Multi-scale interactions among micro-turbulence, macro-MHD, and zonal flow

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THEORY OF FUSION PLASMAS
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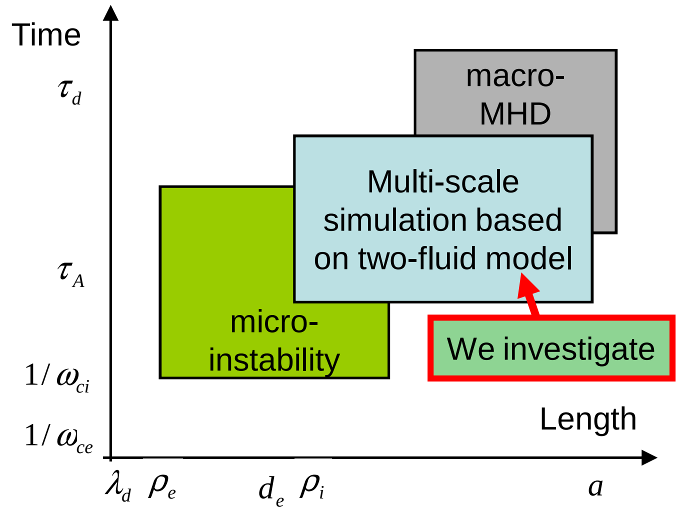
1. Background and motivation

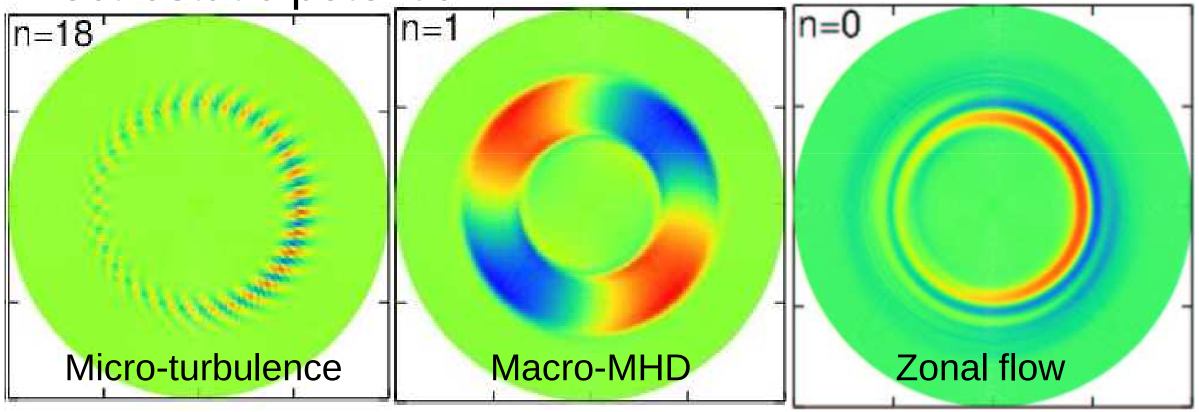
- Effects of MHD instabilities and micro-turbulence on plasma confinement have been investigated separately.
- But these instabilities usually appear in the plasma at the same time.

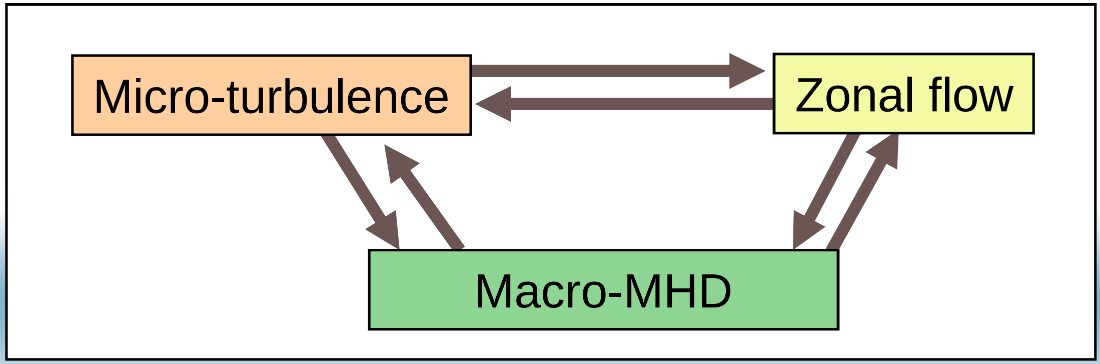


Background and motivation II

- Our goal is to understand multi-scale-nonlinear interactions among micro-turbulence, macro-scale-MHD instabilities and zonal flows.

Electrostatic potential





P.H. Diamond et.al., Phys. Fluids (1984)
D. Biskamp, Plasma Phys. Controlled Fusion (1984)
S.-I. Itoh et. al., Phys. Rev. Lett. (2003)
M. Yagi, et al., Nuclear Fusion (2005)
C.J. McDevitt et.al., Phys. Plasmas (2006)
A. Ishizawa and N. Nakajima, Phys. Plasmas (2007)

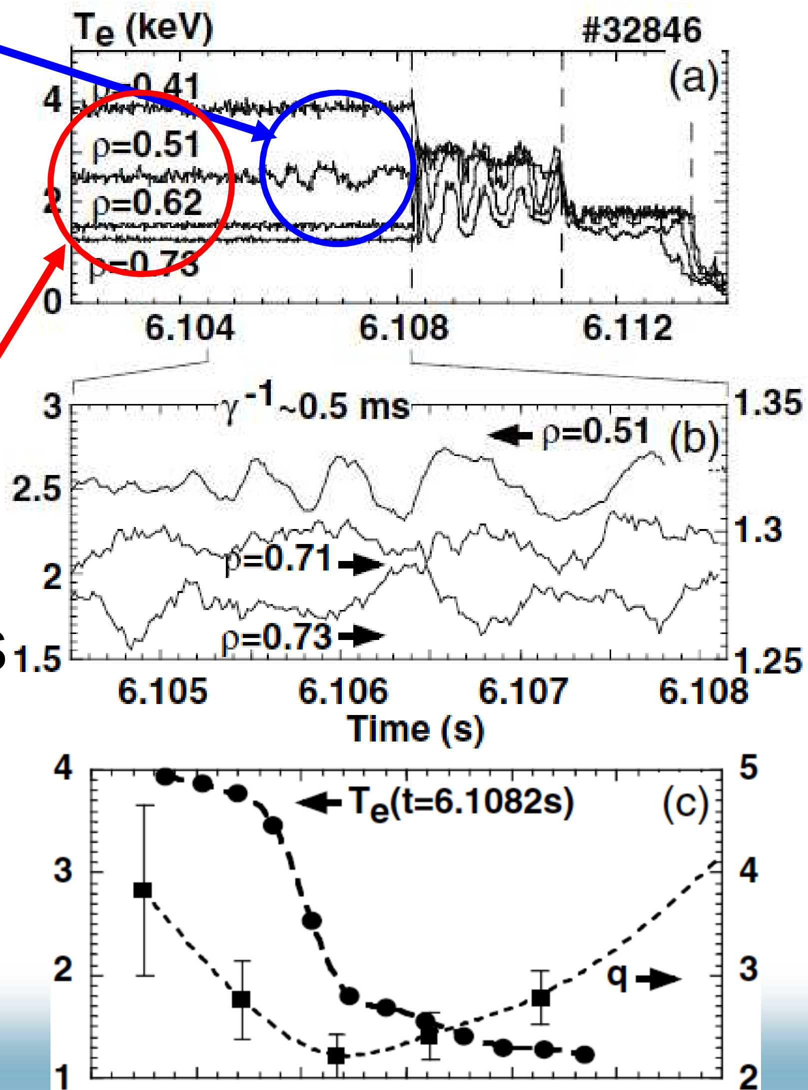
Background and motivation III

- In the experiments
  - Micro-turbulence is observed in Large Helical Device plasmas that usually exhibit MHD activities.

