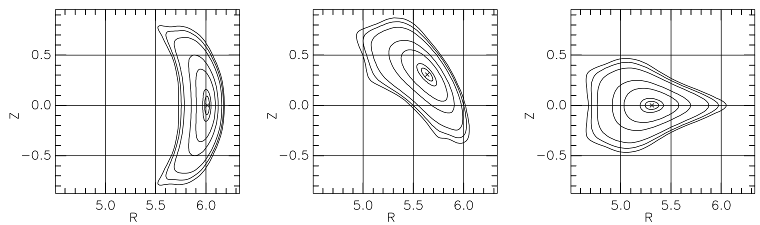


W7-X

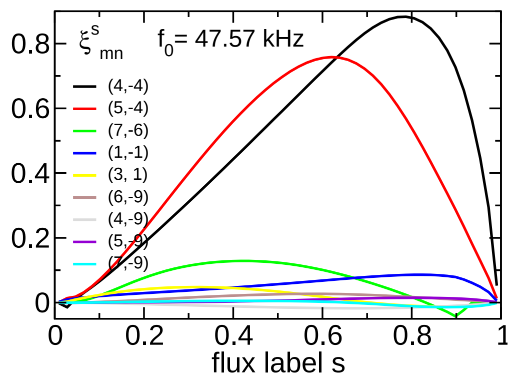
equilibrium:

M. Drevlak et al., Nucl. Fusion, 45, 731 (2005):

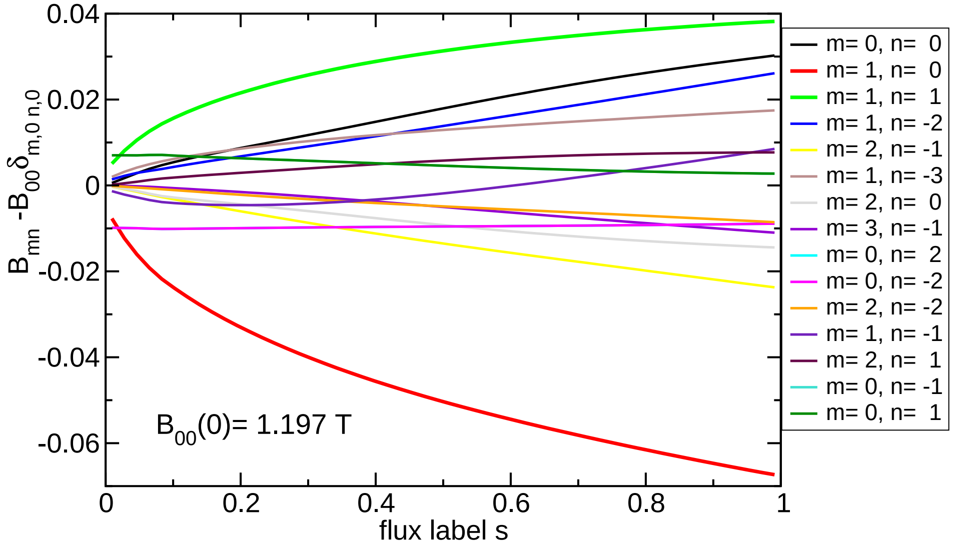
from PIES calculation: practically island free

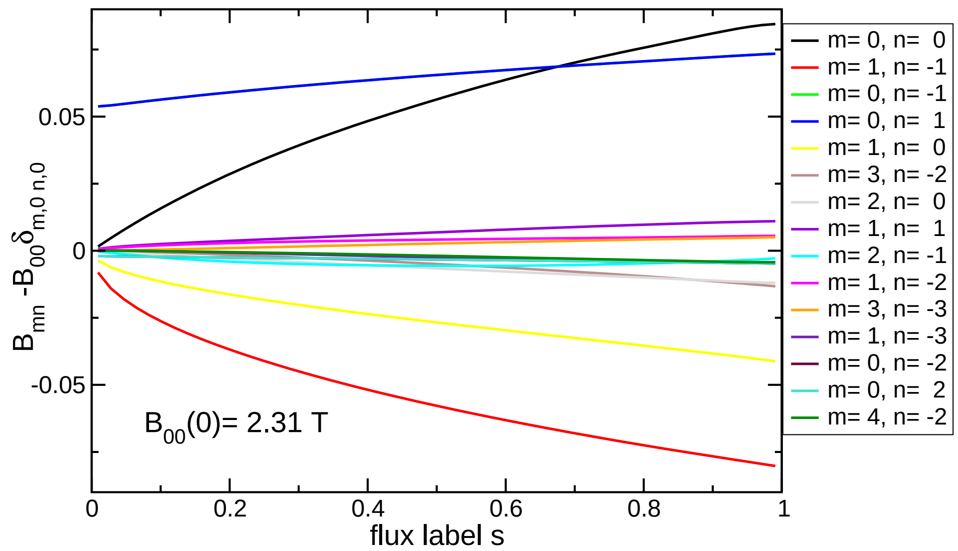


mode:



extract possible coupling from B spectrum



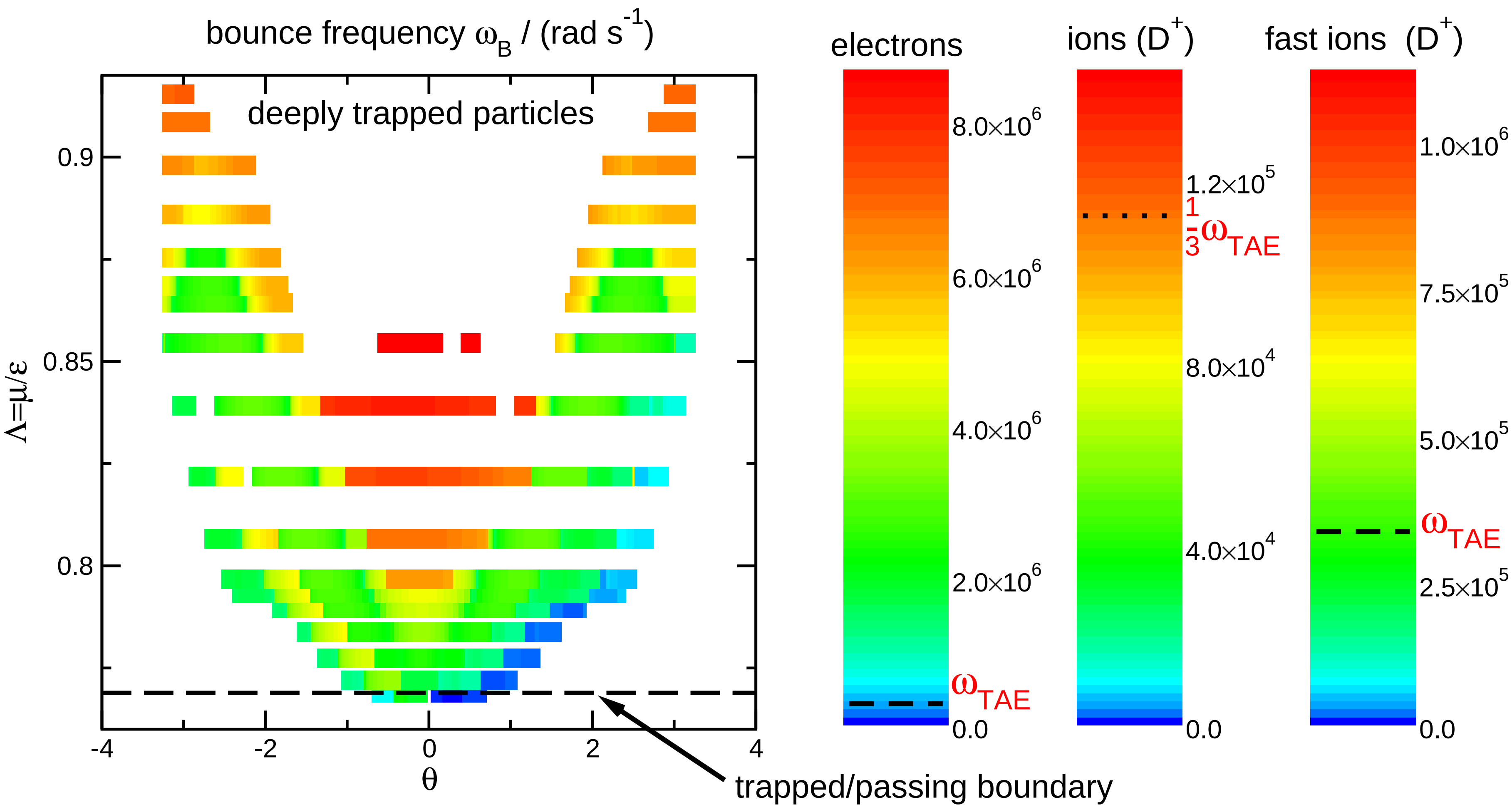


W7-AS **W7-X**

for both discharges:

NBI injection with a slowing down distribution

W7-AS: influence of reflected particles



reminder: Kinetic contribution

particle- wave- energy- exchange by resonant interaction

$$\delta W_s = \frac{\pi}{M_s^2}\left\{\sum_\sigma\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\sum_{\substack{n,m\\n',m'}}\sum_{p=-\infty}^{\infty}e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\times$$

$$\times\left(\frac{\partial F_s}{\partial\epsilon}\right)_\mu\frac{\omega-2\pi(\frac{n}{N_p}J-mI)\omega^*}{m\langle\omega_d^\vartheta\rangle+\frac{n}{N_p}\langle\omega_d^\varphi\rangle+\left\{\begin{matrix}\sigma(p+nq)\\p\end{matrix}\right\}\omega_{\left\{\begin{matrix}t\\b\end{matrix}\right\}}-\omega}L_{m'n'}^{(1)*}\mathcal{M}_{pn}^{m'n'*}L_{mn}^{(1)}\mathcal{M}_{pn}^{mn}$$

definition of $\mathcal{M}_{pn}^{m'n'}$:

for passing particles:

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for reflected particles:

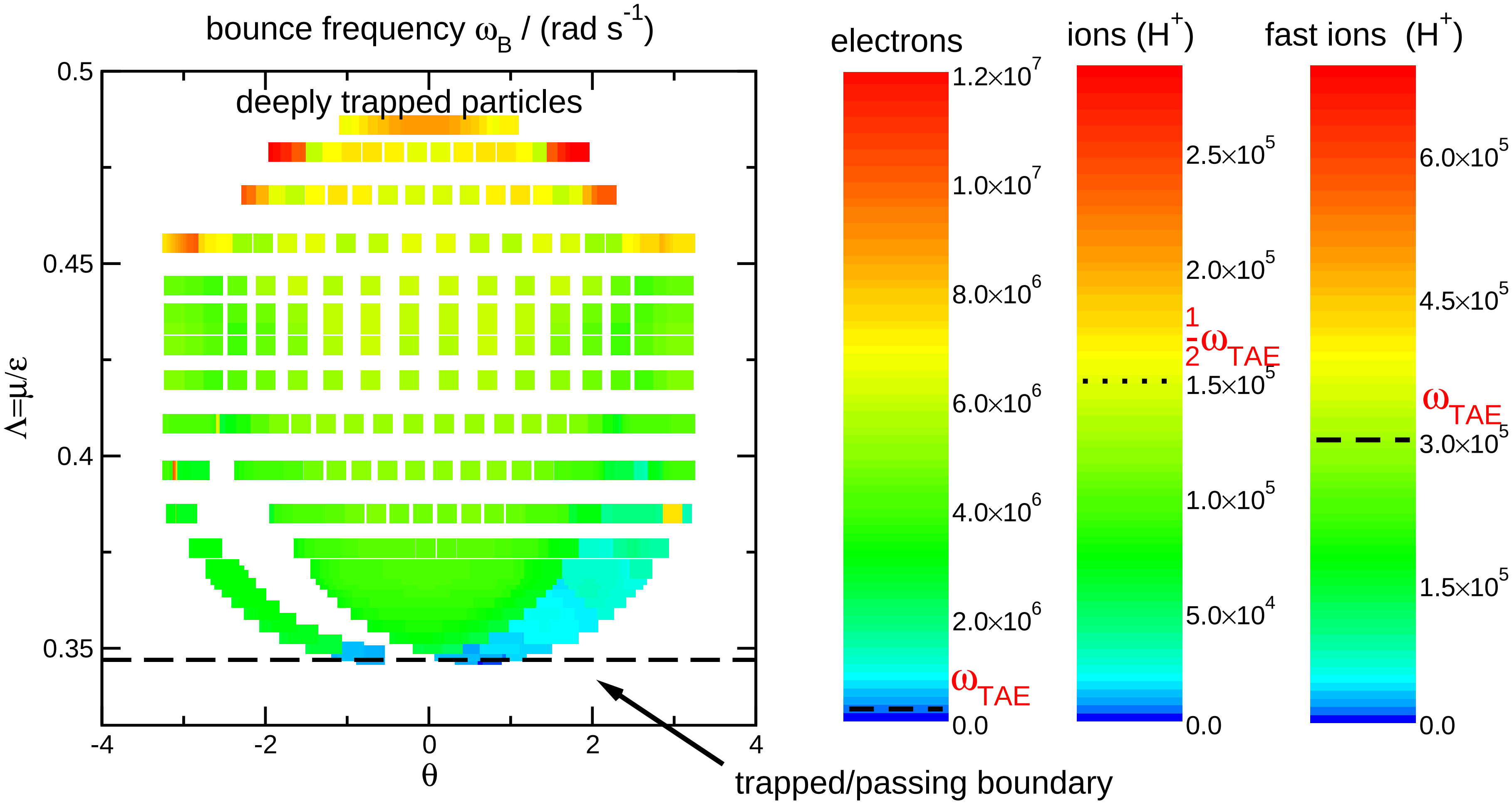
$$\mathcal{M}_{pn}^{m'n'} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}\cos(p\omega_b t'')\right\rangle_{\vartheta''}$$

