Parametric dependence of particle pinch coefficients for electron particle transport in linear gyrokinetic theory

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Outline

- Motivations
- Particle transport
- Linear gyrokinetic theory
- Pinch coefficients calculation
- Comparison with experiments
- Conclusions

Motivations

- Show that **linear gyrokinetic theory** can be used to **interpret** known experimental results and make **concrete predictions**
- Understand the behavior of experimental **stationary density profiles** in different **Tokamak** plasma **scenarios**

Basic particle transport equation

- **Particle continuity equation** for arbitrary species (electrons):

$$\frac{1}{V'}\frac{\partial}{\partial t}(V'n) + \frac{1}{V'}\frac{\partial}{\partial \rho}(V'\Gamma) = S$$

- The **density profile** is determined once the **flux Γ** is known
- The flux is generated by **different physical processes**:

$$\boxed{\Gamma = \Gamma_{\mathrm{neo}} + \Gamma_{\mathrm{turb}} + \cancel{\Gamma_{\mathrm{MHD}}}}$$

Not discussed here

Generic local particle flux

• Generic local form for ***neo***classical and ***turb***ulent **fluxes**:

$$\Gamma_i^{3D} = \underbrace{-D_i \nabla n}_{\text{Diffusion}} + \underbrace{V_i n}_{\text{Convection}}$$

V

$V_i < 0$ → ***inward*** convection / $V_i > 0$ → ***outward*** convection

After **flux-surface averaging**:

$$\boxed{\Gamma_i = -g_{2\rho} n D_i \left[\frac{1}{n}\frac{\partial n}{\partial \rho} - \frac{g_{1\rho}}{g_{2\rho}}\frac{V_i}{D_i} \right]}$$

with metrics: $g_{1\rho} = < |\nabla \rho| >$ and $g_{2\rho} = < |\nabla \rho|^2 >$

Stationary condition: $\Gamma = 0$

- Neglect **core sources** and **neoclassical transport** to have:

$$\frac{1}{V'}\cancel{\frac{\partial}{\partial t}}(V'n) + \frac{1}{V'}\frac{\partial}{\partial \rho}(V'\Gamma) = \cancel{S} \Rightarrow \boxed{\Gamma_{\mathrm{turb}} = 0}$$

0 0 0

$$\Gamma \downarrow \quad \cancel{\Gamma_{\mathrm{neo}}} + \Gamma_{\mathrm{turb}}$$

$$\Gamma_{\mathrm{turb}} = 0 \Rightarrow \boxed{-\frac{1}{n}\frac{\partial n}{\partial \rho} = -\frac{g_{1\rho}}{g_{2\rho}}\frac{V_{turb}}{D_{turb}}}$$

A peaked **density profile** *is sustained by a*

finite inward convection term *driven by turbulence*

V_{turb}/D_{turb} from linear gyrokinetic equation

- **GK** equation: linear, ballooning, ES, spectral *[G. Rewoldt et al 1982]*
- Solve for non-adiabatic part of $\tilde{f}$: $\tilde{f} = \tilde{g} + \frac{e\tilde{\Phi}}{T} F_{\mathrm{M}}$

$$\boxed{\left(-i\omega + iMk_{\|}v_{\|} + i\omega_{\mathrm{d}}\hat{v}_{\mathrm{z}}\right)\tilde{g} = i\tau\left(\omega + \omega^*\right)F_{\mathrm{M}}\tilde{\Phi}}$$

with definitions:

$$\hat{v}_{\mathrm{z}} = \frac{1}{\tau}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)$$

$$M = \sqrt{\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}}}\sqrt{\frac{1}{\tau}}$$

$$\tau = T_{\mathrm{i}}/T_{\mathrm{e}}$$

$$\omega_{\mathrm{d}} = k_{\mathrm{y}}\rho_{\mathrm{i}}\left[\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta\right]$$

$$\omega^* = \frac{k_{\mathrm{y}}\rho_{\mathrm{i}}}{\tau}\left[-\frac{1}{n}\frac{\partial n}{\partial\rho} - \frac{1}{T_{\mathrm{e}}}\frac{\partial T_{\mathrm{e}}}{\partial\rho}\left(E - \frac{3}{2}\right)\right]$$

Gyrokinetic particle flux

- The ES-turbulent flux is by definition (**one mode considered**):

$$\Gamma_{\mathrm{turb}} \propto \Re\left(-ik_{\mathrm{y}}\rho_{\mathrm{i}}\int \tilde{g}\tilde{\Phi}^* d^3v\right) \propto -\frac{1}{n}\frac{\partial n}{\partial \rho} + \frac{g_{1\rho}}{g_{2\rho}}\frac{V}{D}$$

- Substituting the formal solution for $\tilde{g}$ and the expression for ***ω****

$$\frac{\Gamma_{\mathrm{turb}}}{k_{\mathrm{y}}\rho_{\mathrm{i}}|\tilde{\Phi}|^2} \propto \boxed{-\Im\left(\int d^3v F_{\mathrm{M}}\frac{k_{\mathrm{y}}\rho_{\mathrm{i}}}{\omega - Mk_{\|}v_{\|} - \omega_{\mathrm{d}}\hat{v}_{\mathrm{z}}}\right)\frac{1}{n}\frac{\partial n}{\partial \rho}}$$

Diffusion

$$\boxed{-\Im\left(\int d^3v F_{\mathrm{M}}\frac{k_{\mathrm{y}}\rho_{\mathrm{i}}(E-3/2)}{\omega - Mk_{\|}v_{\|} - \omega_{\mathrm{d}}\hat{v}_{\mathrm{z}}}\right)\frac{1}{T_{\mathrm{e}}}\frac{\partial T_{\mathrm{e}}}{\partial \rho}}$$

Thermo-diffusion

$$\boxed{-\tau\Im\left(\int d^3v F_{\mathrm{M}}\frac{\omega}{\omega - Mk_{\|}v_{\|} - \omega_{\mathrm{d}}\hat{v}_{\mathrm{z}}}\right)}$$

Curvature + parallel pinch

V_{turb}/D_{turb} contributions

• Identify **V/D**:

C_T (<0 inward)

$$\frac{V}{D} = -\boxed{\frac{\Im\left(\int d^3v F_M \frac{(E-3/2)}{\omega - M k_{\|} v_{\|} - \omega_d \hat{v}_z}\right)}{\Im\left(\int d^3v F_M \frac{1}{\omega - M k_{\|} v_{\|} - \omega_d \hat{v}_z}\right)}} \frac{1}{T_e}\frac{\partial T_e}{\partial \rho} + \boxed{- \frac{\tau}{k_y \rho_i} \frac{\Im\left(\int d^3v F_M \frac{\omega}{\omega - M k_{\|} v_{\|} - \omega_d \hat{v}_z}\right)}{\Im\left(\int d^3v F_M \frac{1}{\omega - M k_{\|} v_{\|} - \omega_d \hat{v}_z}\right)}}$$

