EXCITATION OF GEODESIC ACOUSTIC MODES BY ITG/FINITE BETA DRIFT WAVE*

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OUTLINE OF TALK

- MOTIVATION
- REVIEW OF OBSERVATIONS/SIMULATIONS
- WHAT IS A GAM?
- THEORY OF PARAMETRIC EXCITATION
  - -LOCAL DRIFT WAVE/ITG PUMP
  - -NONLOCAL THEORY
  - -TIE IN WITH EXPERIMENTS
- CONCLUSIONS

MOTIVATION

• NUMEROUS OBSERVATIONS OF GAMS IN EDGE REGION OF TOKAMAKS

-beam emission spectroscopy DIII-D
-Rogowski coils JET (GAMs or BAE?)
-Multipin probes JFT2M HL-2A
-Doppler reflectometry ASDEX-U
-Heavy Ion Beam Probes T-10, H-1
-Correlation reflectometry TEXTOR CHS

• GAMs observed in numerical simulations of edge plasmas (finite beta Braginskii equations) and fluid/gyrofluid ITG

-Hallatscheck, Scott, Naulin, Miyato, Falchetto

simulations indicate excitation of GAMs by “turbulence” but do not clearly identify the specific Process

• First attempt to provide theory for excitation of GAM by drift waves was by Itoh, Hallatscheck, Itoh 2005
using a wave kinetic approach

• More recent coherent three-wave resonant parametric interaction approach,

-Chakrabarti, Guzdar, Kaw, Singh 2007
- Zonca, Chen (comprehensive study of low frequency modes in toroidal plasmas) 2007

GAMS AN EDGE PHENOMENON
CONWAY ET AL. PPCF 47, 1165 (2005). Doppler reflectometry

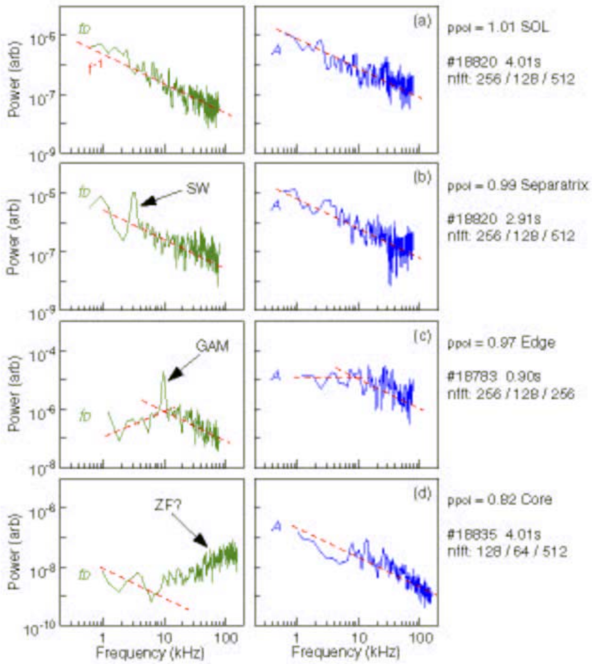


Figure 5. f_D and A spectra from a selection of ohmic discharges recorded at different radial positions, from (*a*) the SOL, (*b*) just inside the plasma boundary, (*c*) in the density pedestal region and (*d*) well inside the pedestal

$$f_D(t) = \frac{\sum f \cdot S(f)}{\sum S(f)} \qquad A(t) = \sum S(f)$$

"To date, no discernable no coherent oscillations have been observed in the core region (at least within the measurable region ρ_{pol}> 0.7)"

"--- the GAM was observed only in the edge density gradient region, i.e. the region of maximum E_r shear And plasma vorticity irrespective of variations in q profile And magnetic shear"

"This implies that the coherent GAM is an edge phenomenon"

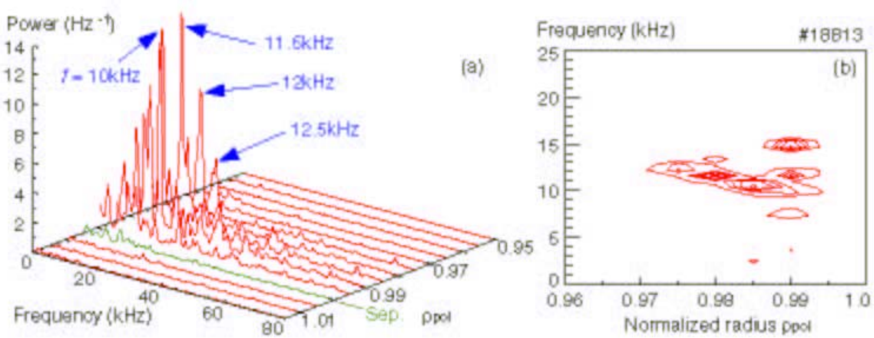


Figure 10. f_D spectral power versus frequency and radial position as (*a*) a perspective plot and (*b*) contour plot for ohmic shot #18813.

JFT-2M,Nagashima et al. PRL **95**,095002(2005)
Reciprocating Langmuir Probes

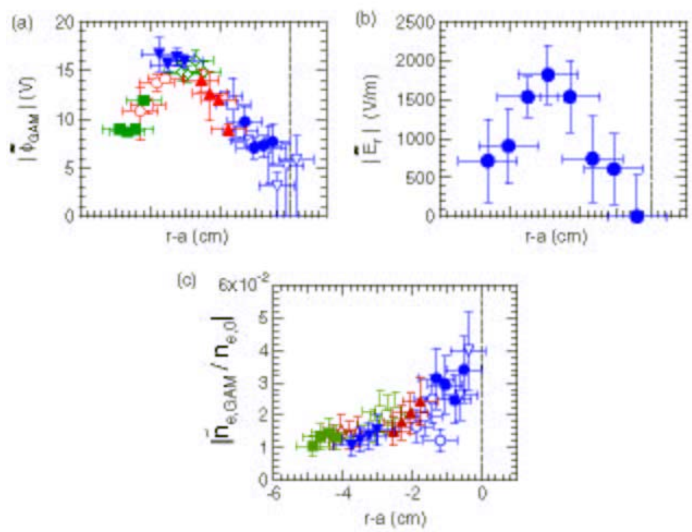


Figure 3. Radial profiles of $\tilde{\phi}_{\rm GAM}$, $\tilde{E}_{r,\rm GAM}$ and $\tilde{n}_{e,\rm GAM}/n_{e,0}$. The horizontal axis is the distance from the separatrix. $r-a=0$ is the location of the separatrix and the negative sign means the inside of the separatrix. The different marks show the data in the different shots.

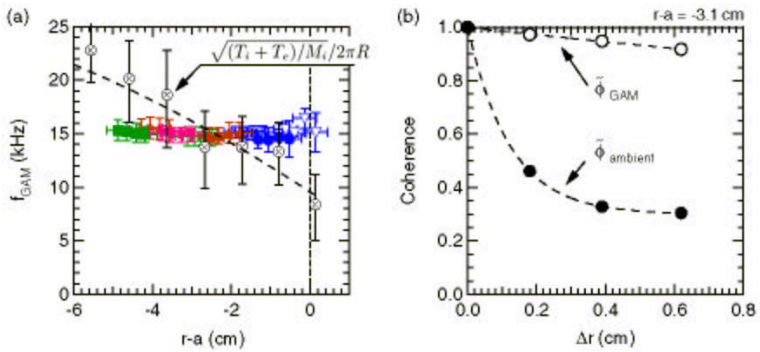


Figure 4. (*a*) The observed GAM frequency. The marks are same as in figure 3. ⊗ show the predicted GAM frequency ($f_{\rm GAM}=\sqrt{(T_i+T_e)/M_i}/2\pi R$), where T_i is measured with CXS and T_e is assumed to be the same as T_i. (*b*) Coherence between $\tilde{\phi}_{\rm GAM}$ signals in separate sample volumes (○) at 3.1 cm inside the separatrix. Coherence between $\tilde{\phi}$ of the ambient fluctuation with the frequency of 30–50 kHz is also shown (●).