    K. Tanaka, et al., Nuclear Fusion (2006)
  - Macro-scale MHD activities are observed in reversed shear plasmas with a transport barrier related to zonal flows and micro-turbulence. Takeji, et.al., Nuclear Fusion (2002)
- In order to understand the macro-MHD instabilities raised above we consider the following situation. Micro-turbulence has already existed before macro-MHD instabilities arise, because micro-instabilities can appear without low q resonant surfaces. Then, macro-MHD mode appears in an equilibrium including turbulence and zonal flow.

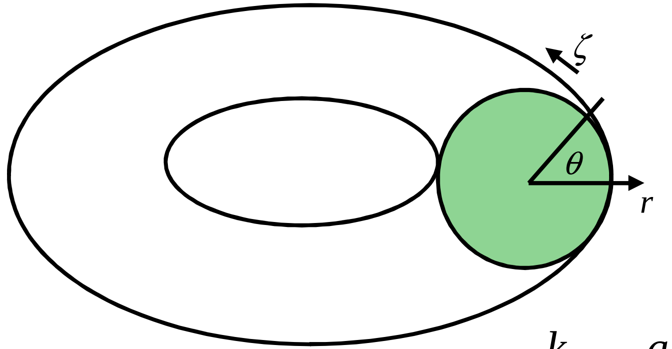
Background and motivation III

- MHD activities are observed in reversed shear plasmas with a transport barrier related to zonal flows and micro-turbulence.
- We will make an initial quasi equilibrium that corresponds to the equilibrium in the experiment. This equilibrium can be formed by a balance between micro-turbulence and zonal flow.



Takeji, et.al., Nuclear Fusion (2002)

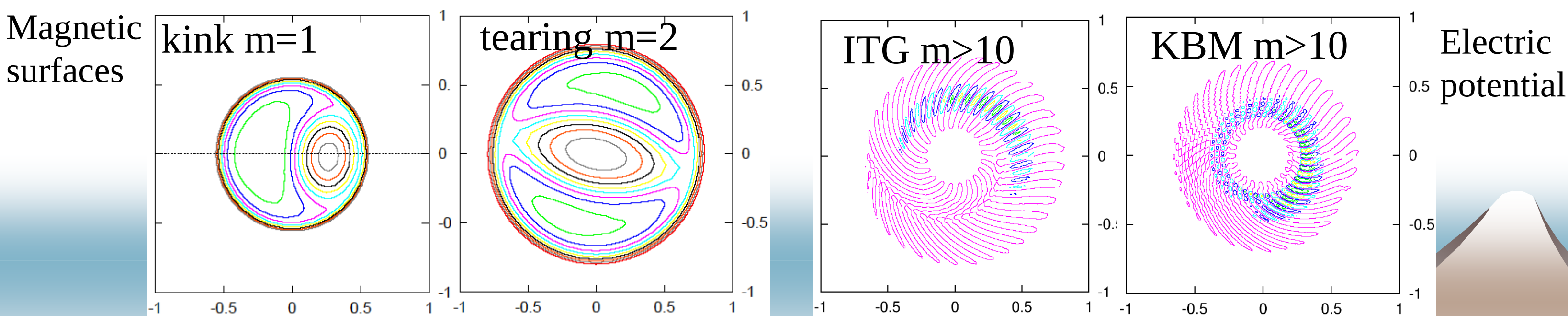
Reduced two-fluid equations



- Basic assumptions
  - Flute approximation
  - Large aspect ratio
  - High-beta ordering

$$\frac{k_{//}}{k_{\perp}} \approx \frac{a}{R} \approx \beta \approx \varepsilon$$

- Extends the standard four-field model by including temperature gradient effects.
- We can describe the nonlinear evolution of tearing modes, interchange modes, ballooning modes and ion-temperature gradient modes.



Reduced two-fluid equations

$$
\begin{aligned}
n_{eq}\frac{d\nabla_\perp^2\Phi}{dt} &= -\nabla_{||}J - K[p] + \tilde{a}\nabla_\perp\cdot[\nabla_\perp\Phi, p_i] + \mu_Q\nabla_\perp^2\nabla_\perp^2\Phi, \\
\frac{dn}{dt} &= -n_{eq}\nabla_{||}v_{e||} + K[n_{eq}\Phi - p_e] + \mu_n\nabla_\perp^2 n, \\
n_{eq}\frac{dv_{||}}{dt} &= -\nabla_{||}p + \mu_v\nabla_\perp^2 v_{||}, \\
\beta\frac{\partial\psi}{\partial t} &= -\nabla_{||}\Phi + \frac{1}{n_{eq}}\nabla_{||}p_e + \eta_L v_{e||} + \eta J, \\
\frac{dT_i}{dt} &= -(\Gamma-1)(T_{eq}\nabla_{||}v_{||} + \kappa_L T_i) \\
&\quad + T_{eq}K[(\Gamma-1)(\Phi + T_i + T_{eq}n/n_{eq}) + \Gamma T_i] + \mu_T\nabla_\perp^2 T_i,
\end{aligned}
$$

$df/dt = \partial f/\partial t + \tilde{a}[\Phi, f]$

$\nabla_{||}f = \epsilon\partial f/\partial\zeta - \beta\tilde{a}[\psi, f]$

$K[f] = 2\epsilon[r\cos\theta, f]$

$\kappa_L = \sqrt{\frac{8T_{eq}}{\pi}}\,\nabla_{||}$

$\eta_L = (\pi\tau m_e/2m_i)^{1/2}|\nabla_{||}|$

$J = \nabla_\perp^2\psi \quad \psi = \psi_{eq} + \tilde{\psi}/\tilde{a} \quad \Phi = \tilde{\Phi}/\tilde{a}$