C_P/R

C_T → Thermodiffusion coefficient

C_P → Pure convection coefficient

Stationary density profile

$\Gamma_{\mathbf{turb}} = \mathbf{0}$ gives:

$$\frac{R}{L_{\rm n}} = -R\frac{V}{D} = -C_{\rm T}\frac{R}{L_{\rm Te}} - C_{\rm P}$$

- **Coefficients depend on plasma equilibrium parameters!**
- $\mathbf{C_x = C_x(\omega, k_y \rho_i, k_{||}, \nu_{eff}, \ldots), \ \omega = \omega(R/L_n, R/L_{Te}, R/L_{Ti}, s, q, \nu_{eff}, \ldots)}$
- Stationary $\mathbf{R/L_n}$ has to satisfy above ***non-linear*** expression
- **Passive** species (i.e. small concentration) → ***linear*** expression

Phase space contributions to C_T and C_P

- For trapped particles: assume $\langle k_{||} v_{||} \rangle_b \sim 0$
- For passing particles: assume $|M k_{||} v_{||}| >> |\omega|$, $|\omega_d v_z|$
- Introduce trapped particle fraction f_t
- Solve imaginary part dividing mode frequency in $\omega = \omega_R + i\,\gamma$

$$C_{\rm T} = \frac{f_{\rm t}\int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma(E-3/2)}{\gamma^2+(\omega_{\rm R}-\omega_{\rm d}\frac{E}{2})^2} + \frac{(1-f_{\rm t})}{4}\frac{1}{Mk_{||}}}{f_{\rm t}\int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\rm R}-\omega_{\rm d}\frac{E}{2})^2} - \frac{(1-f_{\rm t})}{2}\frac{1}{Mk_{||}}}$$

$$C_{\rm P} = -\tau\frac{\omega_{\rm d}}{k_{\rm y}\rho_{\rm i}}\frac{f_{\rm t}\int_0^{+\infty} dE\frac{E}{2}\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\rm R}-\omega_{\rm d}\frac{E}{2})^2} - \frac{(1-f_{\rm t})\omega_{\rm R}}{\pi Mk_{||}}}{f_{\rm t}\int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\rm R}-\omega_{\rm d}\frac{E}{2})^2} - \frac{(1-f_{\rm t})\omega_{\rm d}}{\pi Mk_{||}}}$$

Passing electrons pinch at $f_t \sim 0$

$$C_{\mathrm{T}} = \frac{f_{\mathrm{t}} \int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma(E-3/2)}{\gamma^2+(\omega_{\mathrm{R}}-\omega_{\mathrm{d}}\frac{E}{2})^2} + \frac{(1-f_{\mathrm{t}})}{4}\frac{1}{Mk_{\|}}}{f_{\mathrm{t}} \int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\mathrm{R}}-\omega_{\mathrm{d}}\frac{E}{2})^2} - \frac{(1-f_{\mathrm{t}})}{2}\frac{1}{Mk_{\|}}}$$

$$C_{\mathrm{P}} = -\tau\frac{\omega_{\mathrm{d}}}{k_{\mathrm{y}}\rho_{\mathrm{i}}}\frac{f_{\mathrm{t}} \int_0^{+\infty} dE\frac{E}{2}\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\mathrm{R}}-\omega_{\mathrm{d}}\frac{E}{2})^2} - \frac{(1-f_{\mathrm{t}})\omega_{\mathrm{R}}}{\pi M k_{\|}}}{f_{\mathrm{t}} \int_0^{+\infty} dE\sqrt{E}e^{-E}\frac{\gamma}{\gamma^2+(\omega_{\mathrm{R}}-\omega_{\mathrm{d}}\frac{E}{2})^2} - \frac{(1-f_{\mathrm{t}})\omega_{\mathrm{d}}}{\pi M k_{\|}}}$$

- $C_T = -1/2$ → $N_e \sim T_e^{1/2}$ [F. Miskane *et al.*, *POP* ***7*** 4197 (2000)]
- $C_P \sim -\tau\, \omega_R / k_y\rho_i$ → **inward** for **TEM/ETG ($\omega_R > 0$)**
  outward for **ITG ($\omega_R < 0$)**
- C_P is analogous to the impurity pinch found in
  Angioni and Peeters, PRL 2006

Trapped electrons pinch at $f_t \sim 1$

$$C_T = \frac{f_t \int_0^{+\infty} dE\sqrt{E}e^{-E} \frac{\gamma(E-3/2)}{\gamma^2+(\omega_R-\omega_d\frac{E}{2})^2} + \frac{(1-f_t)}{4}\frac{1}{Mk_{||}}}{f_t \int_0^{+\infty} dE\sqrt{E}e^{-E} \frac{\gamma}{\gamma^2+(\omega_R-\omega_d\frac{E}{2})^2} - \frac{(1-f_t)}{2}\frac{1}{Mk_{||}}}$$

$$C_P = -\tau\frac{\omega_d}{k_y\rho_i}\frac{f_t \int_0^{+\infty} dE\frac{E}{2}\sqrt{E}e^{-E} \frac{\gamma}{\gamma^2+(\omega_R-\omega_d\frac{E}{2})^2} - \frac{(1-f_t)\omega_R}{\pi M k_{||}}}{f_t \int_0^{+\infty} dE\sqrt{E}e^{-E} \frac{\gamma}{\gamma^2+(\omega_R-\omega_d\frac{E}{2})^2} - \frac{(1-f_t)\omega_d}{\pi M k_{||}}}$$

- **C_T (ω_R) → inward** in **ITG**; **smaller/outward** in **TEM**
- **$C_P \sim - \tau\, \omega_d / k_y \rho_i$ →** always **inward ($\omega_d > 0$** for monotonic **q)**
- For **$\omega_d < 0$** the picture is **reversed ($\omega_d < 0$** for very negative **s)**

Quasi-linear rules

- Γ defined for each $\mathbf{k_y \rho_i}$:
- *Quasi-linear* sum:

$$\Gamma_k = A_k \frac{R}{L_n} + B_k \frac{R}{L_{\mathrm{Te}}} + C_k$$

$$C_{\mathrm{T}} = \frac{\sum_k |\tilde{\Phi}_k|^2 B_k}{\sum_k |\tilde{\Phi}_k|^2 A_k}$$

$$C_{\mathrm{P}} = -R \frac{\sum_k |\tilde{\Phi}_k|^2 C_k}{\sum_k |\tilde{\Phi}_k|^2 A_k}$$

- **(1)** We consider mode at highest $\boldsymbol{\gamma / \langle k_\perp^2 \rangle}$ (*Jenko et al., PPCF 2005*)

Other rules:

- (2) Sum using $|\mathbf{\Phi}|^2 \sim (\gamma / \langle k_\perp^2 \rangle)^\alpha$ with $\mathbf{1 \leq \alpha \leq 2}$
- (3) Sum with a particular potential (*Bourdelle et al., POP 2007*)
- … and so on (to be compared with **non-linear** simulations!)