$\langle\dots\rangle$ denotes the transit or bounce average

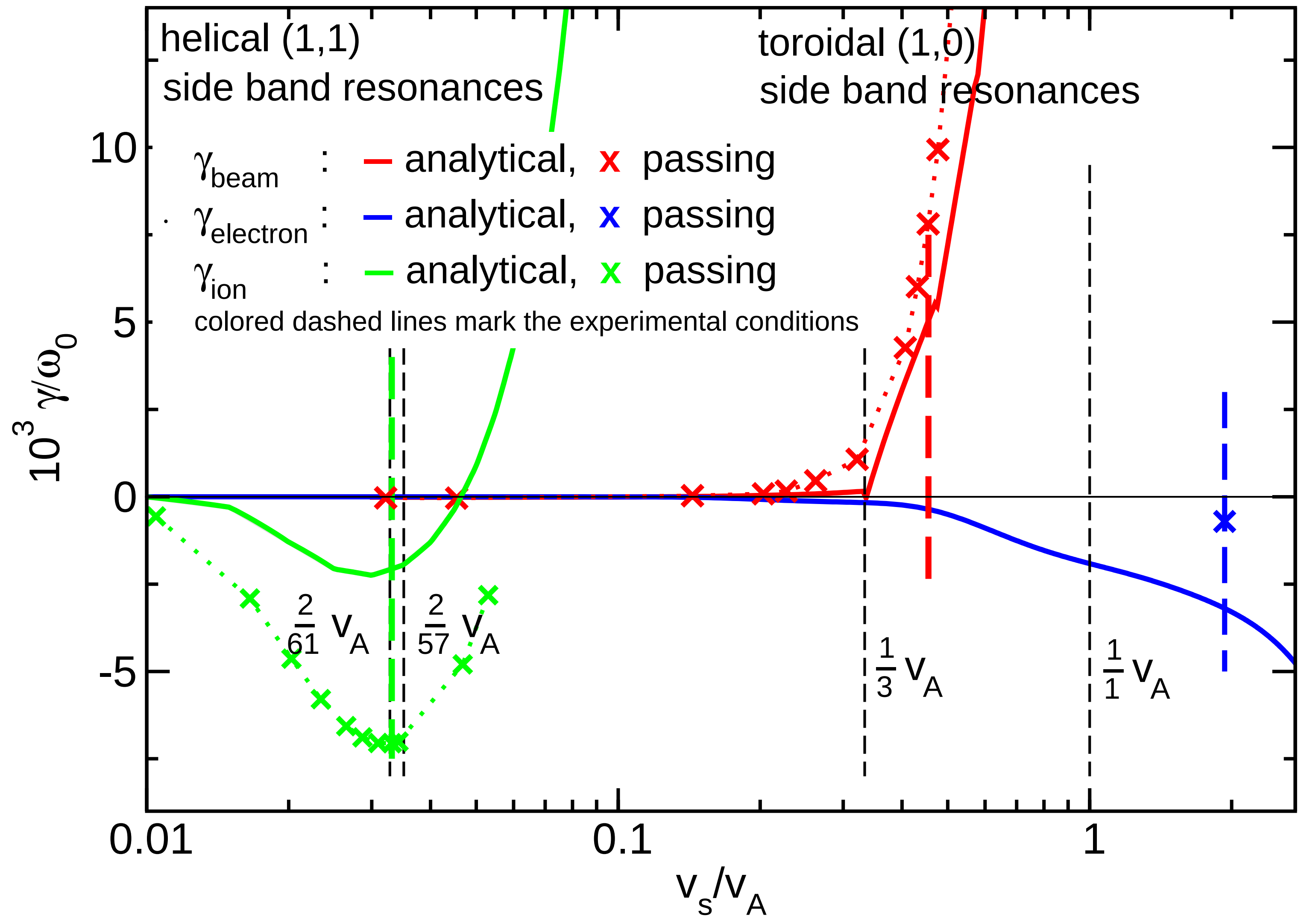
perturbed particle Lagrangian:

$$L^{(1)} = -(Mv_{\|}^2-\mu B)\vec{\xi}_\perp\cdot\vec{\kappa}+\mu B\vec{\nabla}\cdot\vec{\xi}_\perp$$