BICOHERENCE ANALYSIS

Auto bi-spectrum

$$B(f_1, f_2) = \left\langle \Phi_1(f_1)\Phi_1(f_2)\Phi_1^*(f_1 \pm f_2)\right\rangle$$

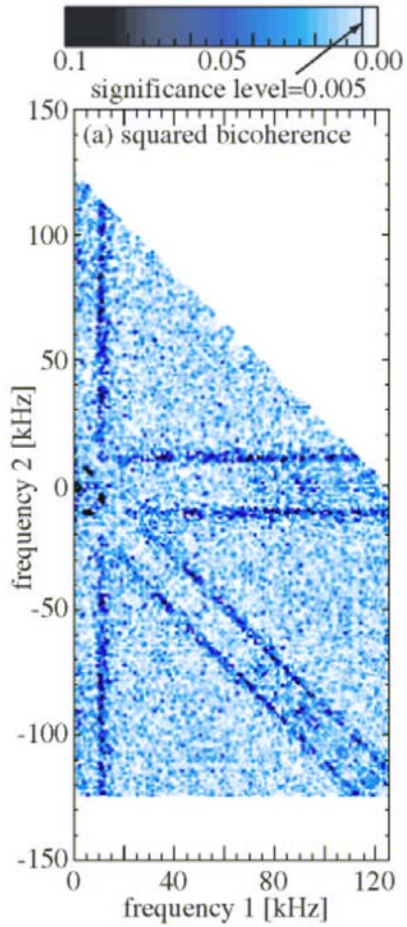
Auto bicoherence

$$b^2(f_1, f_2) = \frac{\left|B(f_1, f_2)\right|^2}{\left\langle\left|\Phi_1(f_1)\Phi_1(f_2)\right|^2\right\rangle\left\langle\left|\Phi_1(f_1 \pm f_2)\right|^2\right\rangle}$$

Nonlinear interaction between nearly "monochromatic" low frequency mode (10kHz/7kHz) with broadband high frequency modes (40 kHz-120 kHz/20-50kHz)
For JFT-2M/HL-7A

JFT-2M
Nagashima et al. PRL **95**,095002(2005)
Reciprocating Langmuir Probes

f_2



f_1

HL-7A,
Yan et al. NF, **47** 1643, 2007
Three-Step Langmuir Probe

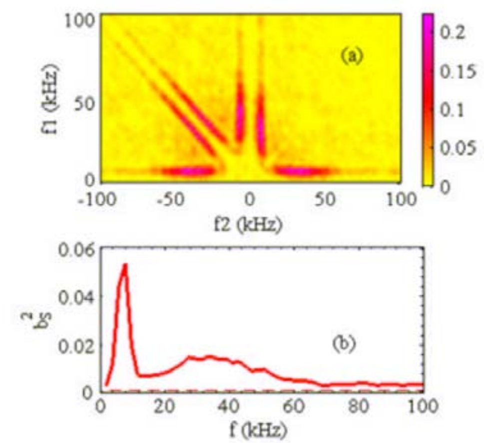
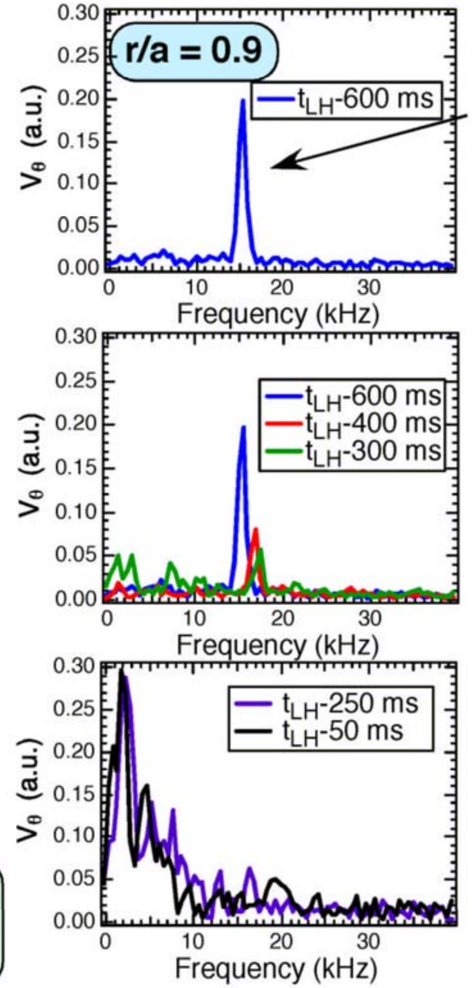


Figure 10. The squared auto-bicoherence $b^2(f)$ of tip 1 for shot 4206. The values of the $b^2(f)$ at f_1–f_2 plane (*a*) and the summed values of the $b_\Sigma^2(f)$ versus coherent frequency (*b*). The $b^2(f)$ valu are higher at the GAM frequency $f = f_1 + f_2 \approx \pm 7$ kHz and $f_2 \approx \pm 7$ kHz than the rest, indicating that the nonlinear three wave coupling is a plausible mechanism for GAMZF generation.

Poloidal Velocity Spectrum Evolves From GAM-Dominated to Low-Frequency Zonal Flow as Plasma Rotation Slows

- **Time-Delay-Estimation (TDE) methods applied to poloidally-separated BES measurements to determine $v_\theta(t)$ (t = 20 μs resolution, 25 kHz)**
- **GAM oscillation identified in $v_\theta(t)$ spectra (E_r oscillation => $v_\theta(t)$)**
- **GAM dominates ZF spectrum at high rotation**
  - *gradually decays in amplitude and disappears as plasma slows*
- **Zero-Mean-Frequency Zonal Flow arises and dominates spectra**
  - *ZMF-ZF power significantly higher than GAM power*
  - *Lower frequency shears more effectively (Hahm-1999)*
  - *More likely to trigger transition?*



Geodesic Acoustic Mode:

- *coherent*
- *m=0, n=0*
- *finite k_r*
- *$f \approx c_s/2\pi R$*

GAM decays with time

ZMF-ZF signature arises at low-frequency prior to LH

E.J. Kim, P.H. Diamond, PRL (2003)

DIII-D NATIONAL FUSION FACILITY

GLOBAL ITG SIMULATIONS-EXCITATION OF GAMS AND SCALING WITH ρ^*
MIYATO, KISHIMOTO AND LI, PPCF, 48, A335 (2006)

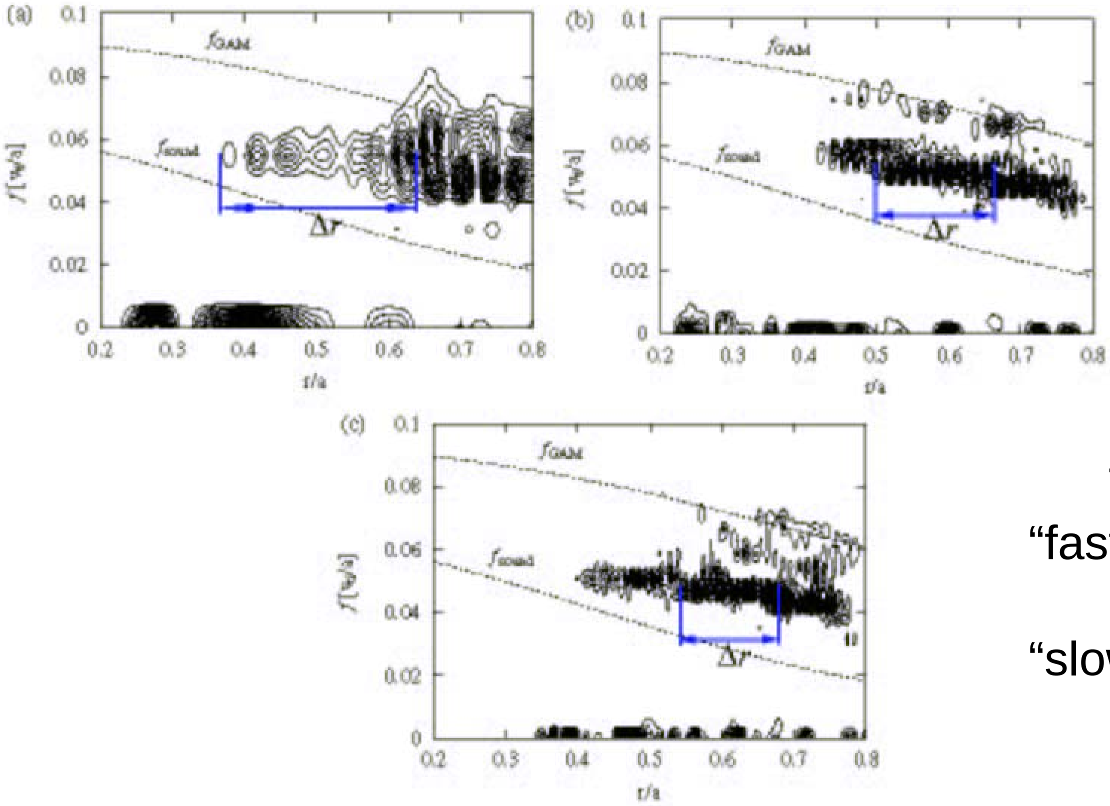


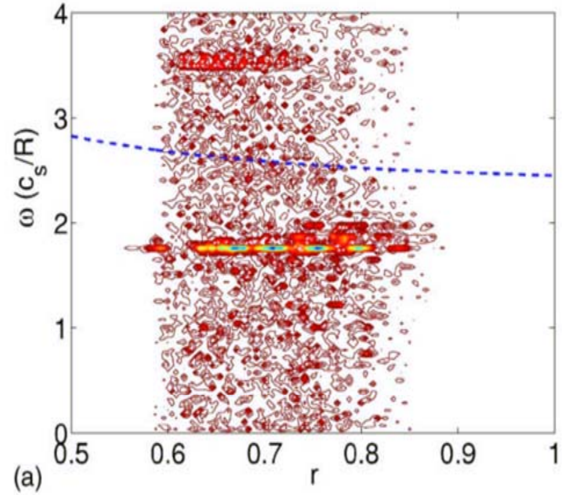
Figure 2. Radial variation of zonal flow frequency spectra for (a) $\rho_* = 0.0125$, (b) 0.005 and (c) 0.003.