Gyrokinetic code

Gyrokinetic flux-tube code **GS2**

[*Kotschenreuther et al., Comp. Phys. Comm. 1995*]

- Linear, electrostatic
- Fully kinetic D-ions and electrons
- ***s**-α* magnetic equilibrium
- No impurities

Parameters scan motivations

(A) Typical **monotonic q profile** cases

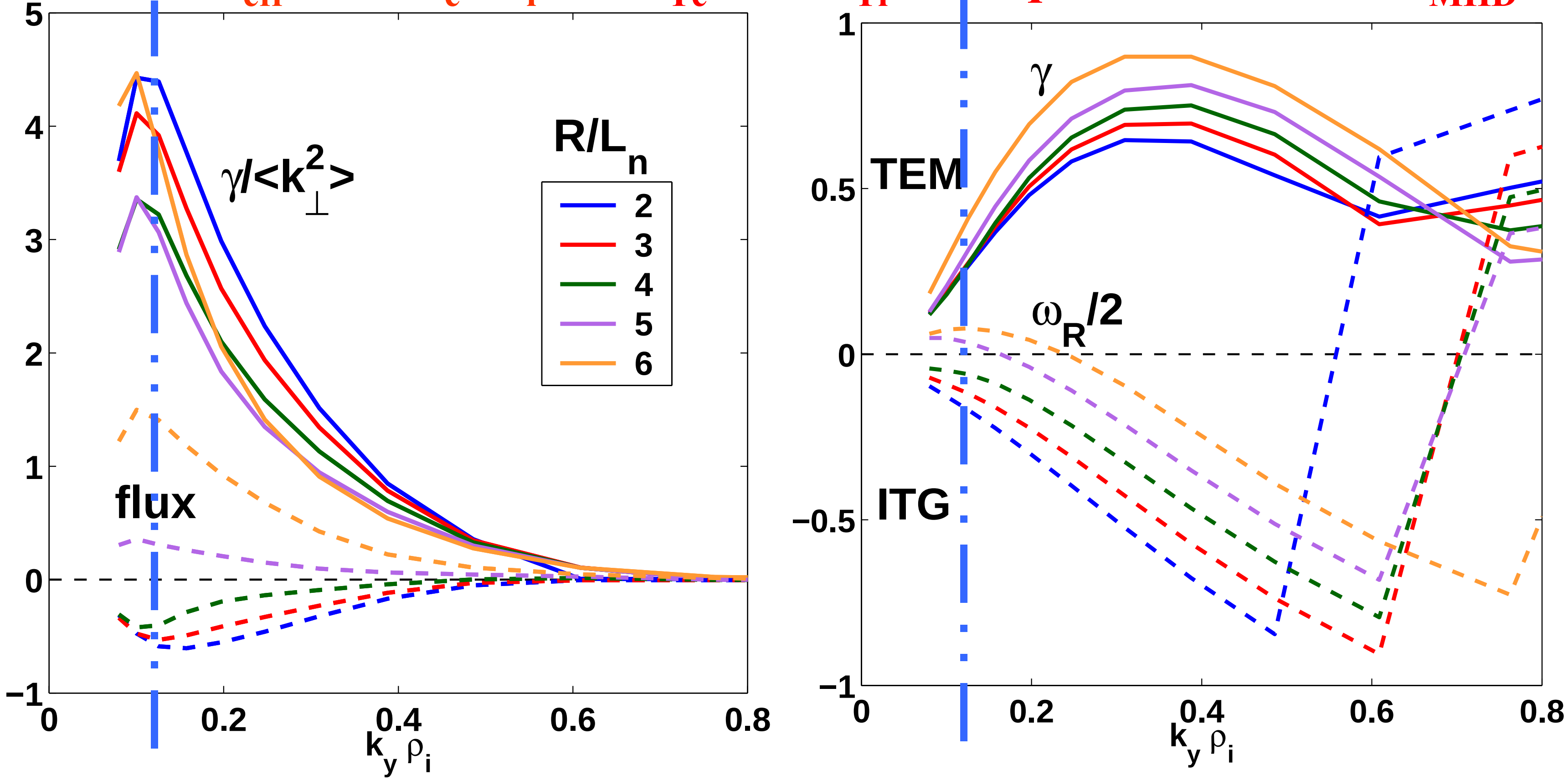
Understand behavior of density peaking and compare with known experimental results

(B) Negative magnetic shear scenario

Understand **thermodiffusion pinch** in **'electron Internal Transport Barriers'** observed in TCV *[E. Fable PPCF 2006]*