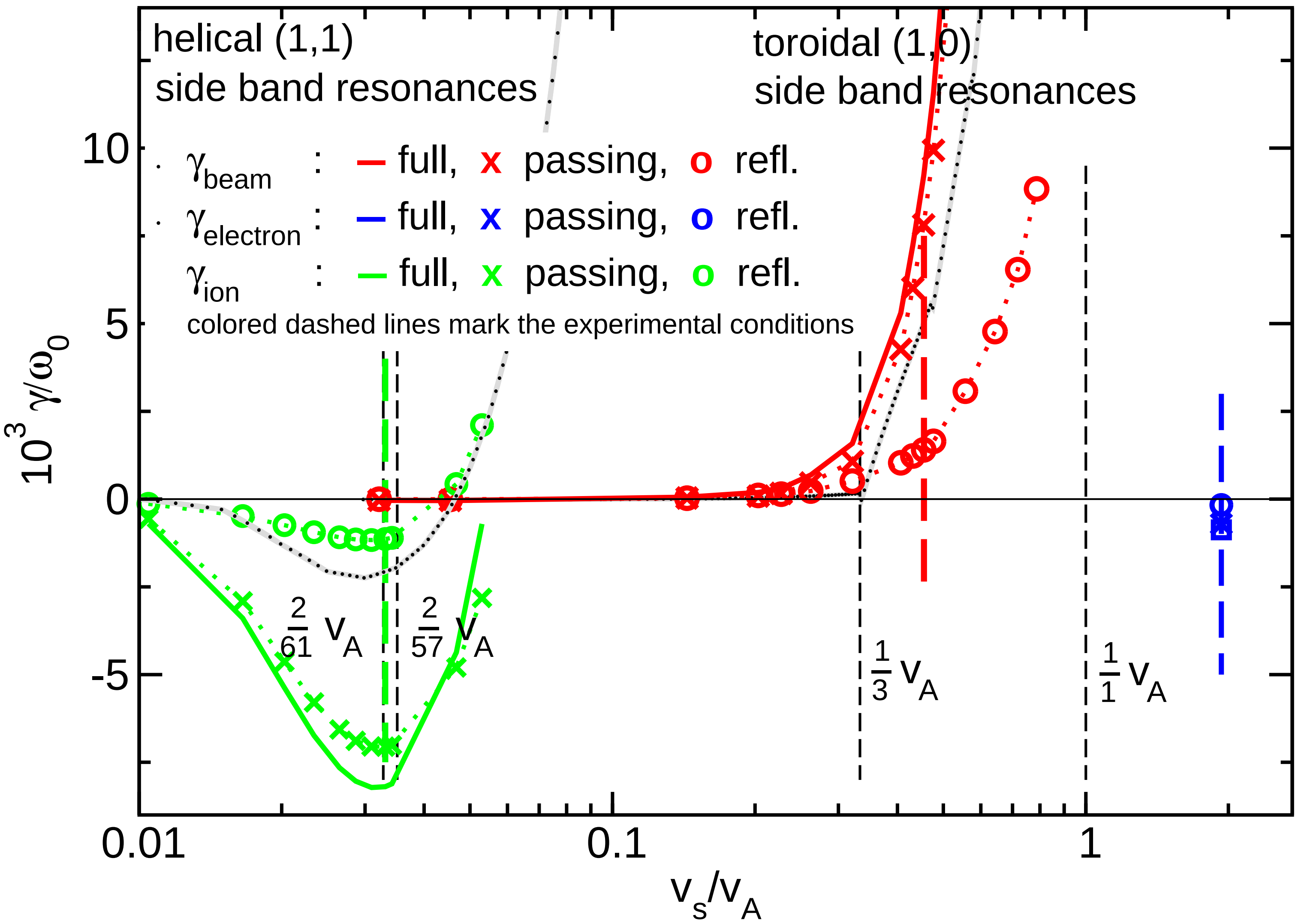
W7-X: influence of reflected particles



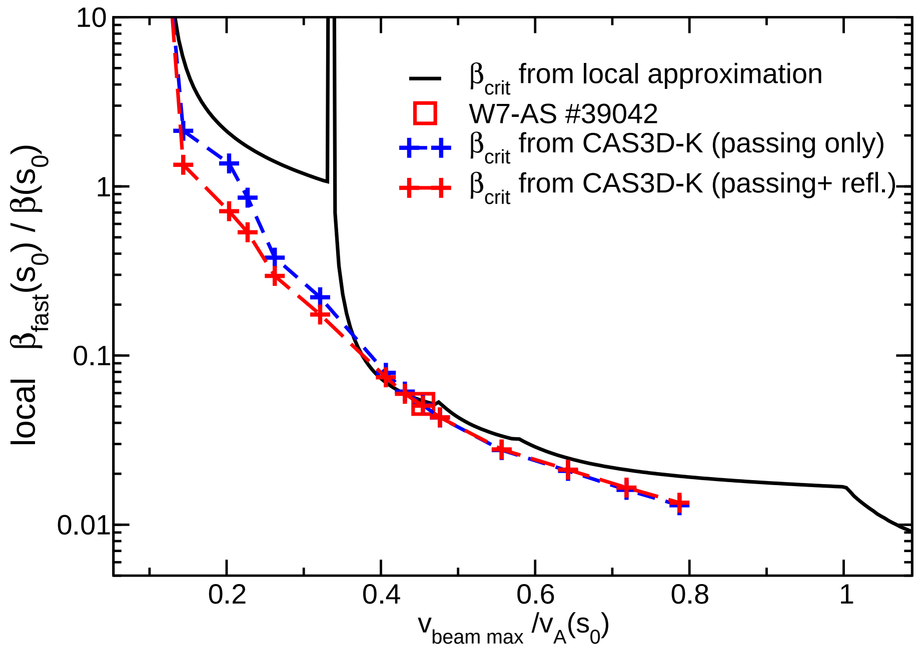
growth and damping rates for TAE in #39042