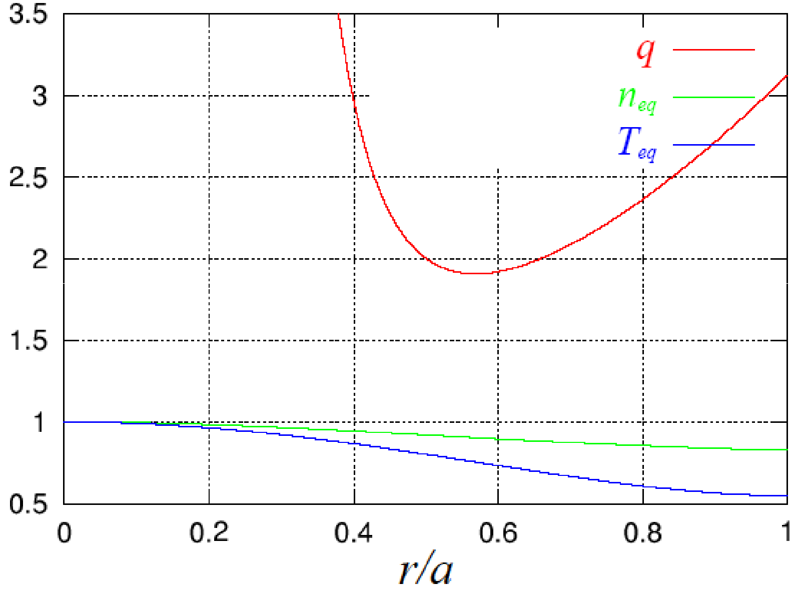
$p = p_i + p_e \quad p_e = \tau n_{eq}T_{eq} + \tau T_{eq}\tilde{n}/\tilde{a} \quad \tilde{a} = a/\rho_i$

$p_i = n_{eq}T_{eq} + T_{eq}\tilde{n}/\tilde{a} + n_{eq}\tilde{T}_i/\tilde{a}$

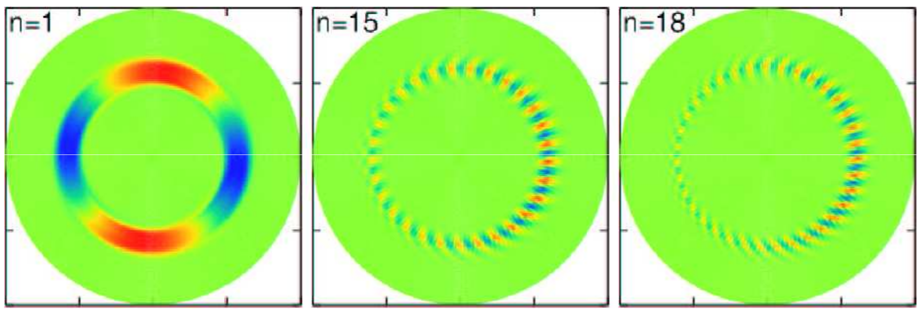
$v_{e||} = v_{||} + J/n_{eq} \quad n = n_{eq} + \tilde{n}/\tilde{a}$

$T_i = T_{eq} + \tilde{T}_i/\tilde{a}$

2. Generation of coherent mode by micro-turbulence



Linear eigen function



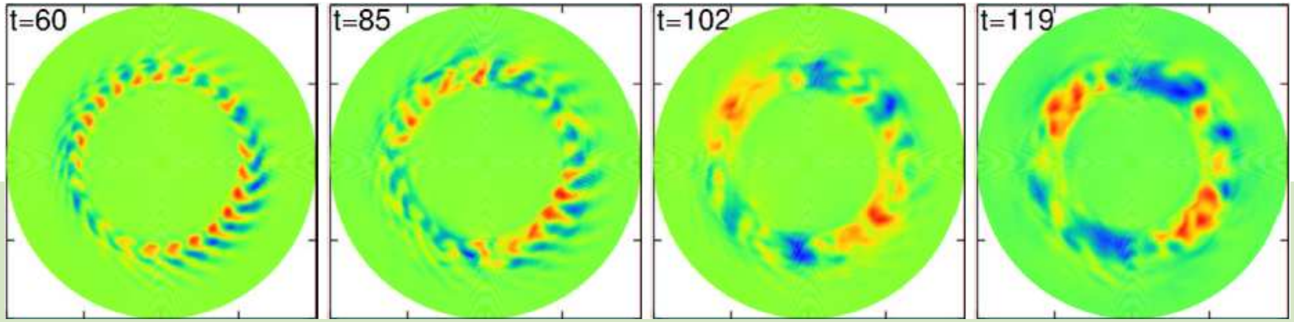
Macro-instability: double tearing mode
Micro-instability: Kinetic ballooning mode

$\beta = 0.01$ $\quad \nu = 2\times10^{-12} m^4$ $\quad N_m \times N_n \times N_r = 256\times128\times256$

$\rho_i / a = 1/80$ $\quad S = 1.6\times10^6$

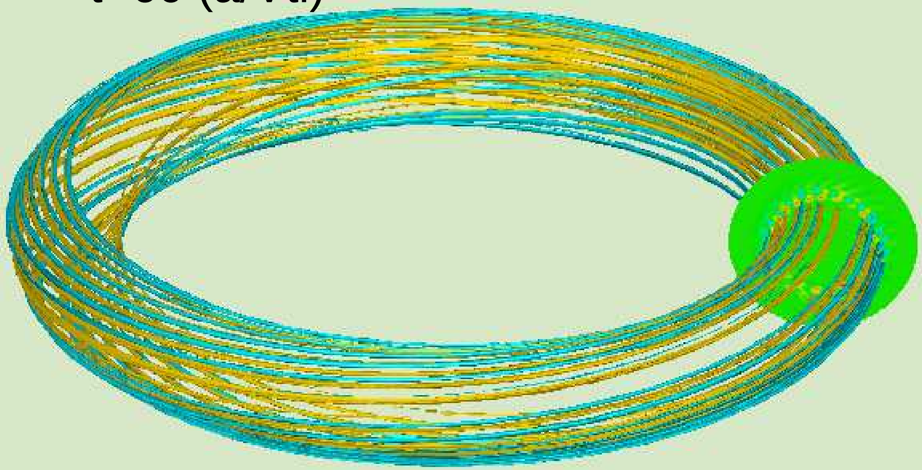
Appearance of macro-MHD mode in micro-turbulence

Electrostatic potential

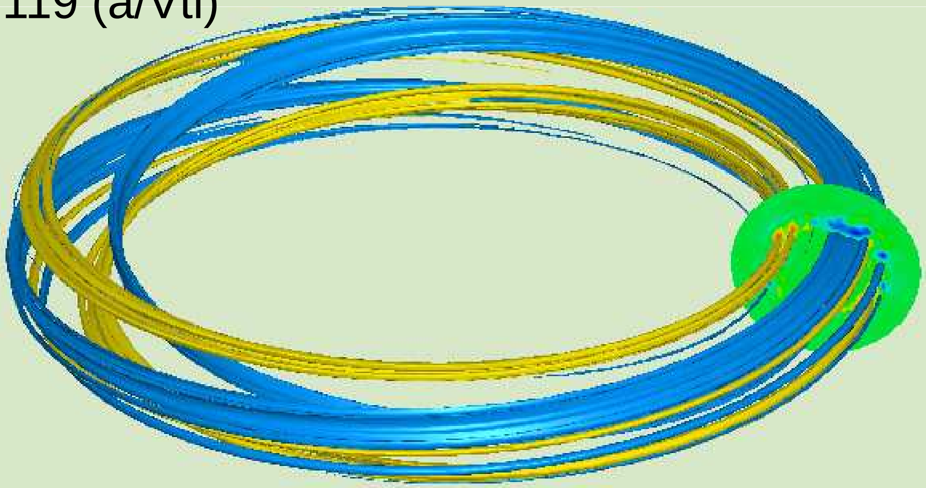


(m,n)=(2,1) macro-MHD appears

t=60 (a/Vti)