Two scales

"fast" scale $k_r\rho_s=1$

"slow"scale $(\rho_i L_T)^{1/2}$

GLOBAL ITG SIMULATIONS-EXCITATION OF GAMS
FALCHETTO ET AL. POP, 14, 082304 (2007)



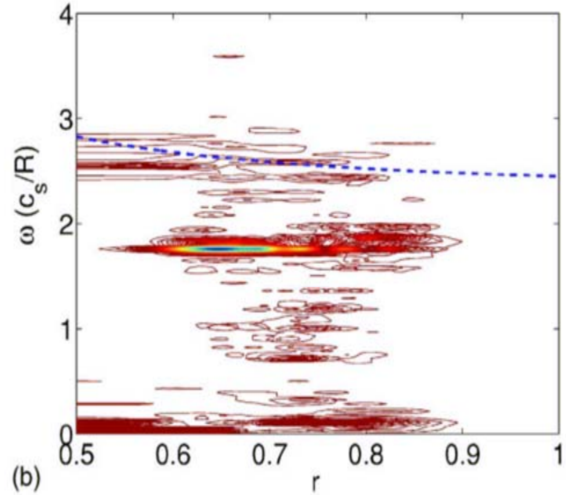


FIG. 12. (Color online) Frequency spectra of the $(n=0, m=0)$ component of the Reynolds' stress (a) and of $|v_\theta|^2$ (b) in the turbulent stationary state of the simulation with $q(r)=2r$ and injected heat flux $F_{in}=0.6$. A well localized peak is present in both spectra, around the same value of frequency $\omega \sim 1.74$ (c_s/R), lower than the GAM one (dashed line).

“Observed” Characteristics of GAMs

- GAMs seem to be an edge phenomenon
- experiments indicate radial structure $k_r\rho_s$ ~(0.02~1)
- bi-coherence study “identify” possible excitation mechanism
  -three wave coupling
- bi-coherence study shows GAM mode in edge almost “monochromatic” excited by “broadband” high frequency turbulence
- simulations show radial structure has two scales, “fast” scale and “slow” scale
- GAMs seen to become stabilized as H mode “threshold” approached (DIII-D)

BASIC EQUATIONS FOR FINITE BETA PLASMA

A. Zeiler, J, F. Drake and B. Rogers, Phys. Plasmas **4**, 2134, 1997

$$\rho_s^2 \nabla_\perp \cdot \frac{d}{dt} \nabla_\perp \phi + 2\frac{\rho_s c_s}{R} \hat{C} n - \nabla_{||} J = 0$$

$$\frac{dn}{dt} + \frac{\rho_s c_s}{L_n} \frac{\partial \phi}{\partial y} - 2\frac{\rho_s c_s}{R} \hat{C} \phi - \nabla_{||} J = 0$$

$$\frac{\partial \psi}{\partial t} + \frac{\rho_s c_s}{L_n} \frac{\partial \psi}{\partial y} - v_A \nabla_{||} \left(\phi - n \right) = 0$$

$$\hat{C} = sin\,\theta \frac{\partial}{\partial x} + cos\,\theta \frac{\partial}{\partial y}$$

$$n = \frac{\tilde{n}}{n_0}, \quad \phi = \frac{e\tilde{\phi}}{T_e}, \quad \psi = \frac{v_A \tilde{\psi}}{\rho_s c_s B_0}, \; J = \rho_S^2 v_A \nabla_\perp^2 \psi$$

WHAT IS A GAM?

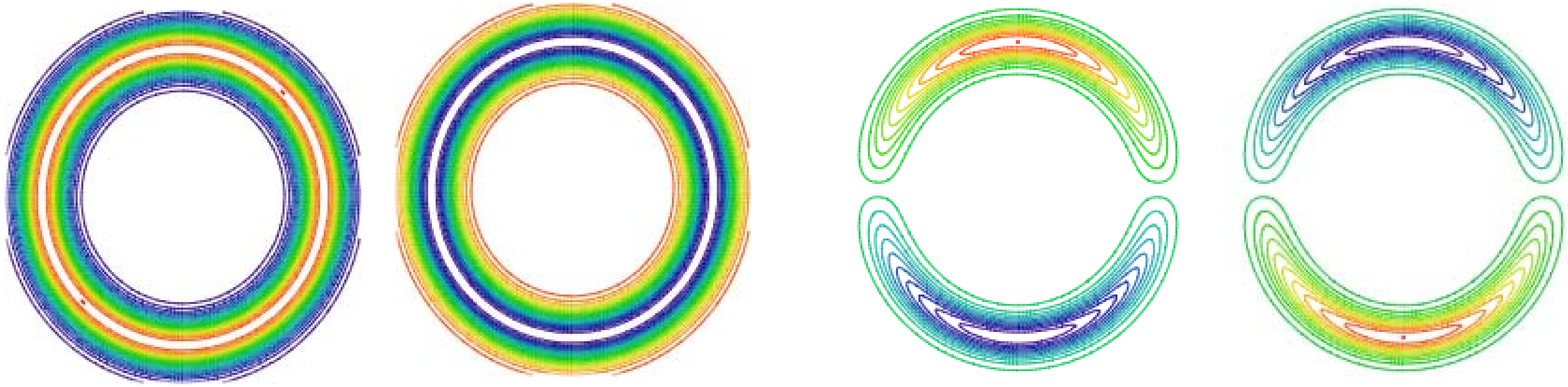
$$\xi_G = (\phi_G, n_G(\theta))e^{-i\omega t + iq_r x}$$

Continuity Equation
$$\omega n_G - \underbrace{2\frac{q_r c_s^2}{\Omega_i R}\phi_G \sin(\theta)}_{Geodesic\ Curvature} = 0$$

Vorticity Equation
$$\omega q_r^2 \rho_s^2 \phi_G - \underbrace{2\frac{q_r c_s^2}{\Omega_i R}\sin(\theta)}_{Geodesic\ Curvature} = 0$$

$$\omega_G{}^2 = 2\frac{c_s^2}{R^2} \quad T_G = \frac{\sqrt{2}\pi R}{c_s}$$

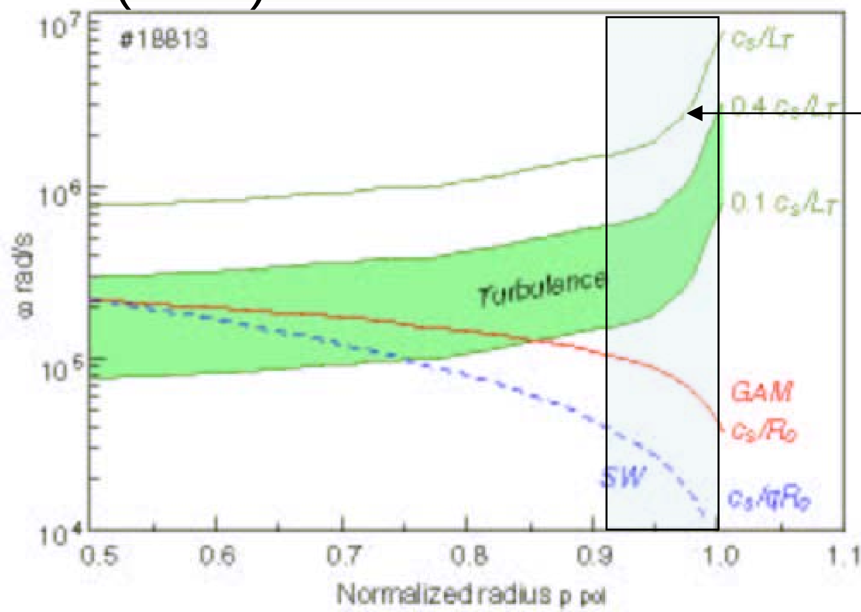
$\Phi_G(t)$ $\Phi_G(t+T_G/2)$ $n_G(\theta)(t)$ $n_G(\theta)(t+T_G/2)$



GAM IS TIME-DEPENDENT ZONAL FLOW.

WHY IS GAM AN EDGE PHENOMENON ?

Conway et al. PPCF **47**, 1165 (2005)



Region where three wave interaction is possible

Figure 14. Radial profiles of predicted GAM frequency c_s/R_0 (——) and SW $c_s/$
generic edge turbulence $(k_\theta \rho_s)\, c_s/L_T$ (green shaded) calculated using T_e from I
CLISTE for ohmic shot #18813.

WHY IS GAM AN EDGE PHENOMENON ?

If drift waves are the primary driver of GAMs, then GAMs will be excited **only if** drift wave frequency exceeds GAM frequency.

$$\omega_0^{max} > \omega_{GAM} \qquad \frac{c_s}{2L_n} > \left[2 + \frac{1}{q^2}\right]^{1/2} \frac{c_s}{R}$$

Conway et al. PPCF **47**, 1165 (2005)

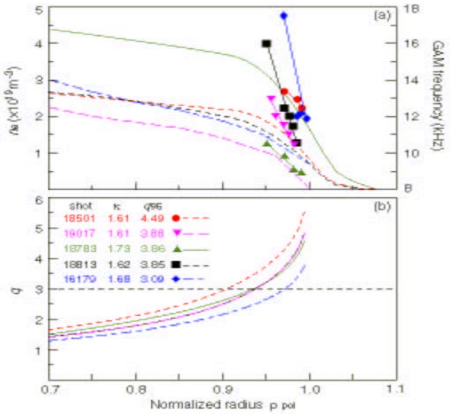


Figure 12. (a) Radial density profiles and f_{GAM} plus (b) q profiles for a selection of ohmic discharges with varying q_{95} and plasma elongation κ.