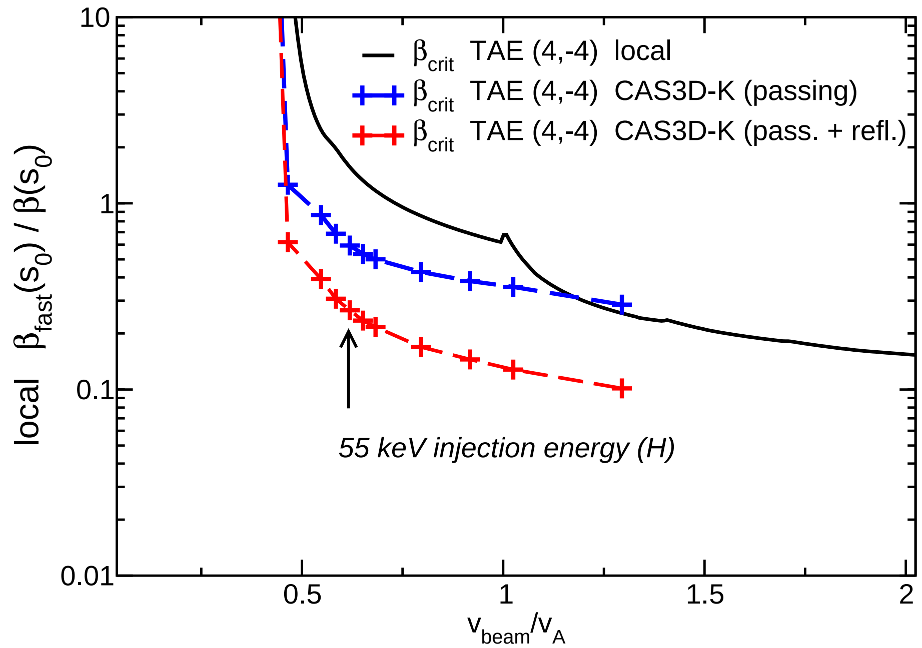
W7-AS: influence of reflected particles



stability diagrams/ critical β

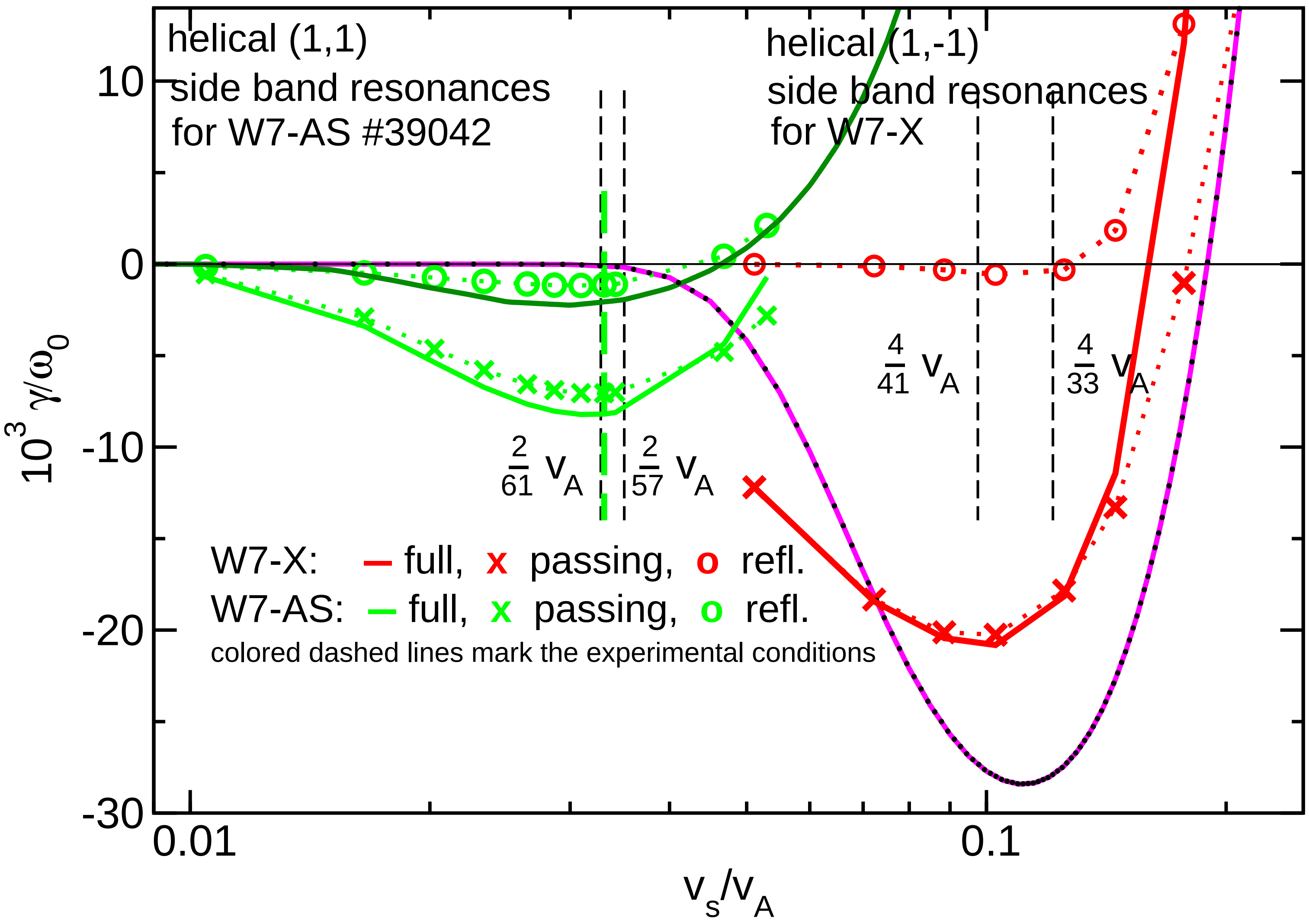


(5,-2),(6,-2) TAE in W7-AS



(4,-4),(5,-4) TAE in W7-X

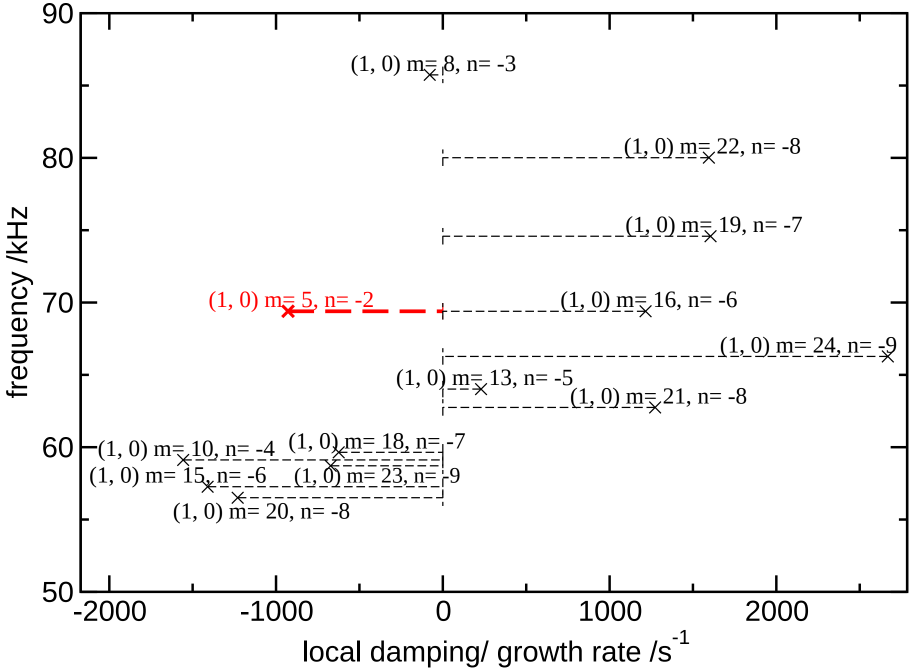
damping by thermal ions