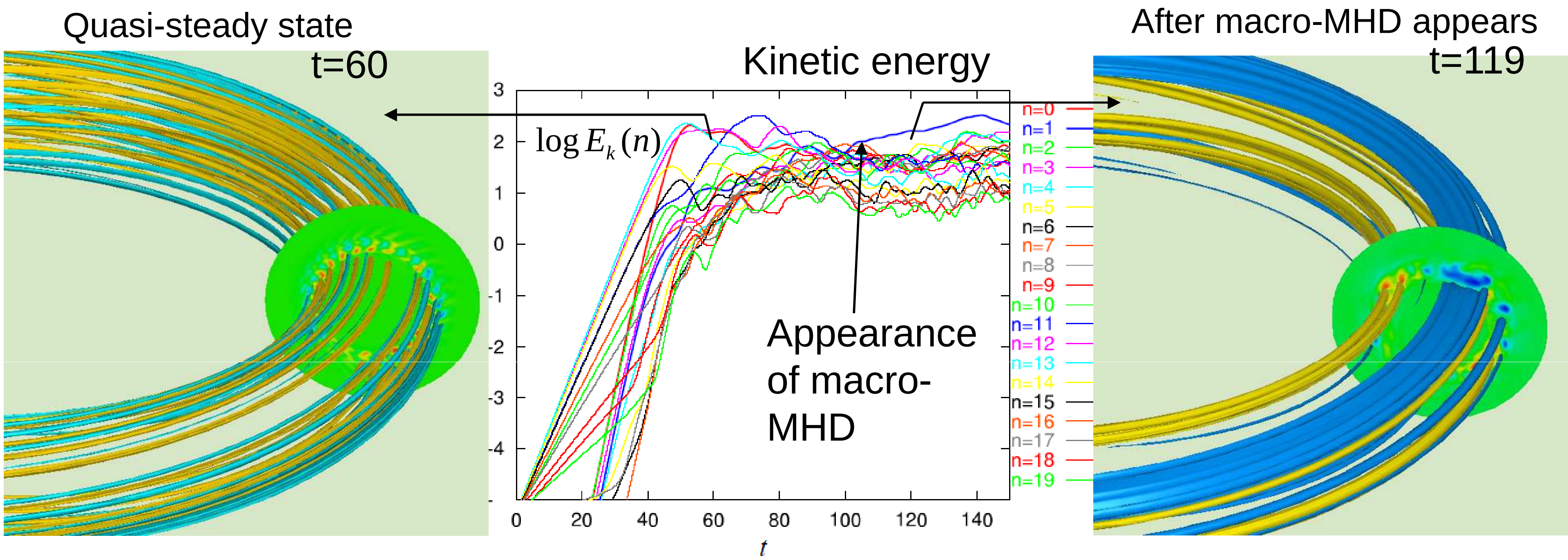


t=119 (a/Vti)



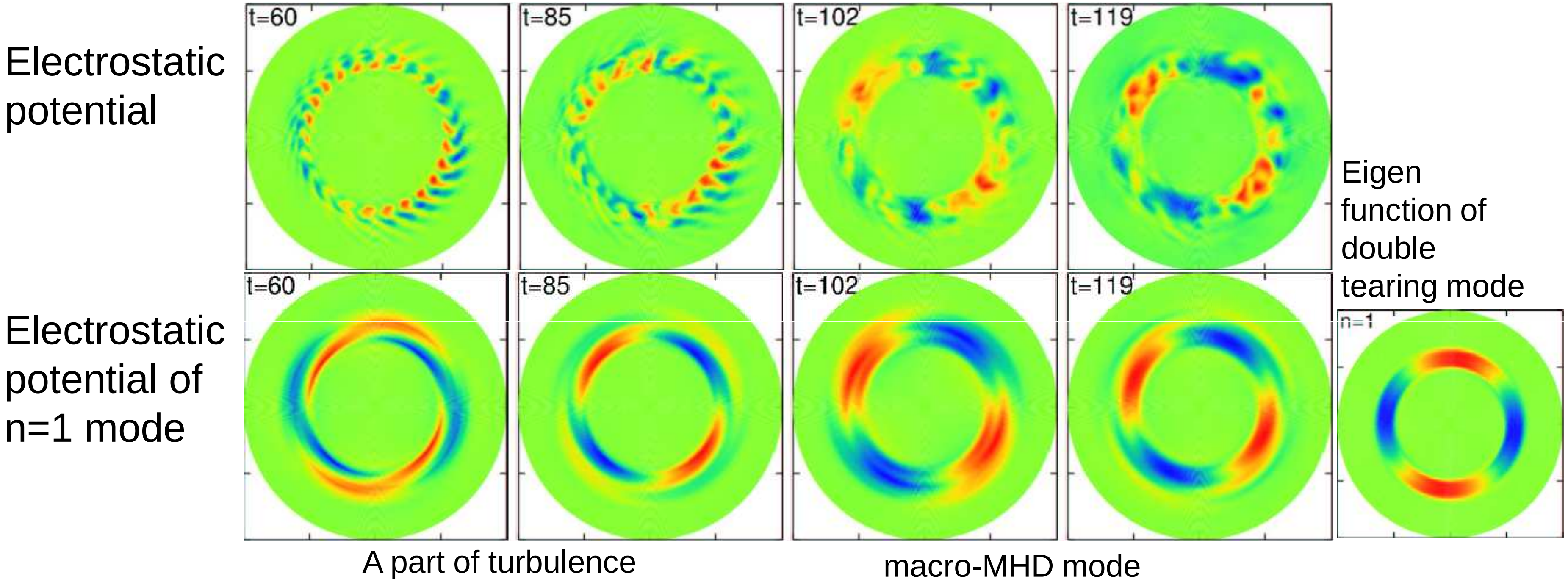
Ballooning structure of potential is altered to macro-MHD mode.

Effect of macro-MHD instability on micro-turbulence



The appearance of macro-MHD instability changes a turbulent state to another turbulence state.