Carlstrom et. al, NF **39**, 1941(1999)

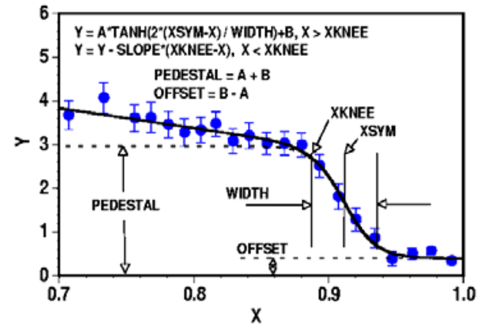
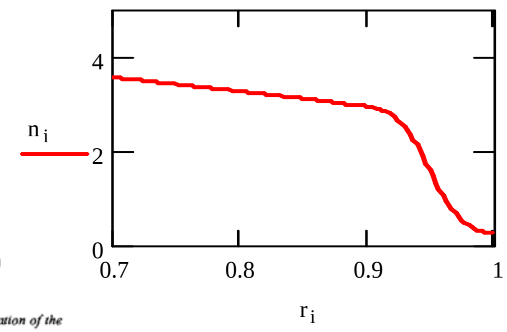
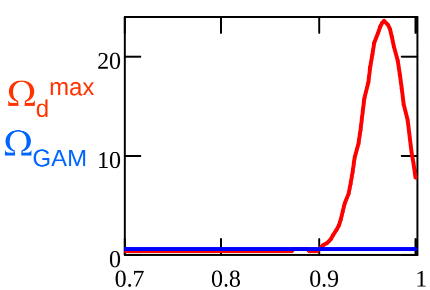
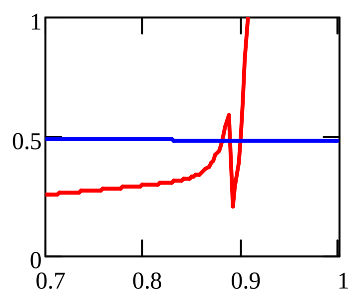


FIG. 3. Modified hyperbolic tangent fit to a typical L-mode edge n_e profile showing the location of the symmetry point, xsym, and the profile knee, xknee.







$\Omega_d^{max} = \omega_0^{max}\, a/c_s = a/2L_n$

$\Omega_{GAM} = \omega_{GAM} a/c_s = (2+1/q^2)^{1/2}\, a/R$

GAM EXCITATION THEORY -0

"PUMP" DRIFT WAVE

$$\xi_0 = \sum_m \xi_m^0 e^{-i\omega_0 t + in\zeta + im\theta}$$

GEODESIC ACOUSTIC MODE

$$\xi_G = (\phi_G, n_G(\theta), \psi_G) e^{-i\omega t + iq_r x}$$

"SIDE-BAND" DRIFT WAVE

$$\xi_s = \sum_m \xi_m^s e^{-i(\omega - \omega_0)t - in\zeta - im\theta + iq_r x}$$

MODE-COUPLING ANALYSIS-1

EXCITATION OF GAM BY PUMP AND SIDEBAND DRIFT WAVES

CONTINUITY EQUATION

$$\omega\overline{n_G Sin\theta}+\frac{q_r c_s^2}{\Omega_i R}\phi_G=-\frac{q_r c_s^2}{2i\Omega_i a}m_0\left[\left(\phi_{m_0}^0 n_{m_0+1}^s-n_{m_0}^0\phi_{m_0+1}^s\right)-\left(\phi_{m_0}^0 n_{m_0-1}^s-n_{m_0}^0\phi_{m_0-1}^s\right)\right]$$

$$+\frac{q_r c_s^2}{2i\Omega_i a}\left[(m_0+1)\left(\phi_{m_0+1}^0 n_{m_0}^s-n_{m_0+1}^0\phi_{m_0}^s\right)-(m_0-1)\left(\phi_{m_0-1}^0 n_{m_0}^s-n_{m_0-1}^0\phi_{m_0}^s\right)\right]$$

VORTICITY EQUATION

$$\omega q_r^2\rho_s^2\phi_G+2\frac{q_r c_s^2}{\Omega_i R}\overline{n_G Sin\theta}=-i\frac{q_r^3\rho_s^3 c_s}{a}m_0\phi_{m_0}^0\phi_{m_0}^s$$

MODE-COUPLING ANALYSIS-1

EXCITATION OF GAM BY PUMP AND SIDEBAND DRIFT WAVES

$$\left[(\omega_0-\omega)(1+k_{\perp,m_0}^2\rho_s^2)-\omega_{*m_0}\right]\phi_{m_0}^s=iq_r\rho_s\frac{m_0c_s}{a}\left[1-(q_r^2-\frac{m_0^2}{a^2})\rho_s^2\right]\left(\phi_{m_0}^0\right)^*\phi_G$$

$$-q_r\rho_s\frac{m_0c_s}{a}\left[\left(\frac{m_0+1}{2m_0}\right)\left(\phi_{m_0+1}^0\right)^*\overline{n_G}-\left(\frac{m_0-1}{2m}\right)\left(\phi_{m_0-1}^0\right)^*\overline{n_G}\right]$$

$$k_{\perp m_0}^2=q_r^2+\frac{m_0^2}{a^2}$$

FINAL COUPLED EQUATIONS

$$\left[(\omega_0-\omega)(1+k_{\perp,m}^2\rho_s^2)-\omega_{*m_0}\right]\phi_{m_0}^s=iq_r\rho_s\frac{m_0c_s}{a}\left[1-(q_r^2-\frac{m_0^2}{a^2})\rho_s^2\right]\left(\phi_{m_0}^0\right)^*\phi_G$$

$$(\omega^2-\omega_G^2)\phi_G=-i\frac{\omega_Gq_r\rho_sm_0c_s}{a}\phi_{m_0}^{(0)}\phi_{m_0}^s$$

GAM EXCITATION THEORY-IV
RESONANCE CONDITION (IN ES LIMIT)

$\omega_0 - \omega_s = \omega_G$

$$\frac{\omega_{*m_0} q_r^2 \rho_s^2}{\left(1+\frac{m_0^2}{a^2}\rho_s^2\right)\left[1+\left(q_r^2+\frac{m_0^2}{a^2}\right)\rho_s^2\right]} = \frac{\sqrt{2}c_s}{R}$$

For $m_0\rho_s/a << 1$ *and* $q_r\rho_s << 1$

$$\frac{m_0}{a} q_r^2 \rho_s^3 = \frac{\sqrt{2}L_n}{R}$$

For $q_r \sim \frac{m_0}{a}$

$$q_r\rho_s = \left(\frac{\sqrt{2}L_n}{R}\right)^{1/3}$$

For DIII-D, L_n=3 cm, R=169 $\quad q_r\rho_s = 0.3$

For ASDEX-U, L_n=5 cm, R=165 $\quad q_r\rho_s = 0.35$

WHY IS FINITE β IMPORTANT IN THE EDGE REGION?

Modes relevant to edge region of tokamak plasmas

$$(1-\frac{\omega_*}{\omega})(\frac{\omega^2}{\omega_A^2}-1)=k_\perp^2\rho_s^2\omega \qquad \omega=\omega_*=\frac{k_y\rho_s c_s}{L_n} \qquad \omega=\omega_A=\frac{V_A}{qR}$$

$$\left(\frac{\omega_*}{\omega_A}\right)^2=\left(k_y\rho_s\right)^2\frac{\beta}{2}\left(\frac{qR}{L_n}\right)^2=\left(k_y\rho_s\right)^2\hat{\beta}$$

$$\beta=4\times10^{-2}\,\frac{n(10^{20}/m^3)T(KeV)}{B(T)^2}$$

$$B=2T,\ n=0.1\ T(KeV)=0.1\text{-}0.2,\ \beta=10^{-4}$$

$$R=2m,\ L_n=0.02-0.1m$$

$$q=3$$

$$\hat{\beta}=5-100$$

INTERESTING POSSIBILITY OF MODE CROSSING WHICH CAN CHANGE DISPERSION CHARACTERISTICS OF MODES

GAM EXCITATION THEORY -1

$$\left[\omega_s\left(1+k_\perp^2\rho_s^2\right)-\omega_*-\frac{\omega_s^2\left(\omega_s-\omega_*\right)}{k_{||}^2 v_A^2}\right]\phi_s = i\Gamma F_1\phi_p^*\phi_G + i\Gamma F_2\phi_p^*\psi_G \quad \Gamma = q_r\rho_s\frac{m_0 c_s}{a}$$

DRIFT WAVE

$$\left(\omega^2-\omega_G^2\right)\phi_G = -i\Gamma\omega_G\left(1-\frac{\omega_0\omega_s}{k_{||}^2 v_A^2}\right)\phi_p\phi_s$$

GAM

$$\omega\psi_G = i\Gamma\frac{\omega_0\omega_s\omega_G}{k_{||}^3 v_A^3}\phi_p\phi_s$$

ZONAL FIELD

$$F_1 = \left[1+\left(k_y^2-k_x^2\right)\rho_s^2\right]-\frac{k_y^2-k_x^2}{k_\perp^2}\left[\frac{\omega_s\left(\omega_s-\omega_*\right)}{k_{||}^2 v_A^2}\right]-\frac{\omega_0\left(\omega_0+\omega_s-\omega_*\right)}{k_{||}^2 v_A^2}$$

$$= F_{1A} + F_{1B}$$

NEW TERMS FROM FINITE BETA

$$F_2 = \frac{\omega_s\omega_0}{k_{||}^2 v_A^2}\left[\frac{\left(\omega_0-\omega_*\right)}{k_{||}v_A}+\frac{k_y^2-k_x^2}{k_\perp^2}\frac{\left(\omega_s-\omega_*\right)}{k_{||}v_A}\right]$$