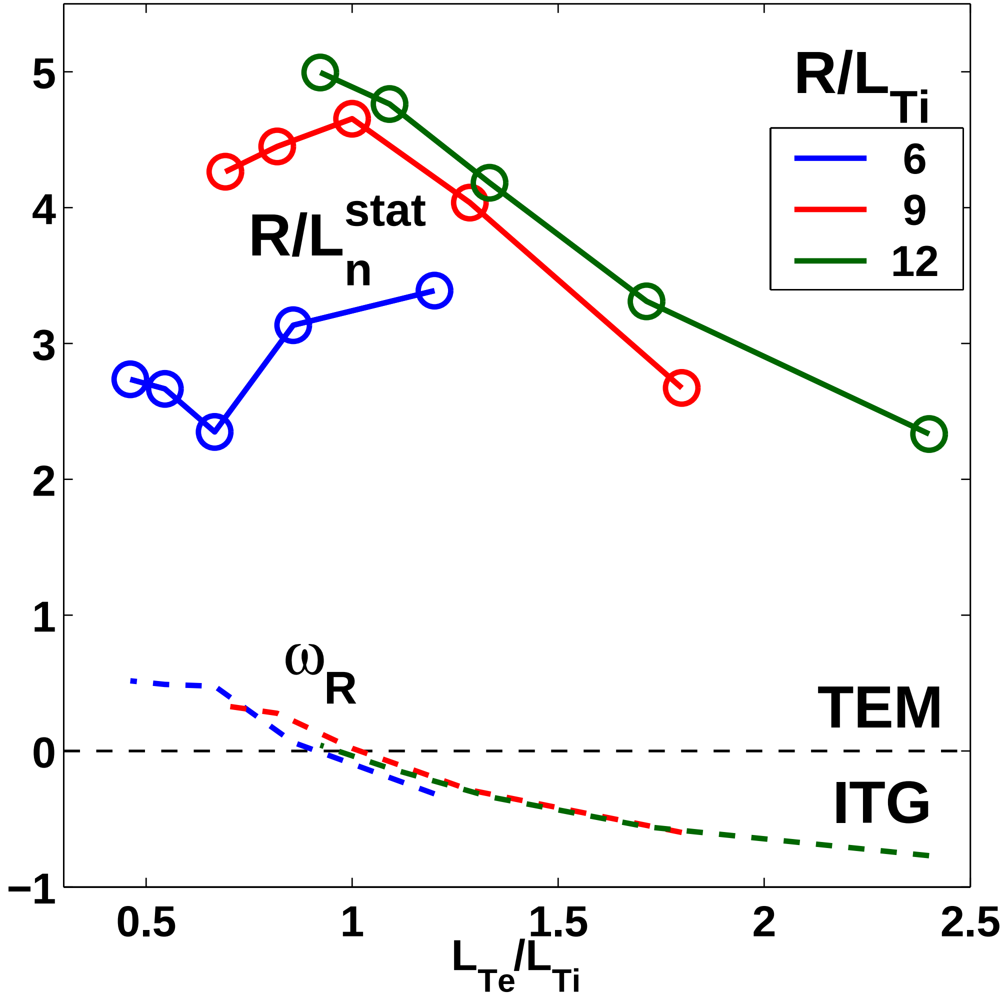
Monotonic q: typical spectrum

$\varepsilon = 0.11$; $\nu_{eff} = 0$; $T_e = T_i$; $R/L_{Te} = R/L_{Ti} = 9$; $q = 1.4$; $s = 0.8$; $\alpha_{MHD} = 0$



- Most of the transport in the range $\mathbf{k_y\rho_i \sim 0.08 - 0.3}$
- Flux goes from **negative (inward)** to **positive (outward)** increasing $\mathbf{R/L_n}$
- Dominant mode, $\mathbf{k_y\rho_i \sim 0.12}$, goes from an ***ITG*** to a ***TEM*** type increasing $\mathbf{R/L_n}$