Generation of n=1 mode by micro-turbulence

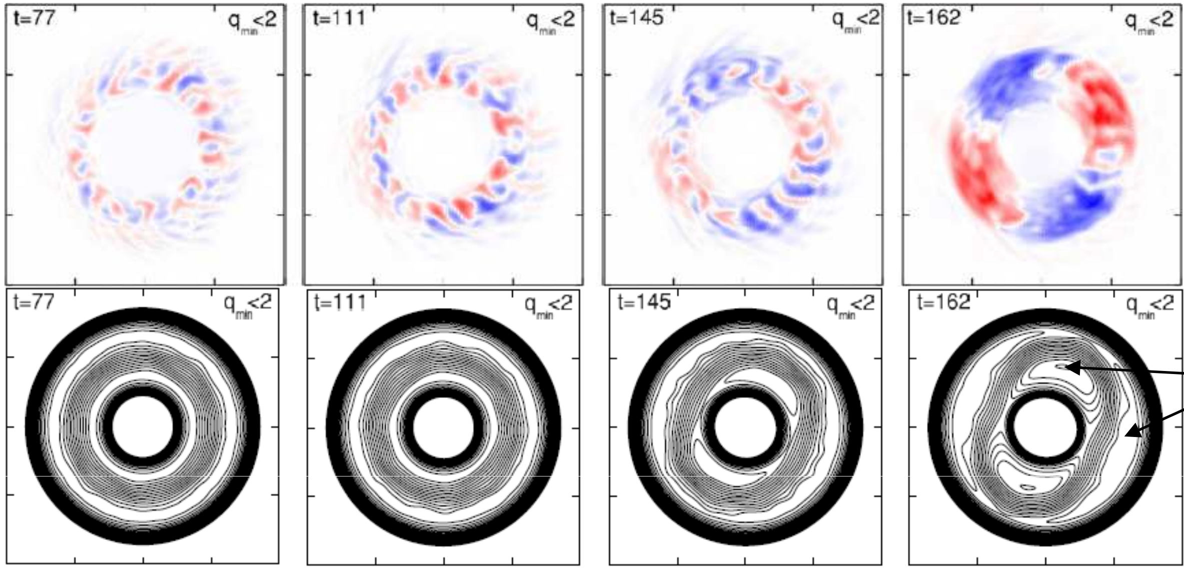


Nonlinear mode coupling of micro-scale turbulence causes macro-scale mode at t=60, but it is different from the macro-MHD instability because the spatial profile of induced macro-scale mode is significantly different from the profile of double tearing mode.