GAM EXCITATION THEORY-II

$$\omega = \omega_G + \delta\omega, \quad \omega_{s0} = \omega_0 - \omega_G$$

$$A = 1 + k_\perp^2 \rho_s^2 - \omega_{s0} \frac{\left(3\omega_{s0} - 2\omega_*\right)}{k_{||}^2 v_A^2} = A_{1A} + A_{1B}$$

$$Resonance\ Condition\quad k_x \rho_s = \left[-1 - k_y^2 \rho_s^2 + \frac{\omega_*}{\omega_{s0}} + \frac{\omega_{s0}^2 \left(\omega_{s0} - \omega_*\right)}{k_{||}^2 v_A^2} \right]^{1/2}$$

$$\delta\omega \phi_s = -i \frac{\Gamma}{A} \left(F_1 \phi_p^* \phi_G + F_2 \phi_p^* \psi_G \right)$$

$$\delta\omega \phi_G = -i \frac{\Gamma}{2} \left(1 - \frac{\omega_0 \omega_s}{k_{||}^2 v_A^2} \right) \phi_p \phi_s$$

Since zonal field ψ_G does NOT modify dispersion relation of GAM it a driven scalar

$$\cancel{\omega}_G \psi_G = i\Gamma \frac{\omega_0 \omega_s \cancel{\omega}_G}{k_{||}^3 v_A^3} \phi_p \phi_s$$

GAM EXCITATION THEORY-III

$$\delta\omega^2 + \frac{\Gamma^2 \left|\phi_p\right|^2 \left(F_{1A} + F_{1B}\right)}{2\left(A_{1A} + A_{1B}\right)} \left(1 - \frac{\omega_0 \omega_{s0}}{k_{\|}^2 v_A^2}\right) = 0$$

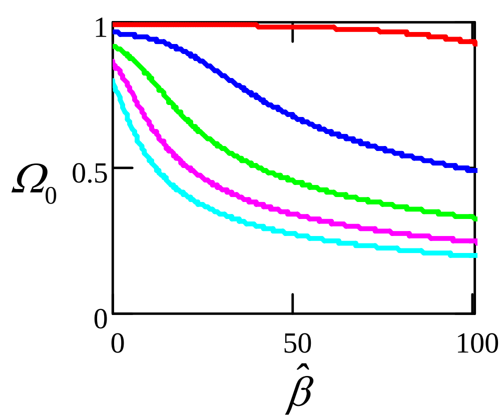
$$\uparrow \qquad \uparrow$$

$$\begin{Bmatrix} Reynold \\ stress \end{Bmatrix} \begin{Bmatrix} Maxwell \\ stress \end{Bmatrix}$$

NORMALIZED EQN

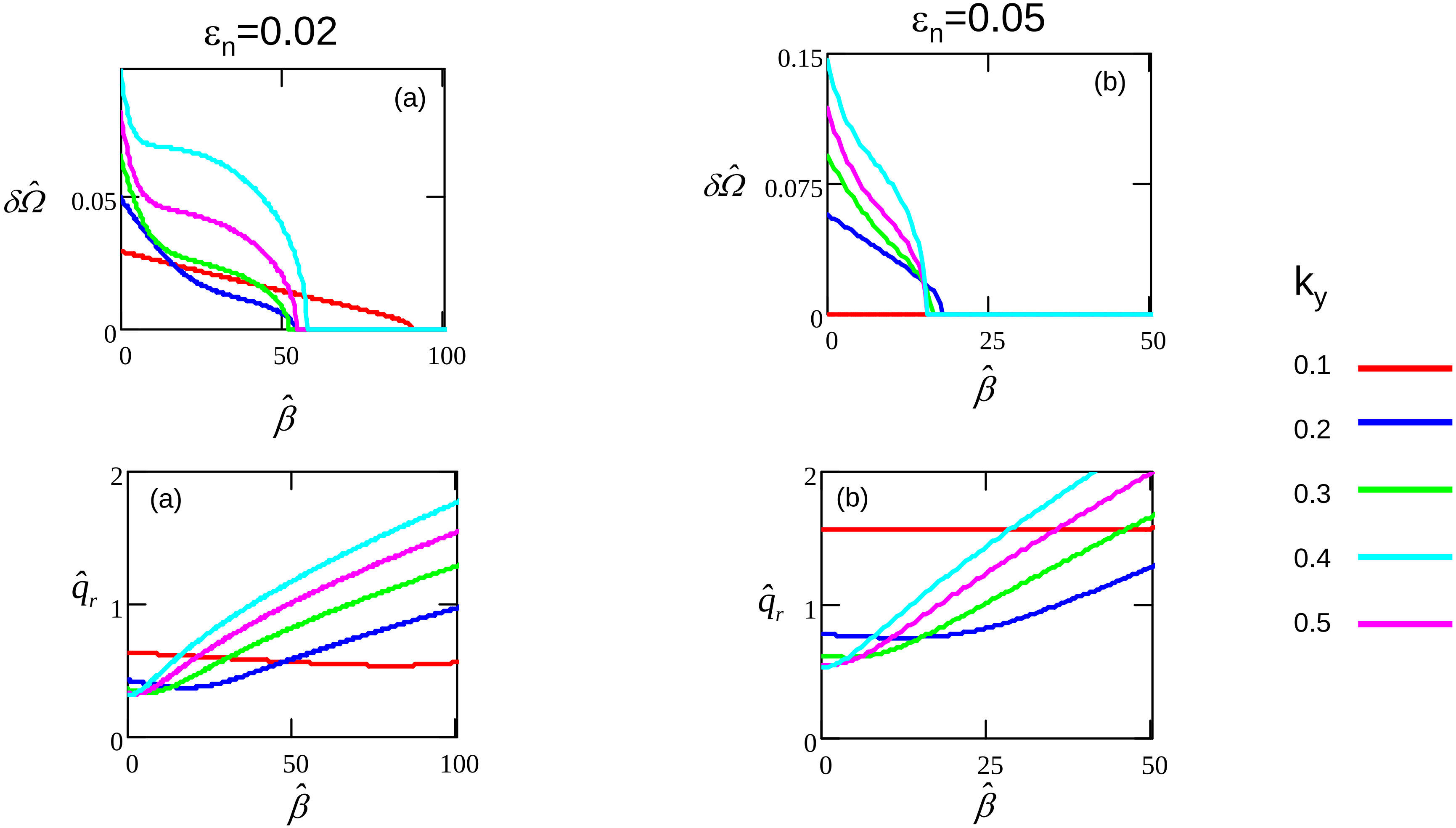
$$\Omega_0[1 + k_y^2 \rho_s^2] - 1 - \Omega_0{}^2 k_y^2 \rho_s^2 \hat{\beta}(\Omega_0 - 1) = 0$$

$$where\ \Omega_0 = \frac{\omega_0}{\omega_*} \quad and \quad \hat{\beta} = \beta \frac{q^2 R^2}{2L_n^2}$$



GROWTH RATE AND WAVENUMBER FOR GAMS

$$\varepsilon_n = \frac{L_n}{R}, \ \frac{L_n}{\rho_s}\Phi = 1$$



Stabilization due to finite beta effects of the pump and side band and cancellation effect of Reynold's stress and Maxwell's stress

BASIC EQUATIONS

$$\frac{c_s^2}{\Omega_i^2}\nabla_\perp\cdot\frac{d}{dt}\nabla_\perp\phi-\frac{2c_s^2}{\Omega_i}b\times\kappa\cdot\nabla p_e-\nabla_\parallel J=0$$

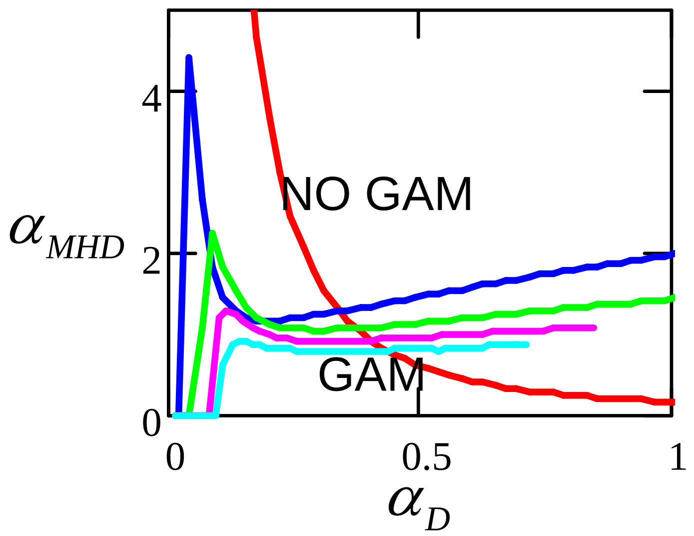
$$x_\perp\to x_\perp/L_0,\ x_\parallel\to x_\parallel/L_\parallel,\ t\to t/t_0 \qquad L_\parallel=2\pi qR,\ n/t_0\sim c_s^2\phi/L_0L_n\Omega_i,\ J\sim\sigma\nabla_\parallel\phi$$

$$1 \quad : \quad \frac{RL_n}{2c_s^2}t_0^2 \quad : \quad \frac{4\pi v_A^2\eta_\parallel t_0L_0^2}{c^2L_\parallel^2}$$

$$\Rightarrow t_0=c_s\left(\frac{2}{RL_n}\right)^{1/2} \qquad L_0=\frac{L_\parallel c}{v_A}\left(\frac{\eta_\parallel}{4\pi t_0}\right)^{1/2}$$

Two dimensionless parameters: (1) $\alpha_D=\omega_* t_0$ (2) $\alpha_{MHD}=\left(\frac{v_A}{qR}t_0\right)^2$

PARAMETRIC STABILITY BOUNDARY FOR GAMS IN (α_{MHD},α_D) SPACE



LH

$\varepsilon_n = 0.01$

$\varepsilon_n = 0.02$

$\varepsilon_n = 0.035$

$\varepsilon_n = 0.05$

ABSENCE OF GAMS IN H-MODES?
HOW IS THIS RELATED TO
MCKEE'S OBSERVATIONS?