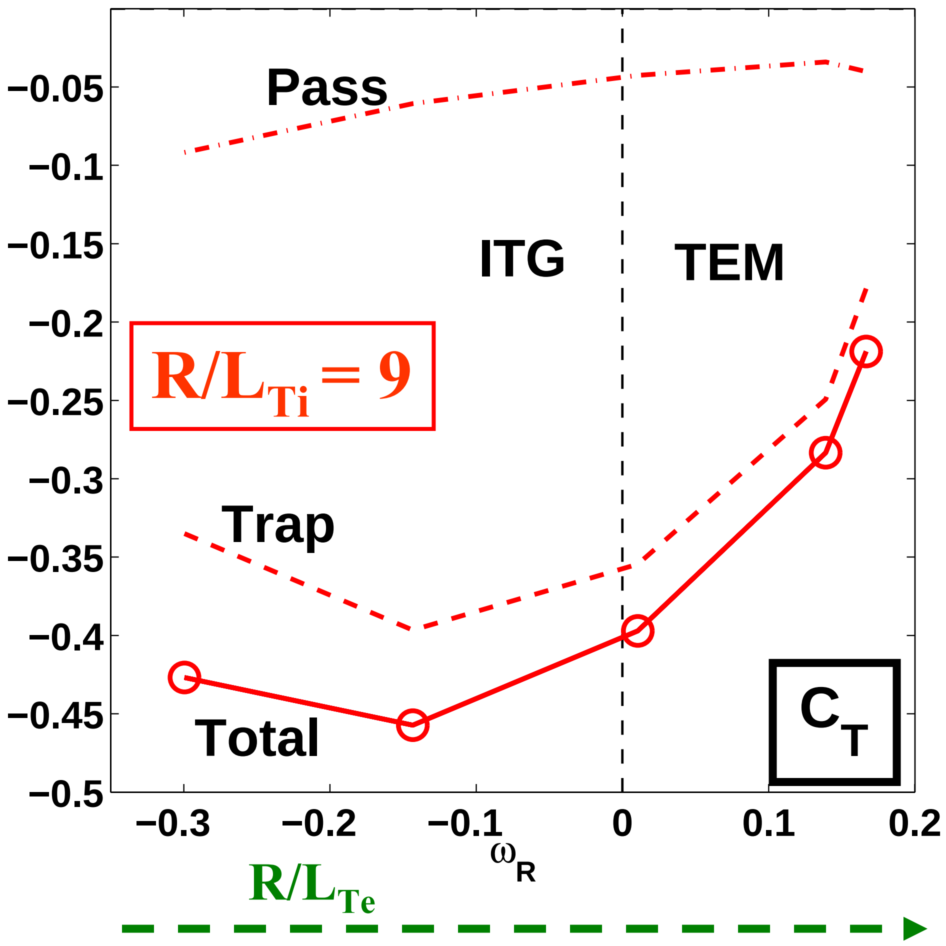
Mode dependence on main gradients

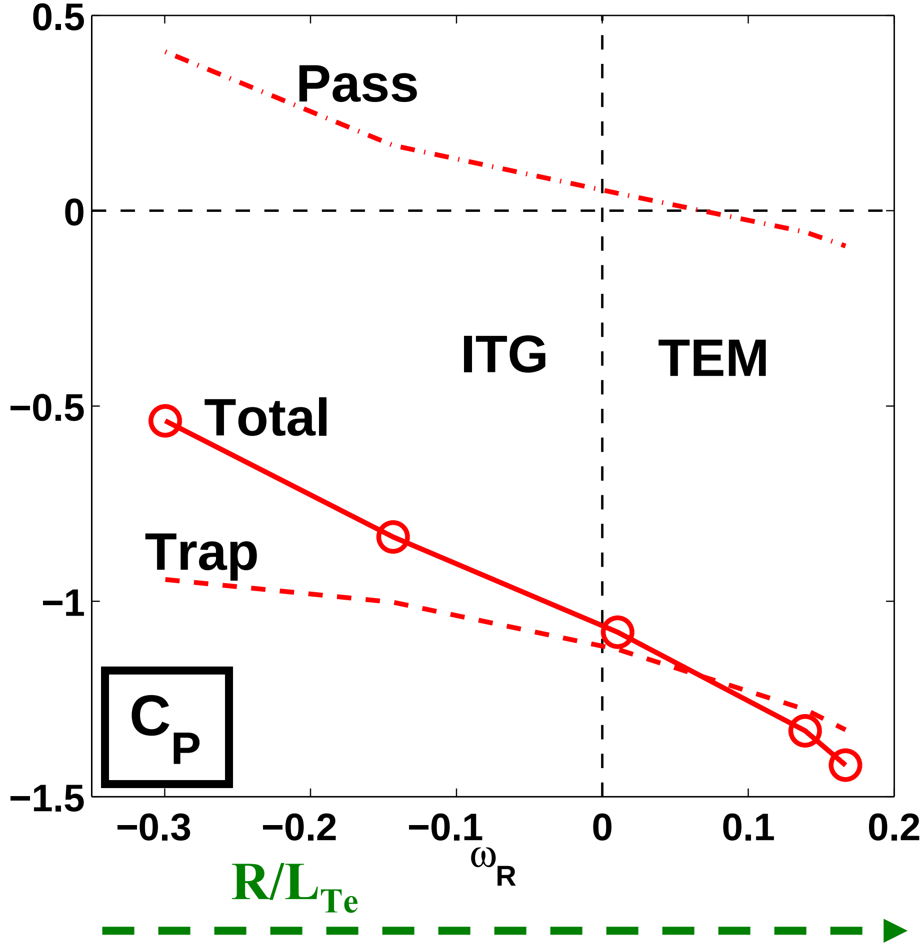


• **TEM** dominant for $\mathbf{L_{Te}/L_{Ti} < 1}$, **ITG** dominant for $\mathbf{L_{Te}/L_{Ti} > 1}$

• Stationary $\mathbf{R/L_n}$ tends to be **maximized** when the $\boldsymbol{\omega_R \sim 0}$