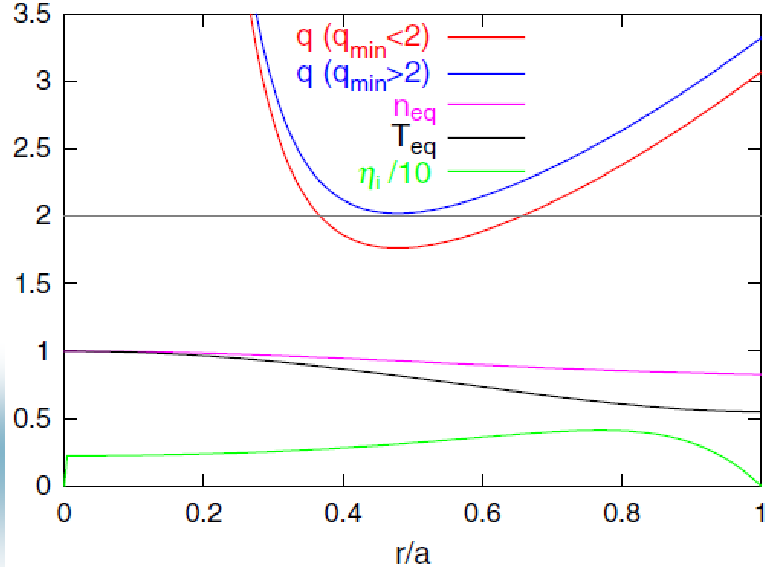
3. Energy transfer to coherent mode

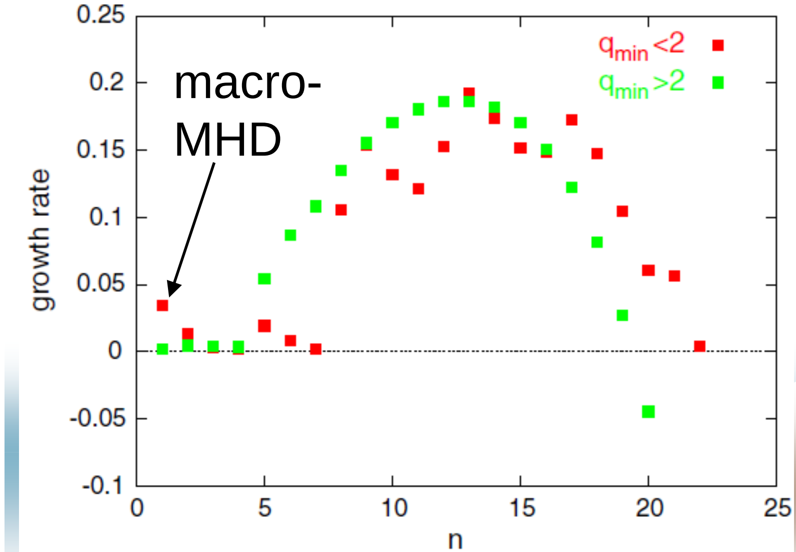
Electrostatic potential

Helical flux function of m/n=2

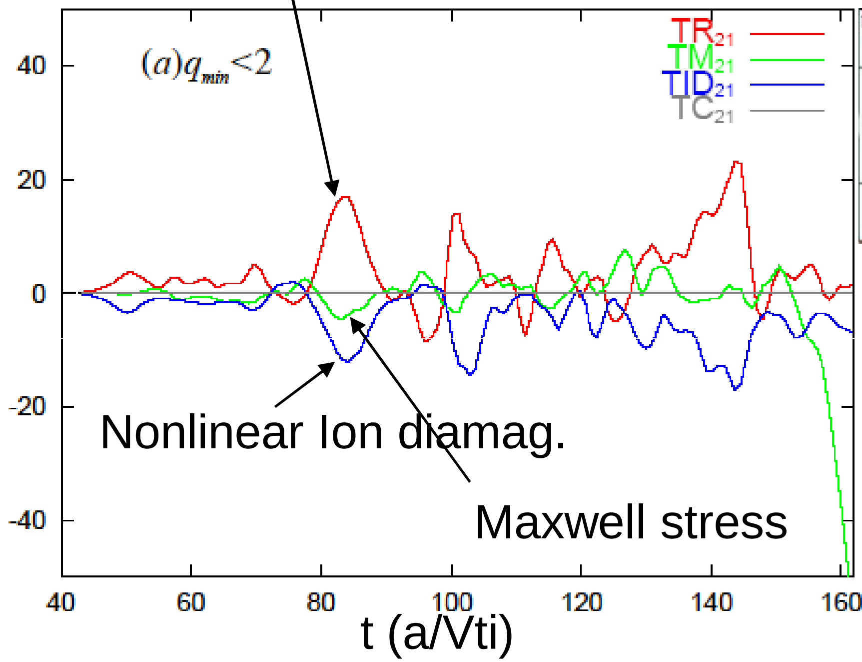


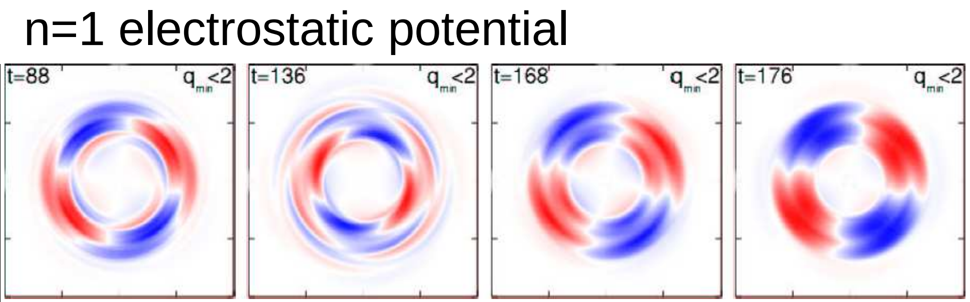
magnetic islands





Energy transfer to (m,n)=(2,1) coherent flow



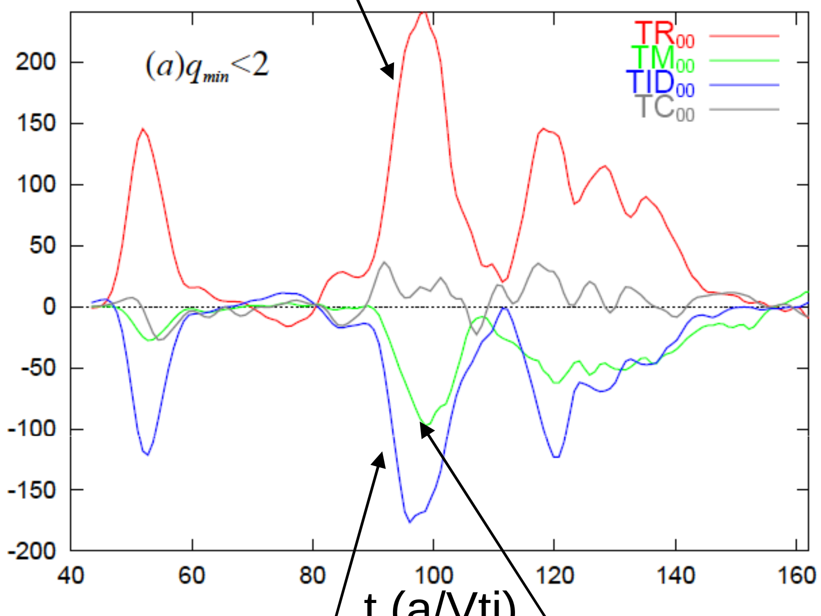


The Reynolds stress tends to drive the flow, while the ion diamagnetic term and the Maxwell stress counteract with the drive.

(m,n) component of EXB flow equation

$$\frac{d}{dt}\int d\mathbf{x}\frac{V_{Emn}^2}{2} = T_{mn}^R + T_{mn}^M + T_{mn}^{ID} + T_{mn}^L$$

Energy transfer to zonal flow



The Reynolds stress drives zonal flow. On the other hand, the ion diamagnetic term and the Maxwell stress suppress the drive.

(m,n) component of EXB flow equation

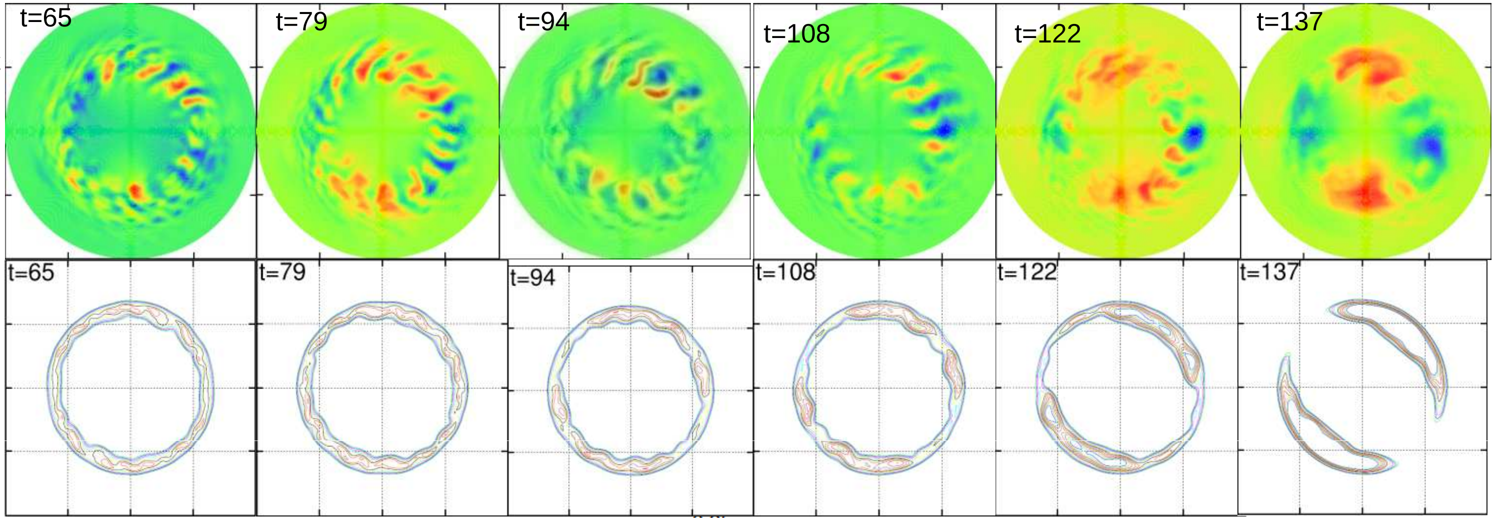
$$\frac{d}{dt}\int d\mathbf{x}\frac{V_{Emn}^2}{2} = T_{mn}^R + T_{mn}^M + T_{mn}^{ID} + T_{mn}^L$$

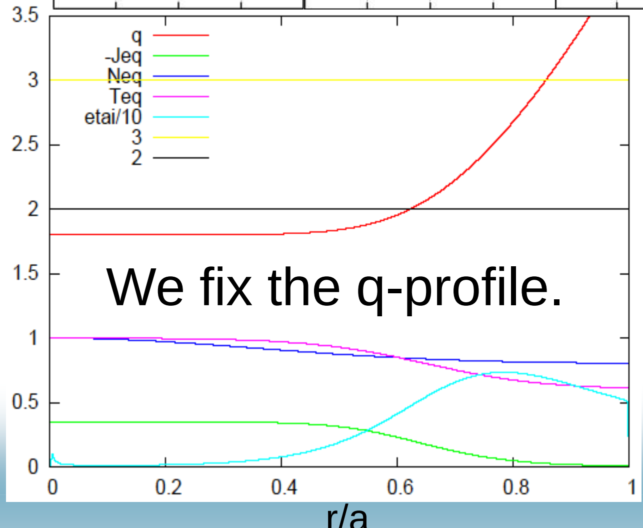
The energy transfer from micro-turbulence to macro-MHD flow is similar to the energy transfer from micro-turbulence to zonal flow. A. Ishizawa and N. Nakajima, Nuclear Fusion (2007)