BASIC EQUATIONS FOR ITG WAVES AND GAMS
Sandberg PoP 2005

$$\frac{\partial n}{\partial t}-\left[\frac{\partial}{\partial t}-\tau\left(1+\eta_i\right)\right]\nabla_\perp^2\phi+\frac{1}{a}\frac{\partial\phi}{\partial\theta}-C\left[n+\tau\left(n+T_i\right)\right]=\left\{\phi,\nabla_\perp^2\phi-n+\tau\nabla_\perp^2\left(n+T_i\right)\right\}$$

$$\frac{\partial T_i}{\partial t}-\frac{5}{3}\tau C T_i+\left(\eta_i-\frac{2}{3}\right)\frac{1}{a}\frac{\partial\phi}{\partial\theta}-\frac{2}{3}\frac{\partial n}{\partial t}=\left\{\phi,\frac{2}{3}n-T_i\right\}$$

$$n=\phi-\overline{\phi}$$

$$n=\delta n L_n / n_0\rho_s\,,\ \phi=e\delta\phi L_n/T_{e0}\rho_s\,,\ T_i=\delta T_i L_n/T_{i0}\rho_s$$

$$\varepsilon_n=2L_n/R,\ \ \eta_i=L_n/L_{T_{i0}}\,,\ \tau=T_{i0}/T_{e0}$$

$$C=\varepsilon_n\left[\frac{Cos\theta}{a}\frac{\partial}{\partial\theta}+Sin\theta\frac{\partial}{\partial x}\right],\ \left\{A,B\right\}=\frac{\partial A}{\partial x}\frac{\partial B}{a\partial\theta}-\frac{\partial B}{\partial x}\frac{\partial A}{a\partial\theta}$$

FINAL DISPERSION RELATION FOR EXCITATION OF GAMS BY ITG MODES

$$\left(\omega^2-\omega_G^2\right)\left(\omega_0-\omega-\omega_{s+}\right)\left(\omega_0-\omega-\omega_{s-}\right)=k_{m_0}^2\frac{\omega\left[\left(\omega_0-\omega\right)F_1+k_{m_0}\varepsilon_n\tau F_2\right]}{1+k_{\perp m_0}^2}\left(q_r^2\Delta_{m_0}^1-k_{m_0}^2\Delta_{m_0}^2\right)\left|\phi_{m_0}^0\right|^2$$

$$\omega_{s\pm}=\omega_{sr}\pm i\gamma_s \qquad \omega_{sr}=-\frac{A_s}{2\left(1+k_{\perp m_0}^2\right)} \qquad \left(\omega_{sr}=\omega_0-\omega_G\right)$$

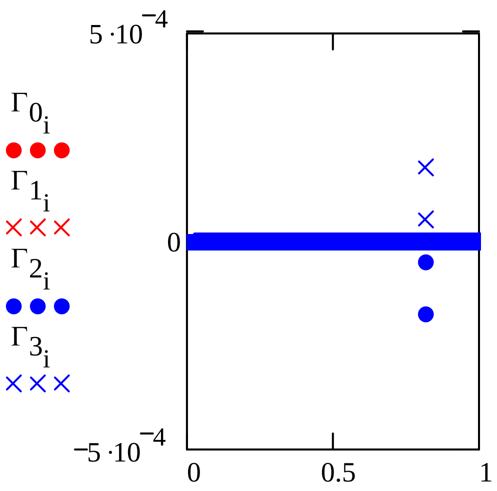
RESONANCE CONDITION

$$q_r\rho_s=\frac{1}{\left(k_{m_0}\rho_s\right)^{\frac{1}{2}}}\left\{\frac{2^{\frac{3}{2}}\left(\frac{5}{3}+\tau\right)^{\frac{1}{2}}\frac{L_n}{R}}{1+\tau\left(1+\eta_i\right)-2\frac{L_n}{R}\left[1+\frac{5}{3}\tau+\frac{1}{k_{m_0}\rho_s}\left(\frac{10}{3}+2\tau\right)^{\frac{1}{2}}\right]}\right\}^{\frac{1}{2}}\left[1+k_{m_0}^2\rho_s^2\right]$$

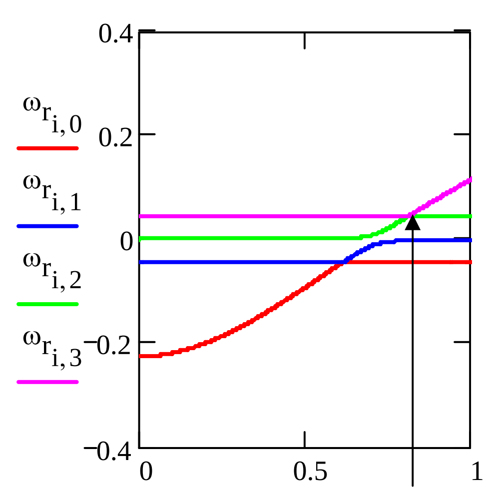
NUMERICAL RESULTS
GROWTH RATE, RADIAL WAVE NUMBER OF RESONANT MODES, AND REAL FREQUENCY OF GAMS EXCITED BY MARGINAL ITG MODES

EDGE-LIKE PARAMETERS

ε_n=0.04, τ=1, η_i=2/3, q=3 $k_y\rho_s$=0.3 Φ=0.001



$q_r\rho_s$

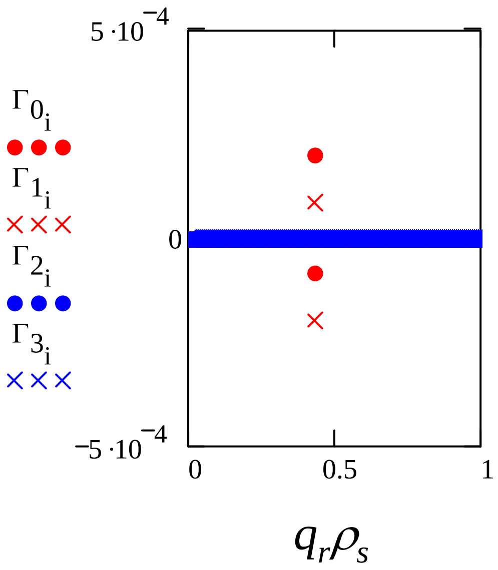


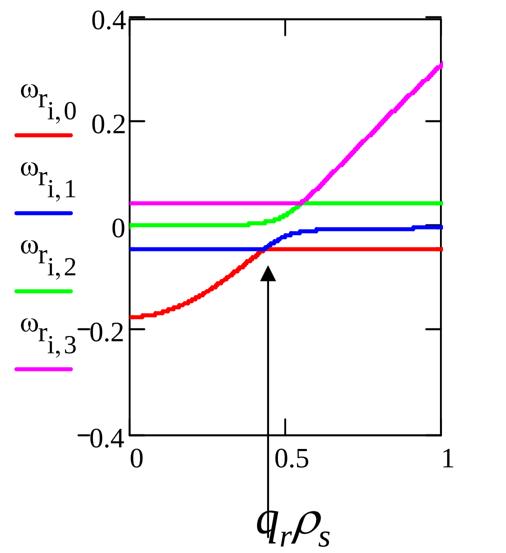
$q_r\rho_s$ ITG pump interacts with ITG and GAM

NUMERICAL RESULTS
GROWTH RATE, RADIAL WAVE NUMBER OF RESONANT MODES, AND REAL FREQUENCY OF GAMS EXCITED BY MARGINAL ITG MODES

EDGE-LIKE PARAMETERS

ε_n=0.04, τ=1, η_i=2/3, q=3 $k_y\rho_s$=0.6 Φ=0.001





ITG pump interacts with DW and GAM

NOVEL THREE WAVE COUPLINGS BETWEEN DIFFERENT BRANCHES

NONLOCAL EIGENVALUE-I

$$\left[\omega^2-\omega_G^2(x)\right]\psi_G=-\Gamma\omega_G\rho_s^2\frac{d^2\phi_s}{dx^2}\quad \Gamma=k_y c_s\left|\phi_0\right|\quad \psi_G=\rho_s\frac{d\phi_G}{dx},\ \omega_G^2=2\frac{c_s^2}{R^2}$$

$$\left\{(\omega_0-\omega)\left[1-\rho_s^2\left(\frac{d^2}{dx^2}-k_y^2\right)\right]-\omega_*(x)\right\}\phi_s=\Gamma\psi_G$$

$$\omega=\omega_{G0}+\delta\omega,\ \ \phi_s=e^{ik_{0x}x}\Phi_s,\ \psi_G=e^{ik_{0x}x}\Psi_s$$