General mechanism dependence on mode type

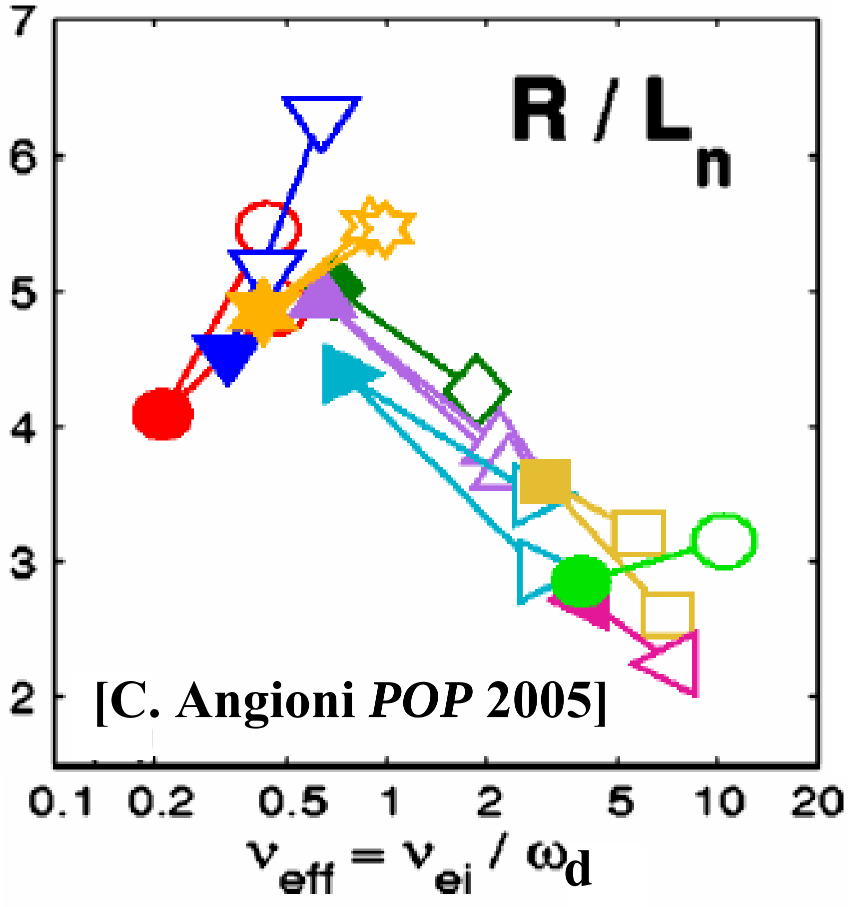


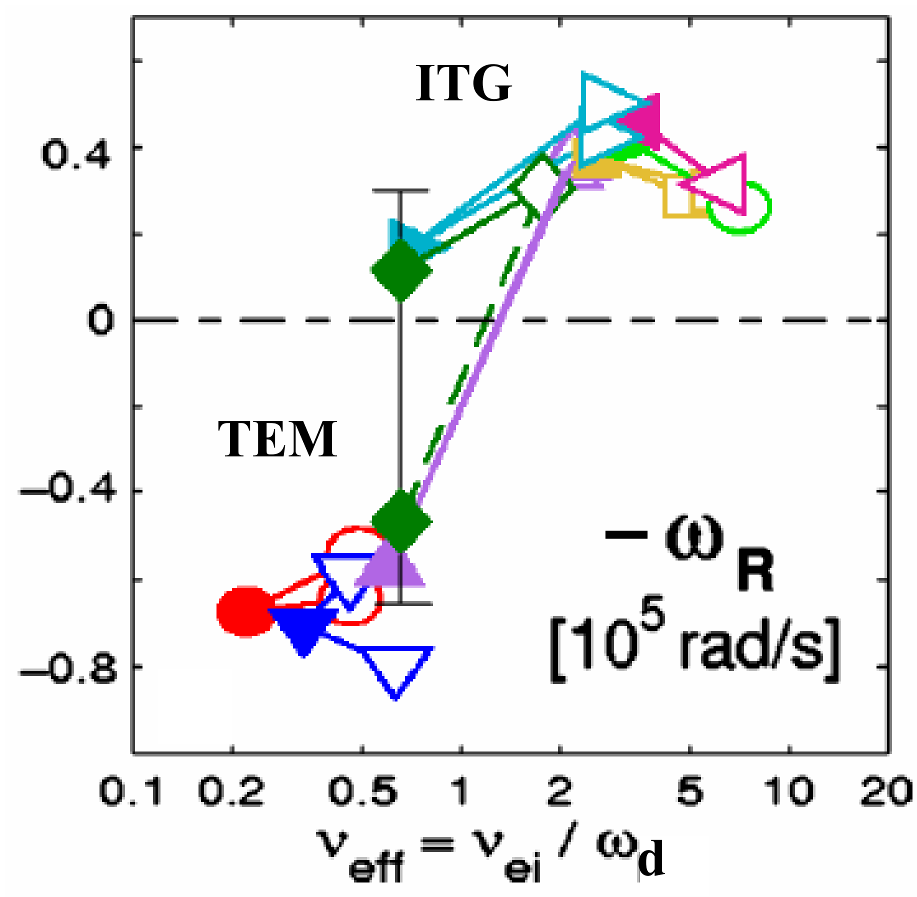


- **Thermodiffusion (C_T): inward**; $|C_T|$ higher in **ITG, smaller** in **TEM**
- **C_P: Inward;** $|C_P|$ higher in **TEM**