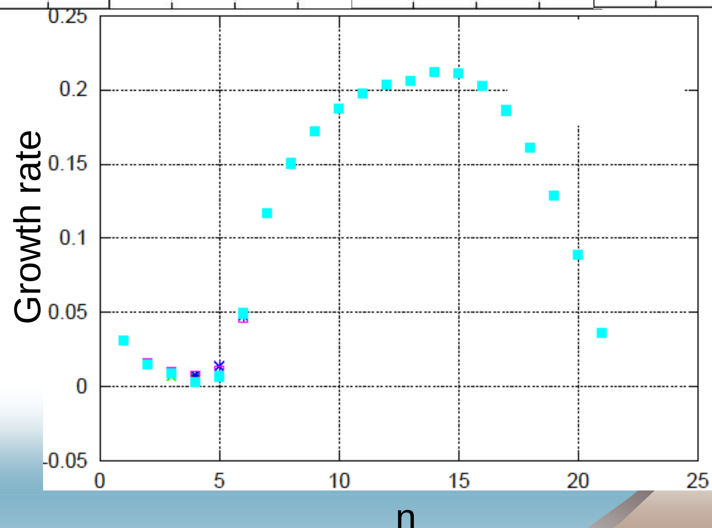
4. Effect of turbulent mixing on macro-MHD

Electrostatic potential

Helical flux function of m/n=2



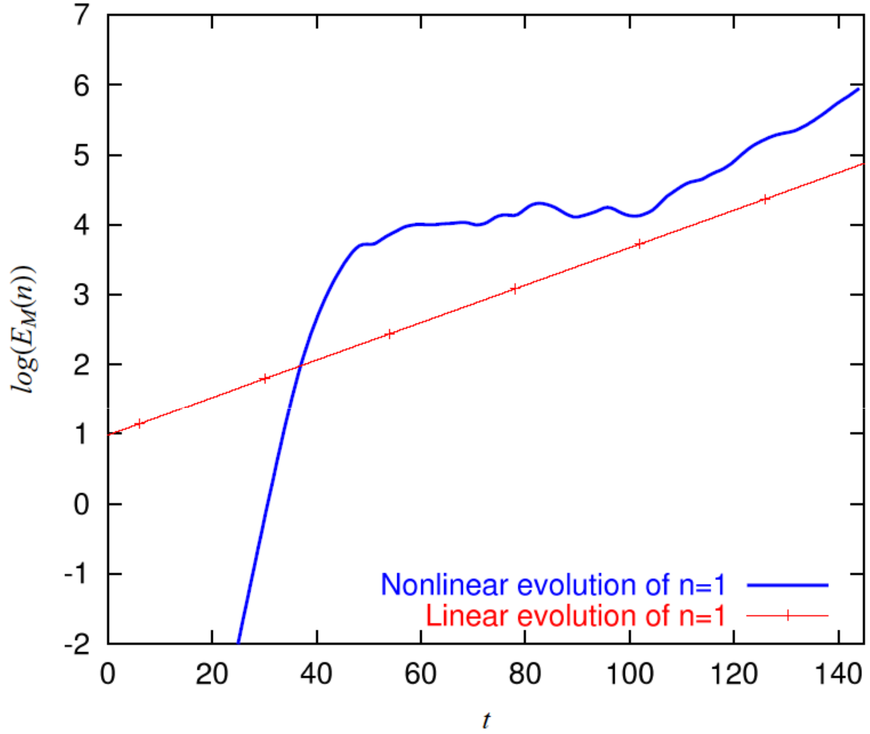




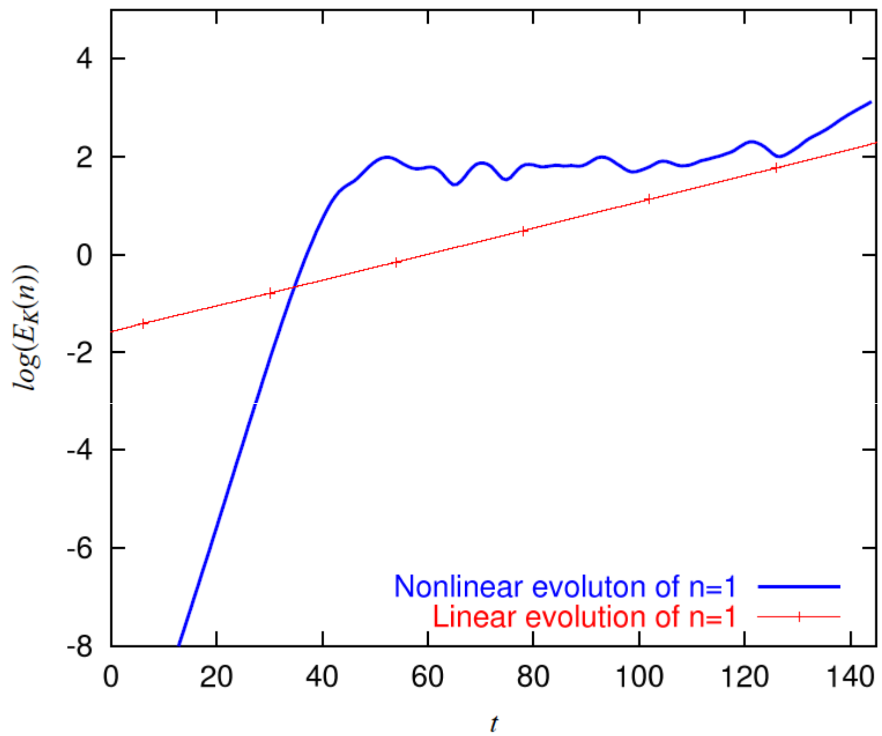
Micro-instability: Kinetic ballooning mode, Macro-instability: tearing mode

Growth of macro-MHD

Magnetic energy of n=1 mode



Kinetic energy of n=1 mode



Nonlinear n=1 mode grows faster than its linear growth. This suggests that micro-turbulence affects the growth.

Turbulent mixing of magnetic flux

$$\beta \frac{\partial \psi}{\partial t} = -\nabla_{//}\Phi + \frac{1}{n_{eq}}\nabla_{//} p_e + \eta_L v_{e//} + \eta J$$

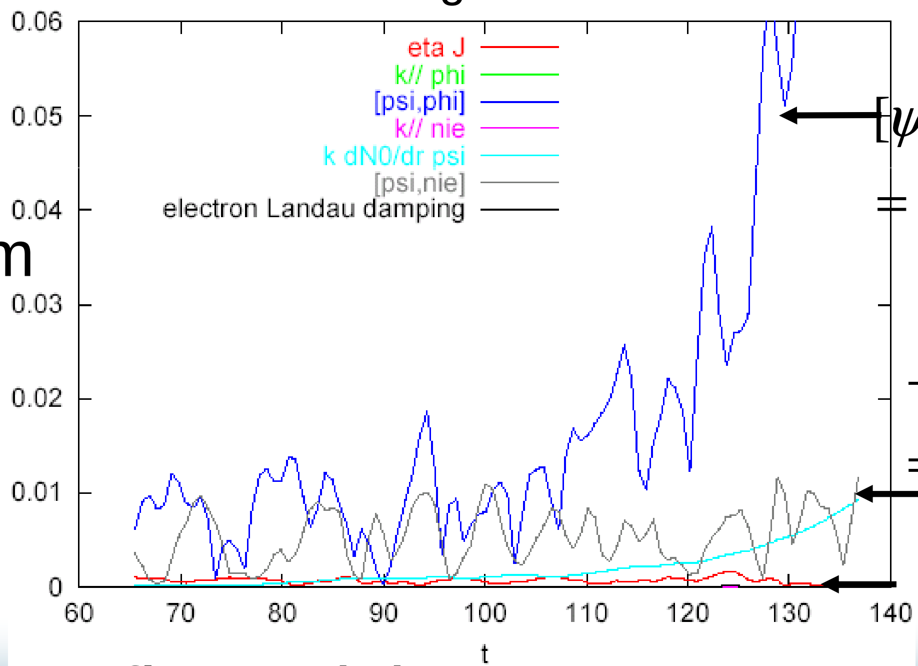
Equation of (m,n)=(2,1) mode on the resonant surface of tearing mode