Resonance Condition

$$k_{0x}\rho_{s0}=\left[-1-k_y^2\rho_{s0}^2+\frac{\omega_{*0}}{\omega_0-\omega_{G0}}\right]^{1/2}$$

$$\left[\delta\omega+\frac{\omega_G^2(0)-\omega_G^2(x)}{2\omega_G(0)}\right]\Psi_G=-\frac{\Gamma\rho_s^2\omega_G}{2\omega_G(0)}\left(\frac{d}{dx}+ik_{0x}\right)^2\Phi_s$$

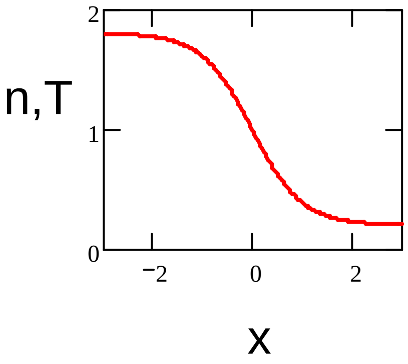
$$\left\{(\omega_0-\omega_G(0))\left[\rho_s^2\left(\frac{d^2}{dx^2}+2ik_{0x}\frac{d}{dx}\right)+\left(\rho_{s0}^2-\rho_s^2\right)k_{\perp0}^2\right]+\omega_*(x)-\omega_*(0)\right\}\Phi_s$$

$$+\Gamma\Psi_G=-\delta\omega\left[1-\rho_s^2\left(\frac{d}{dx}+ik_{0x}\right)^2+k_y^2\rho_s^2\right]\Phi_s$$

NONLOCAL EIGENVALUE II

$$n(x) = n(0)\left[1 - \frac{n_1 - n_2}{2n(0)} tanh\left(\frac{x}{L_N}\right)\right] \quad n(0) = \frac{n_1 + n_2}{2}$$

$$T(x) = T(0)\left[1 - \frac{T_1 - T_2}{2T(0)} tanh\left(\frac{x}{L_T}\right)\right] \quad n(0) = \frac{T_1 + T_2}{2}$$



Characteristic Scale Lengths

$$x_0 = \left[\frac{2k_{x0}\rho_{s0}}{C_1\left(1 + k_{\perp 0}^2 \rho_{s0}^2\right)}\right]^{1/2} \left(\rho_{s0} L_N\right)^{1/2} \quad C_1 = \frac{T_1 - T_2}{2T_0}\frac{L_N}{L_T} - \frac{n_1 - n_2}{2n_0}$$

For $C_1 = 0$

$$x_0 = \left[\frac{2k_{x0}\rho_{s0}}{\left(1 + k_{\perp 0}^2 \rho_{s0}^2\right)}\right]^{1/3} \rho_{s0}{}^{1/3} L_N{}^{2/3}$$

NONLOCAL EIGENVALUE-III

$$\left[F_1\left(\frac{d^2}{dx^2}+2i\hat{k}_{0x}\frac{d}{dx}\right)+F_2\right]\hat{\phi}_s+\hat{\gamma}_0\hat{c}_s\hat{\psi}_G=-\frac{\delta\hat{\Omega}}{1+k_{\perp 0}^2\rho_{s0}^2}\left[1+\hat{k}_y^2\hat{\rho}_s^2-\frac{\hat{\rho}_s^2}{\hat{L}^2}\left(\frac{d}{dx}+i\hat{k}_{0x}\right)^2\right]$$

$$\delta\Omega\hat{\psi}_G=\left[\frac{\hat{\Omega}_G^2-1}{2}\right]\hat{\psi}_G-\frac{\hat{\gamma}_0\hat{c}_s^2\hat{\rho}_s^2}{\hat{k}_{0x}^2}\left(\frac{d}{dx}+i\hat{k}_{0x}\right)^2\hat{\phi}_s$$

$$\hat{c}_s=\frac{c_s}{c_{s0}},\ \ \hat{\rho}_s=\frac{\rho_s}{\rho_{s0}},\ \hat{L}=\frac{L}{\rho_{s0}},\ \hat{k}_{0x}=k_{0x}L,\ \hat{\gamma}_0=\frac{\gamma_0}{\omega_{G0}}$$

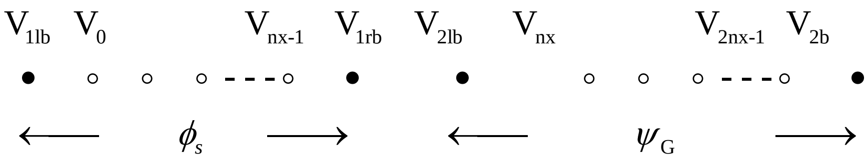
$$F_1=\frac{\hat{\Omega}_0-1}{1+k_{\perp 0}^2\rho_{s0}^2},\ \ F_2=\frac{\left(\rho_{s0}^2-\rho_s^2\right)k_{\perp 0}^2}{1+k_{\perp 0}^2\rho_{s0}^2}+\frac{\hat{\Omega}_*-\hat{\Omega}_{*0}}{1+k_{\perp 0}^2\rho_{s0}^2}$$

$$\hat{\Omega}_0=\frac{\omega_0}{\omega_{G0}},\ \hat{\Omega}_*=\frac{\omega_*}{\omega_{G0}},\quad \hat{\Omega}_G=\frac{\omega_G}{\omega_{G0}},\ \delta\hat{\Omega}=\frac{\delta\omega}{\omega_{G0}}$$

Four dimensionless parameters

$$(1)\hat{\gamma}_0=\frac{\gamma_0}{\omega_{G0}},\ \ (2)\hat{L}=\frac{L}{\rho_{s0}},\ (3)\varepsilon_n=\frac{\sqrt{2}L}{R},\ (4)k_y\rho_{s0}$$

NONLOCAL EIGENVALUE-IV



Use simple finite differencing to create matrix equation ($2n_x$ X$2n_x$) to cast coupled equations into standard eigenvalue problem

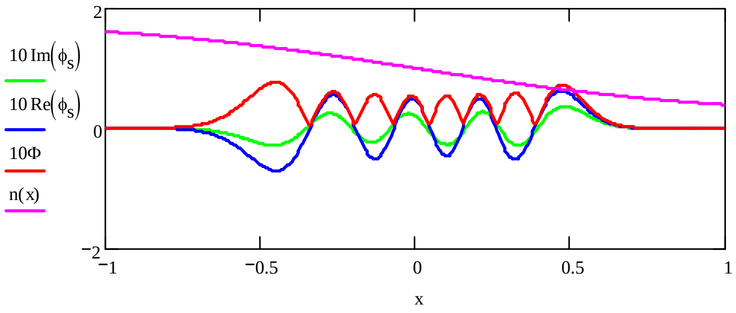
$$\mathbf{A}V=\delta\Omega\mathbf{B}V$$

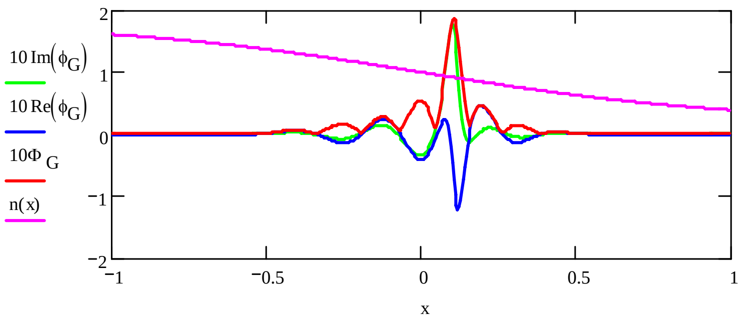
$$\mathbf{B}^{-1}\mathbf{A}V=\mathbf{D}V=\delta\Omega V$$

Use package like mathcad to solve the eigenvalue problem

Typical n_x =400-500.

NONLOCAL EIGENVALUE-V

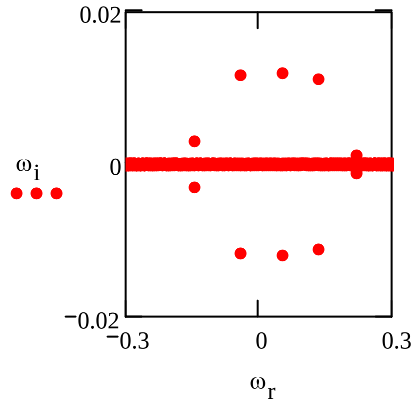




$$\frac{L}{\rho_{s0}} = 50,\ \varepsilon_n = 0.02,\ \Gamma = 0.05,\ k_y \rho_{s0} = 0.1$$

$$k_{0x}\rho_{s0} = 0.584$$

$$\Omega_r = -0.041,\ \ \Omega_i = 0.012$$
$$\Omega_r = 0.054,\ \ \Omega_i = 0.012$$
$$\Omega_r = 0.137,\ \ \Omega_i = 0.0011$$