  Passing electrons **C_P outward** for **ITG** turbulence

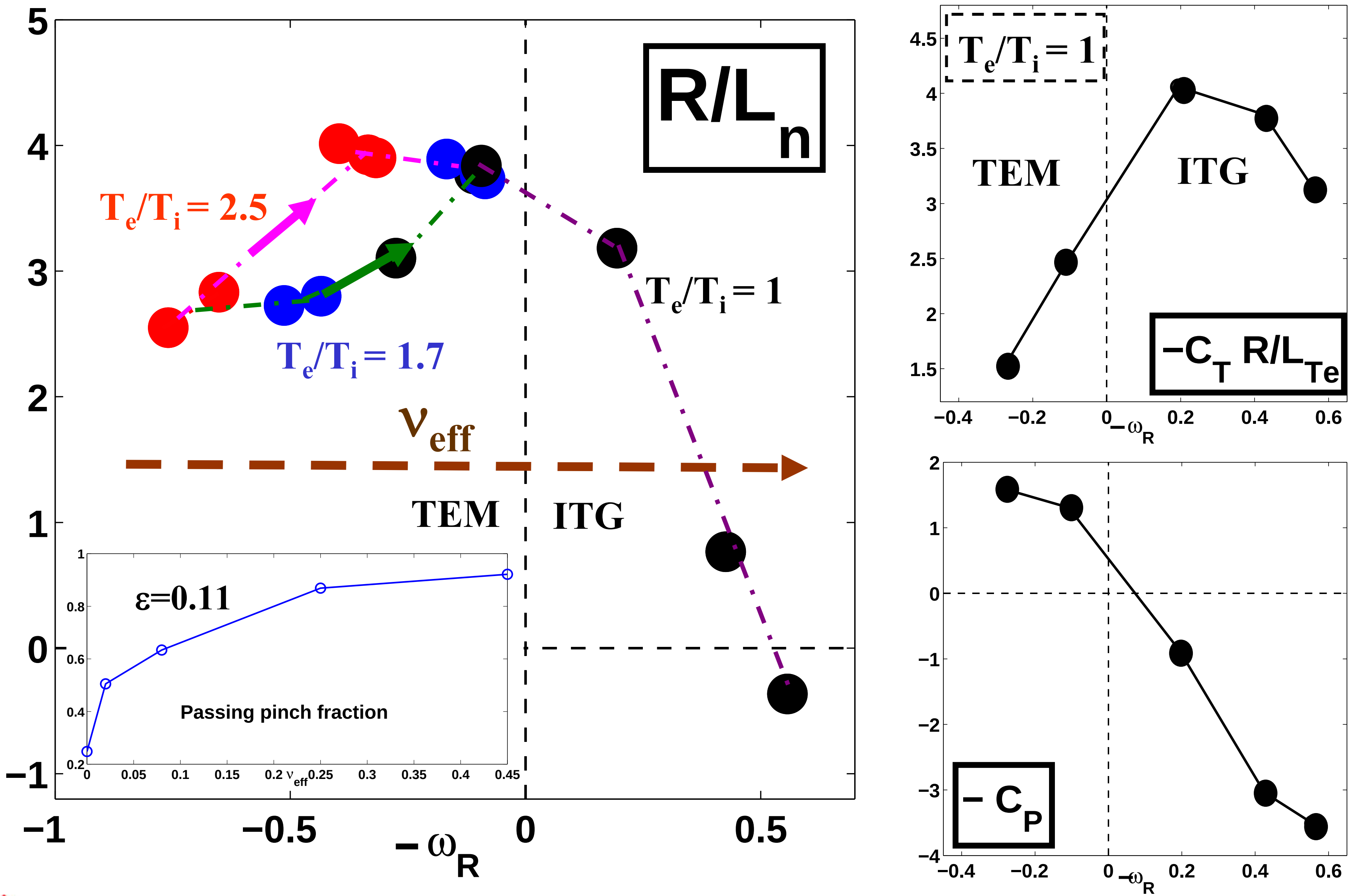
Experimental behavior vs mode type



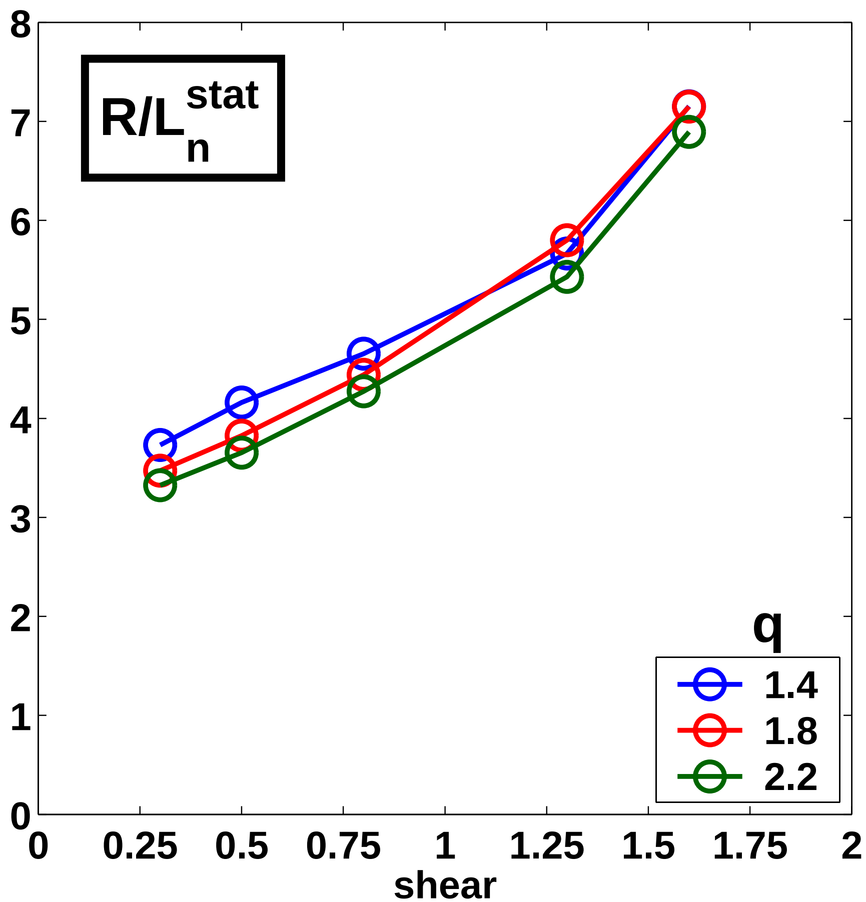


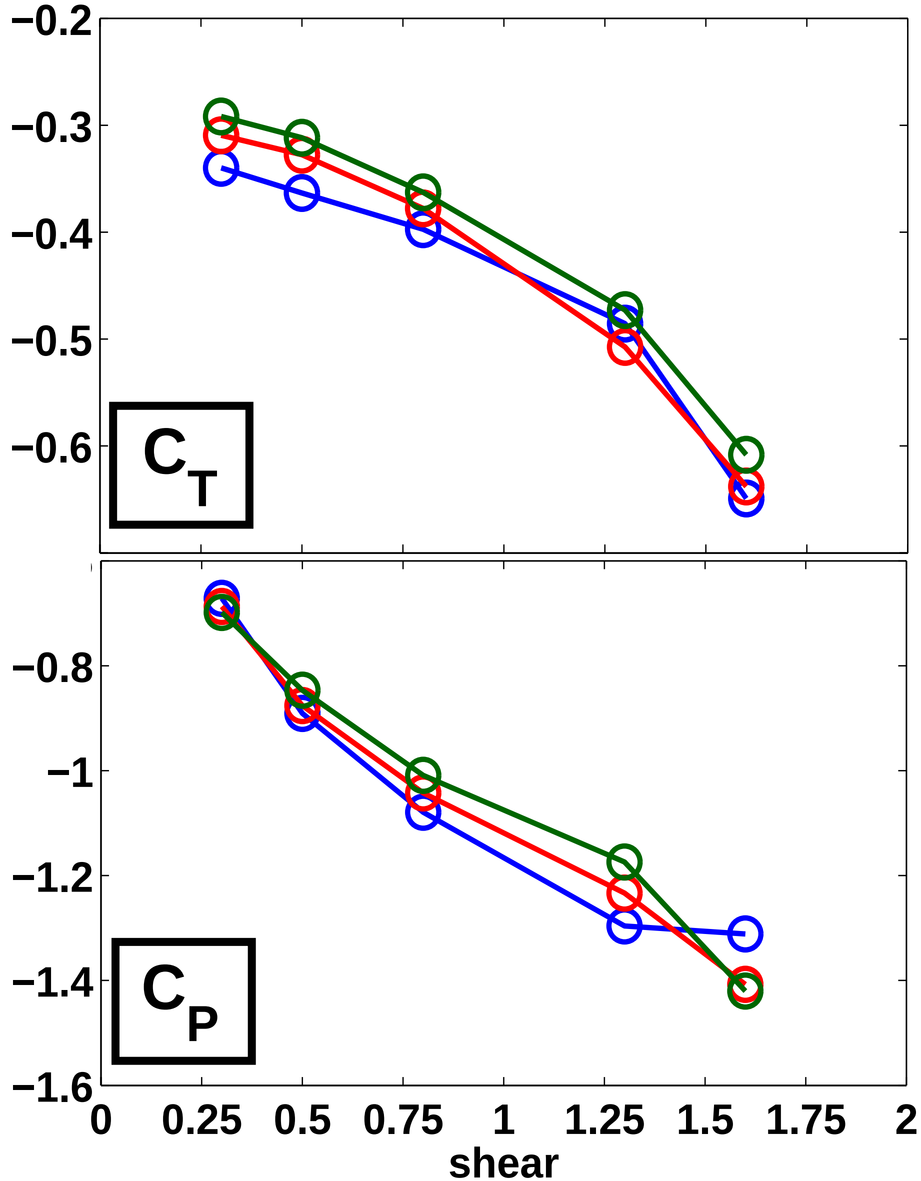
- $\mathbf{R/L_n}$ reaches maximum at ν_{eff} where $\omega_R \sim \mathbf{0}$, i.e. **TEM → ITG**
- Similar to $\mathbf{L_{Te}/L_{Ti}}$ scan, in this case 'mode' driving parameter is collisionality
- Underlying 'universal' mechanism in terms of $\mathbf{C_T}$ and $\mathbf{C_P}$?