$$\beta \frac{\partial \tilde{\psi}_{21}}{\partial t} = \underbrace{\beta[\tilde{\psi},\tilde{\Phi}]_{21} - \frac{\beta}{n_{eq}}[\tilde{\psi},\tilde{p}_e]_{21}}_{\text{Turbulent mixing terms}} + \underbrace{\beta[\tilde{\psi},\tilde{\Phi}_{00}]_{21} - \beta\tilde{a}[\tilde{\psi}, p_{eeq}]_{21}}_{\text{Rotation of island}} + \eta\tilde{J}_{21}$$

$$f = f_{eq} + \tilde{f}/\tilde{a}$$

$$\tilde{a} = a/\rho_i$$

Absolute value of each term



$$[\psi,\Phi]_{21} = -(\mathbf{v}_{ExB}\cdot\nabla\psi)_{21}$$

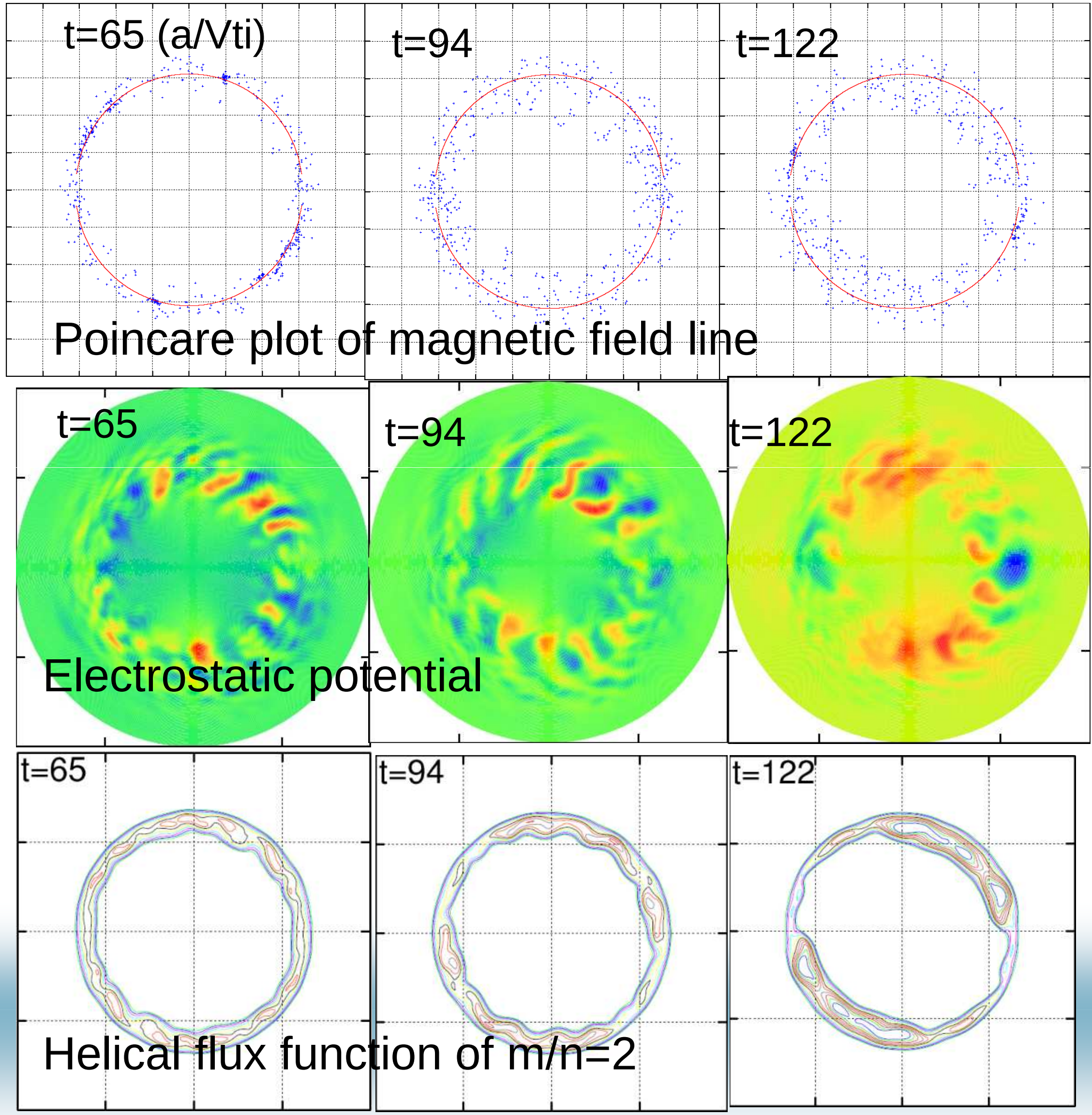
$$-[\psi, p_e]_{21} = (\mathbf{v}_{De}\cdot\nabla\psi)_{21}$$

Turbulent mixing

ηJ_{21} Normal resistivity

Turbulent flow mixing terms are much larger than normal resistivity on the resonant surface

Stochastic field lines: Small scale reconnections take place on the resonant surface



Magnetic surface is broken and magnetic field lines are stochastic at the resonant surface of macro-MHD.
This is similar to anomalous growth of tearing modes reported in Waddell (1978), Diamond (1984).

5. Summary

- We have found that macro-scale MHD appears in a quasi-equilibrium including micro-turbulence and zonal flow.
- Effect of turbulence on macro-MHD instability
  - Development of coherent flow
    - Nonlinear mode coupling of micro-scale turbulence causes macro-scale mode, but it is different from the macro-MHD instability because the spatial profile of induced macro-scale mode is significantly different from the profile of eigen-function of tearing mode. A. Ishizawa and N. Nakajima, Phys. Plasmas (2007)
    - The energy transfer from micro-turbulence to macro-MHD flow is similar to the energy transfer from micro-turbulence to zonal flow. A. Ishizawa and N. Nakajima, Nuclear Fusion (2007)
  - Development of coherent magnetic field
    - Turbulent flow mixing terms are much larger than normal resistivity term and play an important role in the growth of non-ideal macro-MHD mode.