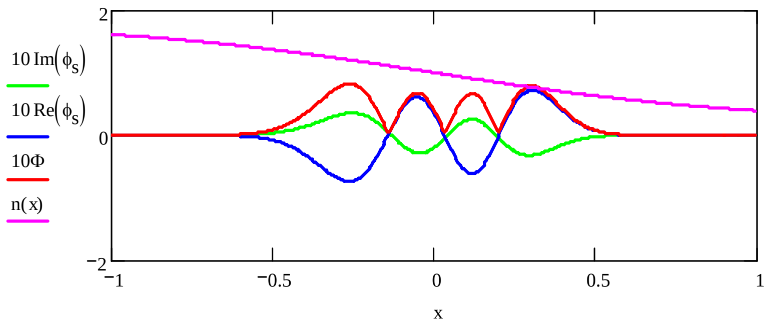
APPEARANCE OF “FAST” SCALE AND “SLOW” SCALE

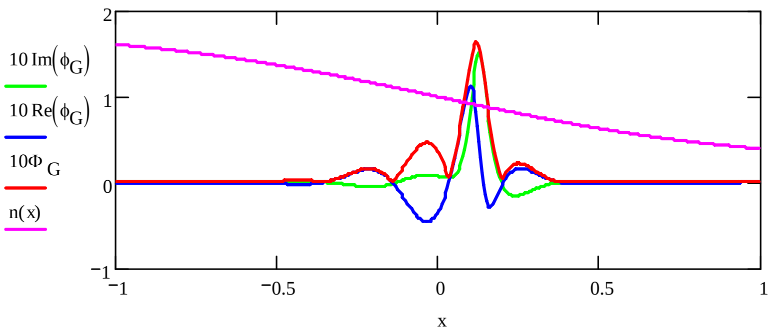
NONLOCAL EIGENVALUE-VI

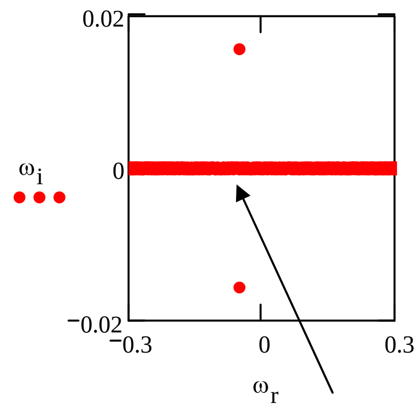
$$\frac{L}{\rho_{s0}} = 50,\ \varepsilon_n = 0.02,\ \Gamma = 0.05,\ k_y \rho_{s0} = 0.2$$

$$k_y \rho_{s0} = 0.394$$

$$\Omega_r = -0.048,\ \ \Omega_i = 0.016$$







STABLE CONTINUUM MODES

NONLOCAL EIGENVALUE-VII

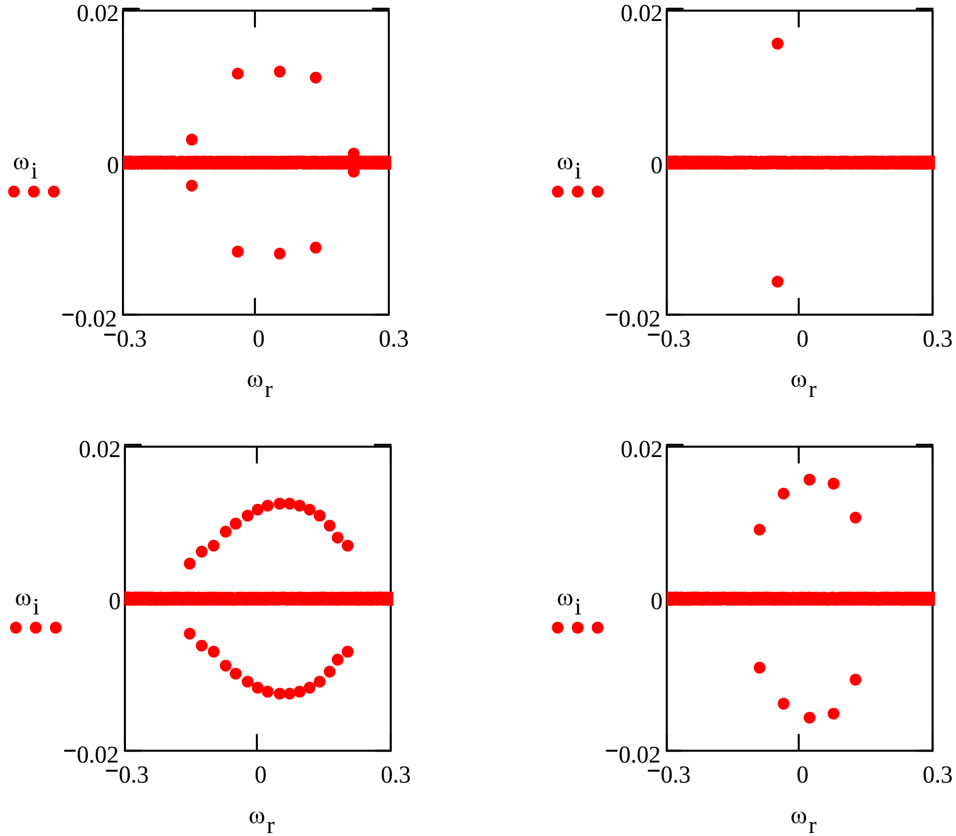
$\epsilon_{\parallel} = 0.02,\ \Gamma = 0.05$

$k_y\rho_{s0} = 0.1$
$k_{0x}\rho_{s0} = 0.584$

$k_y\rho_{s0} = 0.2$
$k_{0x}\rho_{s0} = 0.394$

$L/\rho_{s0} = 50$

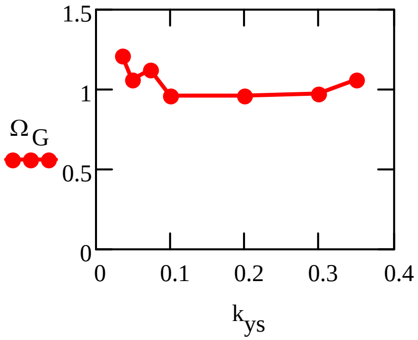
$L/\rho_{s0} = 200$



ROBUSTNESS IN WIDTH OF SPECTRUM OF UNSTABLE MODES $\Delta\omega/\omega \sim 0.3$

NONLOCAL EIGENVALUE-VIII

Frequency of maximally growing modes as a function of $k_y\rho_s$



• For 500% spread in drift frequency($k_y\rho_s$), there is only a 20% spread in the frequency of the excited GAM mode (with maximal growth)

• Thus broad spectrum of drift waves can excites almost "monochromatic" GAM as seen in the bicoherence spectrum

BISPECTRUM

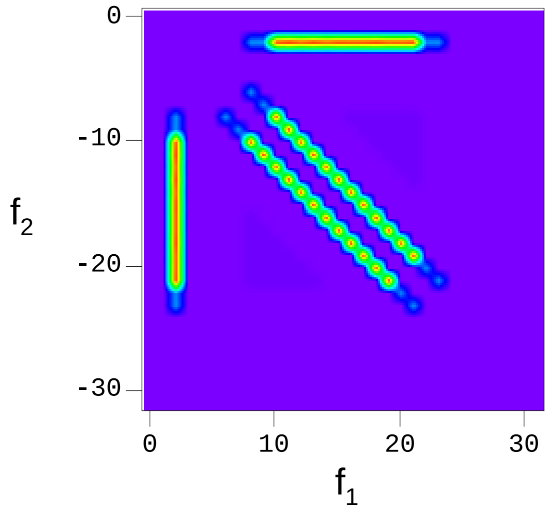
$$f_G = f_p - f_s$$

$i_{max} := 16$

$$f(t) := \left(i_{max} + 1\right)\sin\left(4\pi\cdot t\right) + \sum_{i=0}^{i_{max}} \sin\left[2(8+i)\,\pi\cdot t\right] + \sum_{i=0}^{i_{max}} \sin\left[2\cdot(8+i-2)\,\pi\cdot t\right]$$

GAM PUMP SIDE-BAND

$B(f_1, f_2)$



CONCLUSIONS

•DEVELOPED LOCAL THEORY FOR EXCITATION OF GAMS BY PARAMETRIC THREE-WAVE COUPLING TO DRIFT/ITG MODES

•STUDIED FINITE BETA EFFECTS ON EXCITATION OF GAMs BY DRIFT WAVES

•FINITE BETA LEADS TO STRONG STABILIZING OF GAM EXCITATION AS $\hat{\beta} = \beta q^2 R^2 / 2L_n^2$ INCREASES

• ARE GAMS IMPORTANT FOR L-H TRANSITION ? PRELIMINARY STUDY SEEMS TO INDICATE THEY ARE SUPRESSED BEFORE PLASMA REACHES L-H THRESHOLD, MORE WORK NEEDED EXPERIMENTAL AND THEORY

• WITH BOTH ITG/DRIFT BRANCHES PRESENT POSSIBILITY OF THREE-WAVE MODE COUPLING BETWEEN DIFFERENT BRANCHES

•NONLOCAL ANALYSIS SHOWS THE EXISTENCE OF TWO SPACE SCALES, DIFFERENT MEASUREMENT TECHNIQUES MEASURE “FAST” AND “SLOW” SCALES HENCE VERY LARGE SPREAD IN WAVE NUMBERS REPORTED

•GAM FREQUENCY NEAR STEEPEST TEMP/DENSITY GRADIENT DETERMINES EIGENFREQUENCY AND BROADBAND SPECTRUM OF DRIFT WAVES EXCITE GAMS WITH SMALL BANDWIDTH