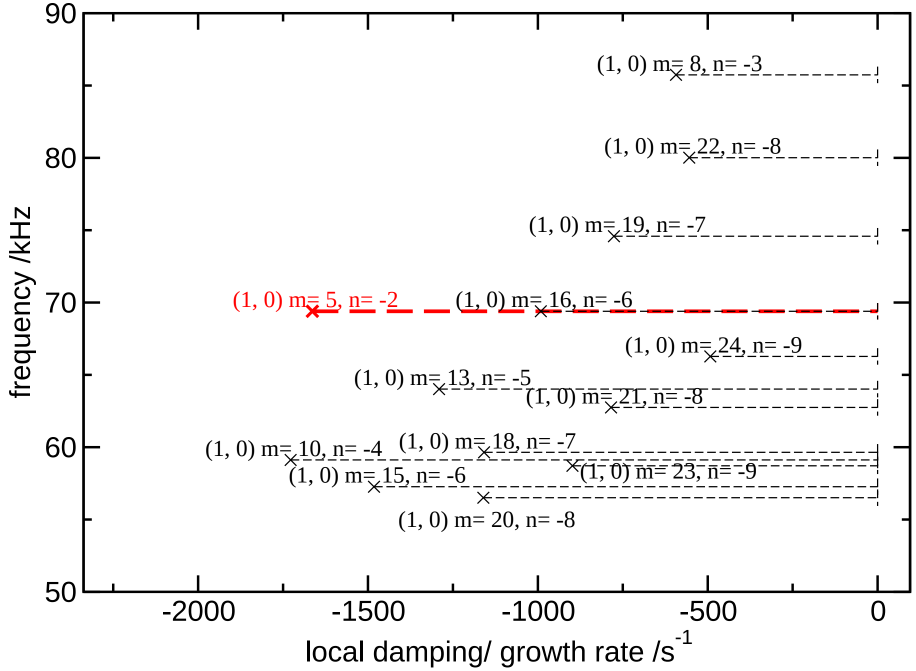
destabilization by temperature gradients

TAE mode frequencies and growth/ damping rates from a local computation

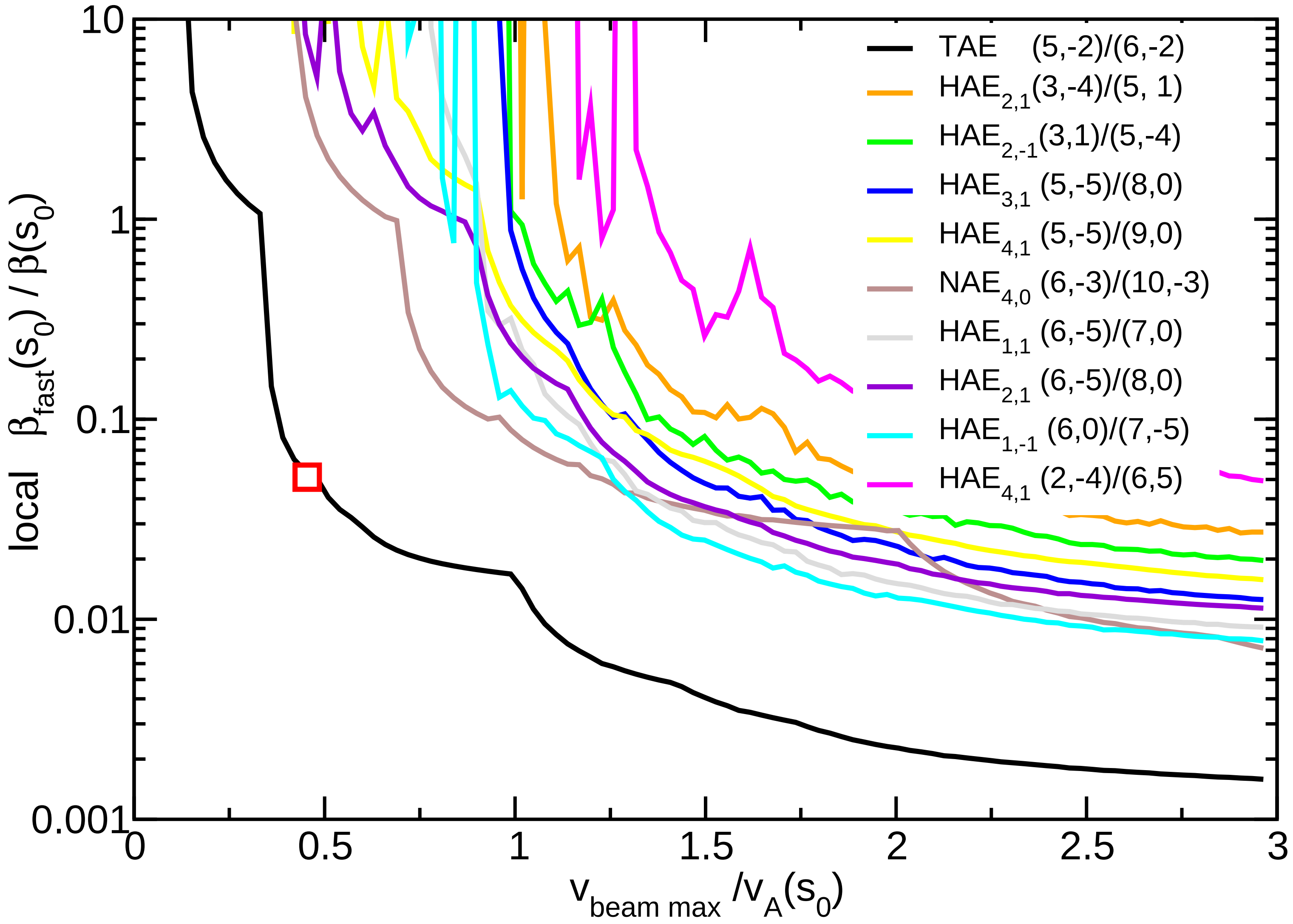
with a temperature gradient:



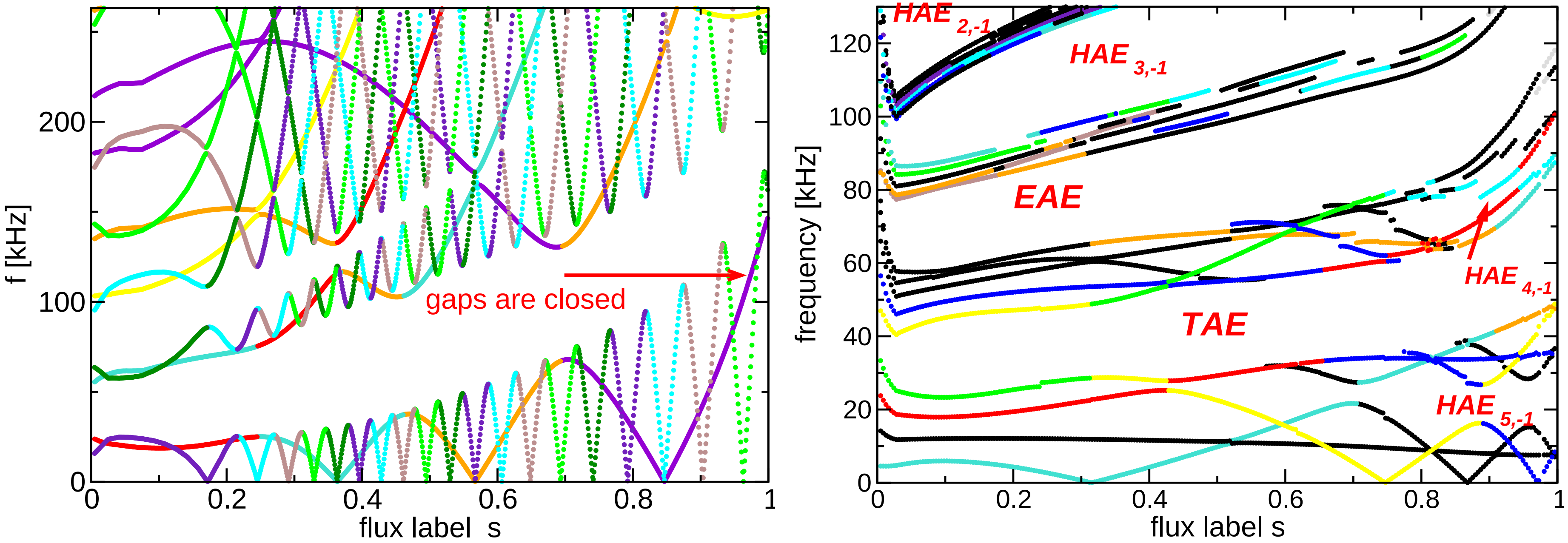
without a temperature gradient:



W7-AS: most unstable mode at given m (LGRO)



limitation of the model



LHD **W7-X**

continuum interaction ⇒ singularities in ideal MHD

include FLR/$E_{||}$ effects – under development

fully kinetic approach - PIC code (GYGLES)

- Global linear 2D fully gyrokinetic δf code
- Slab, pinch and tokamak geometries are available
- The code solves linearized gyrokinetic Vlasov-Maxwell system

$$\frac{\partial}{\partial t}\delta f + \{\delta f, H_0\} = -\left\{F_0, e\left\langle \phi - v_\parallel A_\parallel \right\rangle\right\}$$

$$n_i = n_e \ , \quad -\nabla_\perp^2 A_\parallel = \mu_0 (j_{\parallel i} + j_{\parallel e})$$

- The "Klimontovich" representation for the distribution function

$$\delta f = e^{iS(\vec{x})} \sum_{\nu=1}^{N_p} w_\nu \delta(z - z_\nu)$$

- The "Ritz-Galerkin" representation for the fields (using B splines)

$$\phi(\vec{x}) = e^{iS(\vec{x})} \sum_{k=1}^{N_{\rm FE}} \phi_k \Lambda_k(\vec{x}) \ , \quad A_\parallel(\vec{x}) = e^{iS(\vec{x})} \sum_{k=1}^{N_{\rm FE}} a_k \Lambda_k(\vec{x}) \ ,$$

Performance optimization for GYGLES

- load imbalance problem resolved:
  randomizing radial position of the markers over the different processors
- whole code structure reorganized to avoid redundant calculation of intermediate data (merging time integration)
- data structure of particle array adapted: focus on cache performance

⇒ speed-up of nearly a factor of three
parallel efficiency 97% up 4096 cores on Blue Gene/P (weak scaling)

details of numerical algorithm:
R. Hatzky, A. Könies, and A. Mishchenko, J. Comp. Phys. 255, 568 (2007)
(GYGLES originally written by M. Fivaz (Lausanne) 1998)

cancellation problem

parallel Ampére's law:

$$\left(\frac{\beta_i}{\rho_i^2} + \frac{\beta_e}{\rho_e^2} - \nabla_\perp^2\right) A_\parallel = \mu_0 \left(\bar{j}_{\parallel i} + \bar{j}_{\parallel e}\right)$$

gyro-center current:

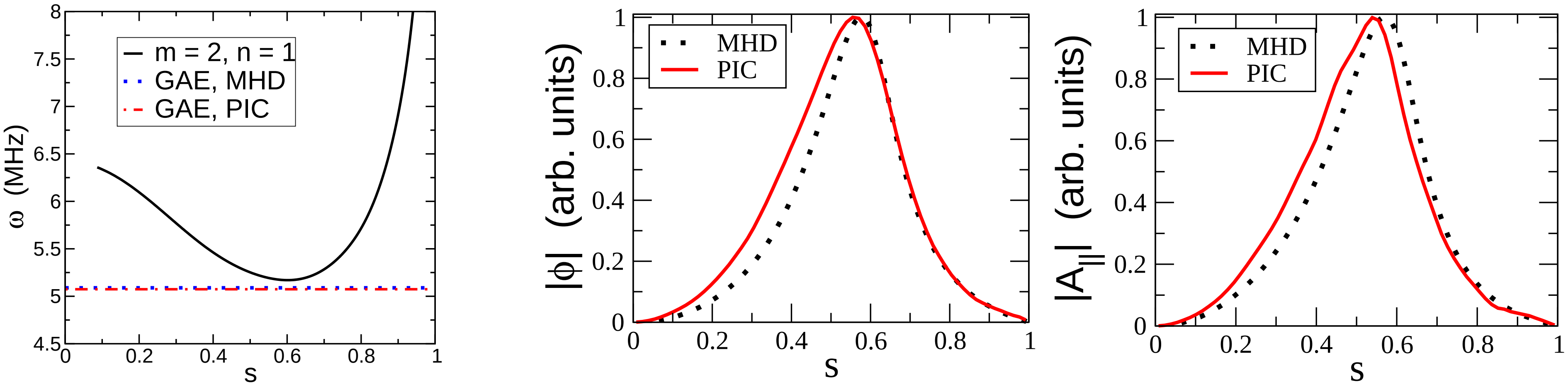
$$\bar{j}_{\parallel s} = q_s \int \mathrm{d}f^6 Z\, F_s\, v_\parallel\, \delta(\mathrm{R} + \rho - \mathrm{x})$$

thermal gyro-radius: $\rho_s = \sqrt{m_s T_s}/(eB)$
plasma beta: $\beta_s = \mu_0 n_0 T_s / B_0^2$

parts of $\bar{j}_{\parallel s}$ (noisy) have to cancel analytic $\frac{\beta_i}{\rho_i^2} \gg k_\perp^2$

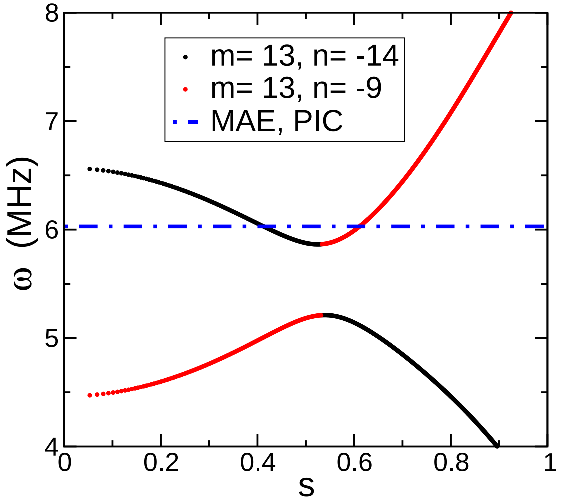
Note: In practice, in the quasi-neutrality equation and in Ampére's law the finite extent of the velocity space has taken into account.

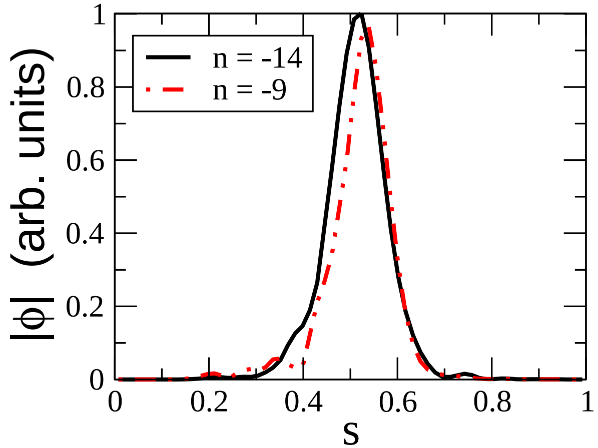
cylindrical GAE from the GYGLES code

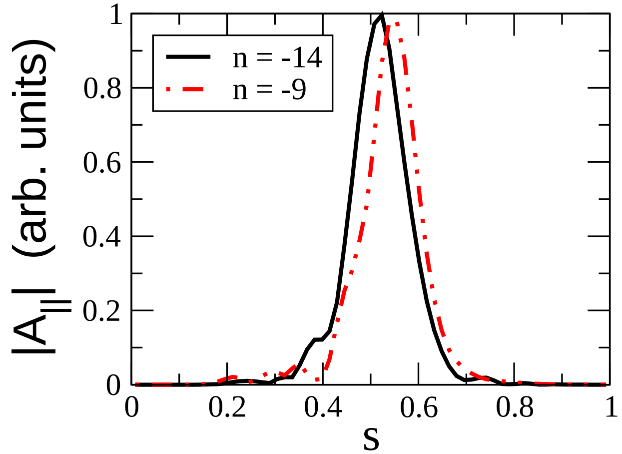


$5 \cdot 10^5$ ions,
$5 \cdot 10^5$ electrons,
ideal GAE mode resides below the Alfven continuum (calculated with the CONTI code),
very good agreement between MHD and kinetic result

Electro-magnetic gyro-kinetic PIC simulation



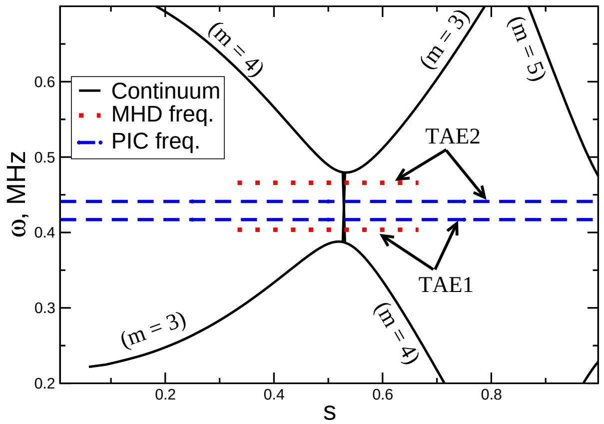


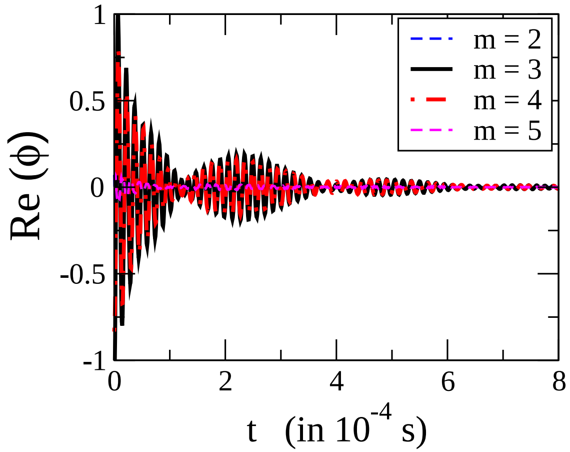


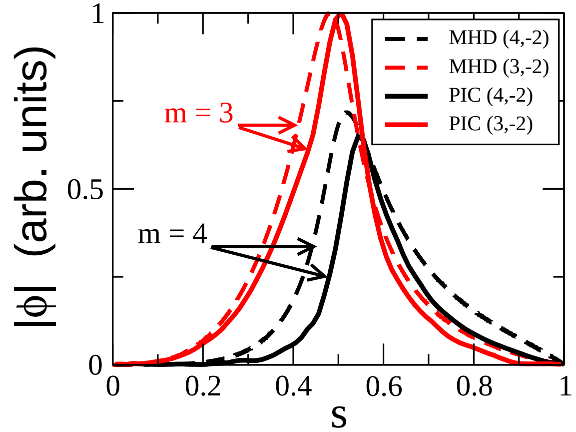
(kinetic) Mirror Alfven Eigenmode in a bumpy pinch

$4 \cdot 10^6$ ions, $4 \cdot 10^6$ electrons,
no modes have been found with the ideal MHD codes: CKA and CAS3D

Electro-magnetic gyro-kinetic PIC simulation (tokamak)







Toroidal Alfven Eigenmode in a tokamak (GK PIC against MHD)

time signal analysis (Prony's method):

TAE coexists with other Alfven modes (e.g. KTAE)

Late stage: being least damped, even TAE dominates the spectrum

$4 \cdot 10^6$ **ions,** $16 \cdot 10^6$ **electrons,**

(details see Poster of Mishchenko et al.)

Summary

- numerical tools for wave-particle interaction in 3D:
  - CONTI: ideal MHD, Alfvén + sound continuum
  - LGRO: local growth rates for 3D gap modes
    * for high plasma beta, the TAE may be driven unstable by thermal ions
  - CAS3D-K: linear drift-kinetic equation for reflected and passing particles, growth and damping rates, neoclassical application
    * stability diagrams shown allow a direct comparison with the experiment
    * damping is mainly due to ions and caused by the helical resonances (genuine 3D effect)
  - CKA: kinetic Alfvén waves, under construction ideal MHD runnig
- first principle numerical ansatz for Alfvén waves in 2D:
  - GYGLES: linear electro-magnetic global gyro-kinetic PIC code damping rates, mode structure of Alfvén modes (pinch, bumpy pinch, tokamak)

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