Predicted stationary R/L_n versus ν_{eff}



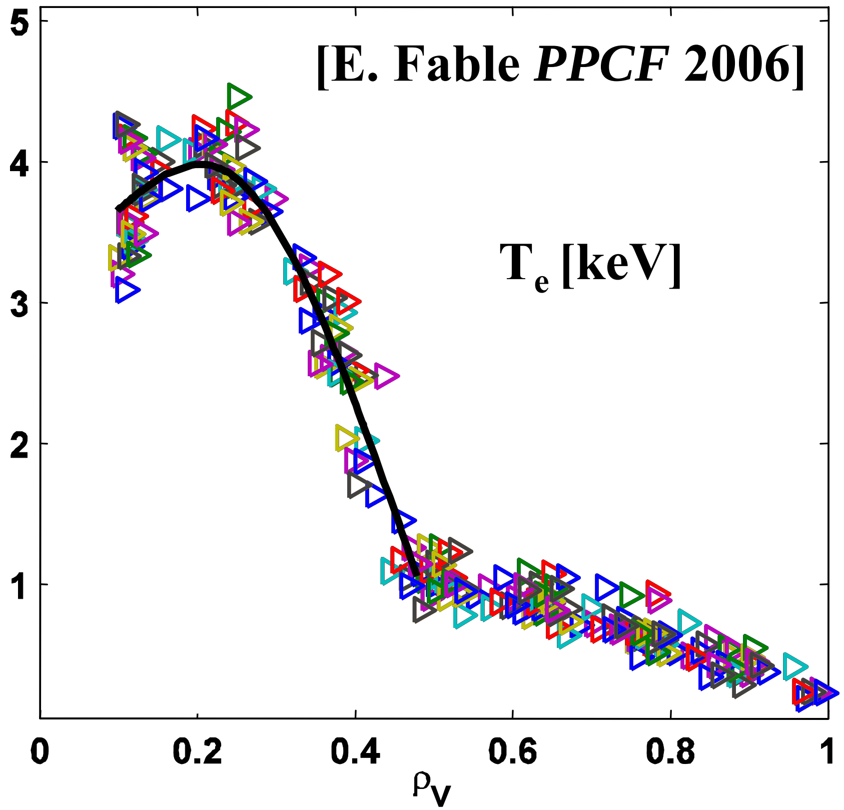
Shear dependence of density peaking

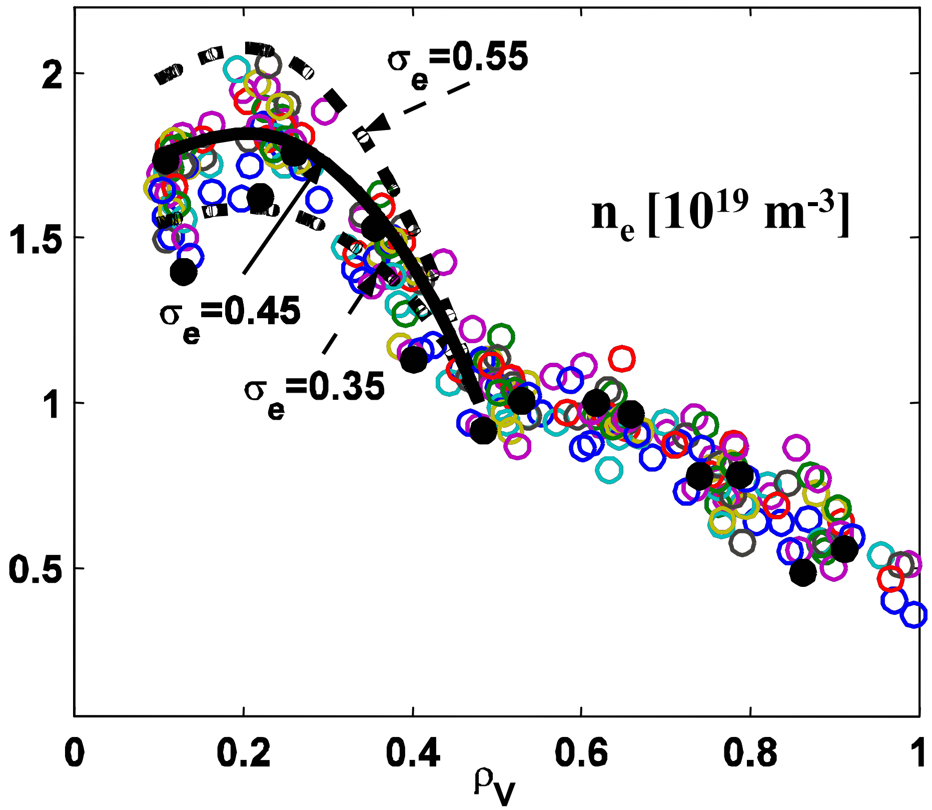




- **shear** ↑ →increase density peaking ($\omega_R \sim 0$)
- **q** → negligible effect
- C_T & C_P proportional to shear
- Trend found in [H. Weisen *et al.*, *Nuclear Fusion* (2005)]

eITB thermodiffusive pinch in TCV

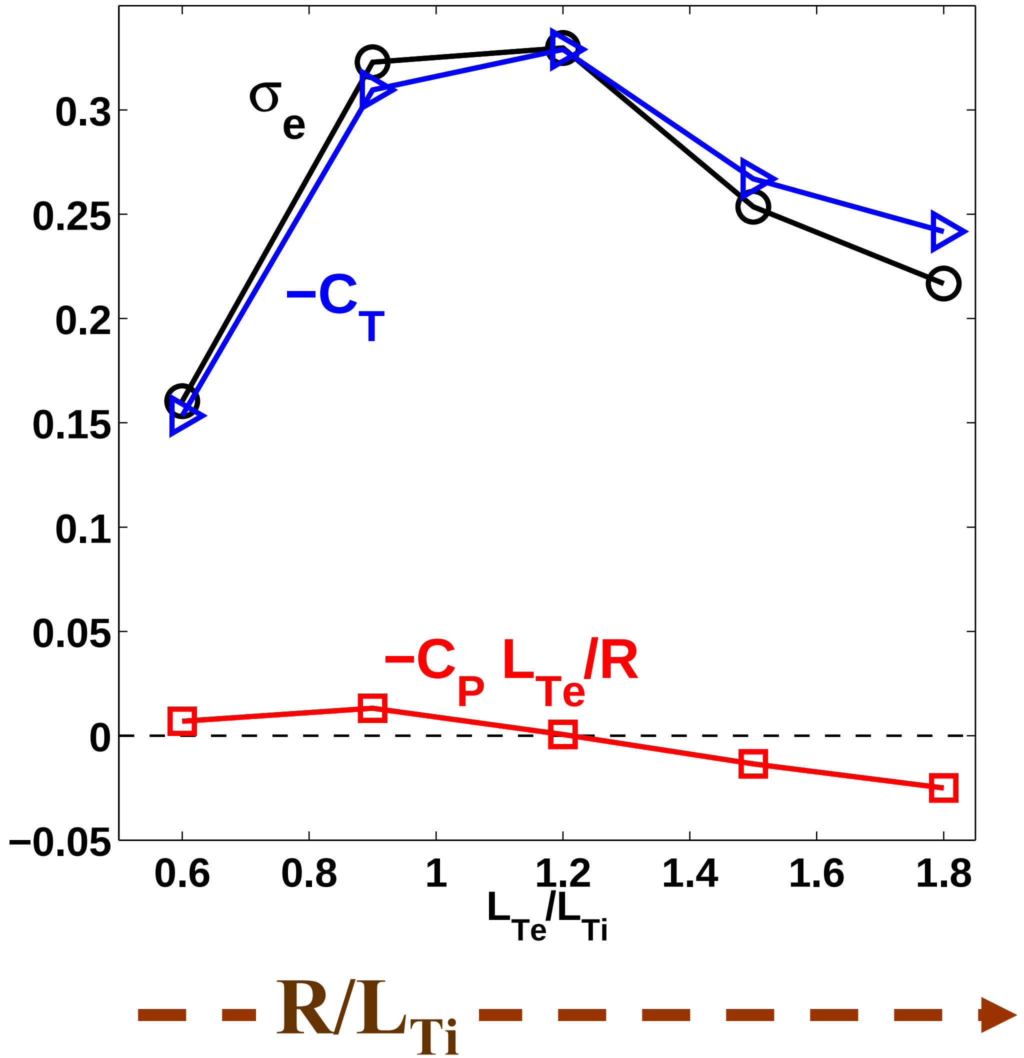




- Fully non inductive plasmas obtained with off-axis co-current drive
- Core **negative magnetic shear** [M. Henderson *et al.*, *PRL* (2004)]
- $\mathbf{T_e}$ and $\mathbf{n_e}$ barriers appear with similar structure, **correlation between gradients**
- In high gradient region, $\boldsymbol{\sigma_e = 1/\eta_e \sim 0.45}$
- This value decreases if confinement is deteriorated (barrier disappears)

Dominant thermodiffusive pinch

$\varepsilon = 0.11$; $\nu_{eff} = 0$; $T_e/T_i=2.8$; $R/L_{Te} = 20$; $q = 3$; $s = -0.7$; $k_y\rho_i = 0.23$



- Stationary state (black)

  $\sigma_e = L_{Te}/L_n$ **maximized** at

  $L_{Te}/L_{Ti} \sim 1$

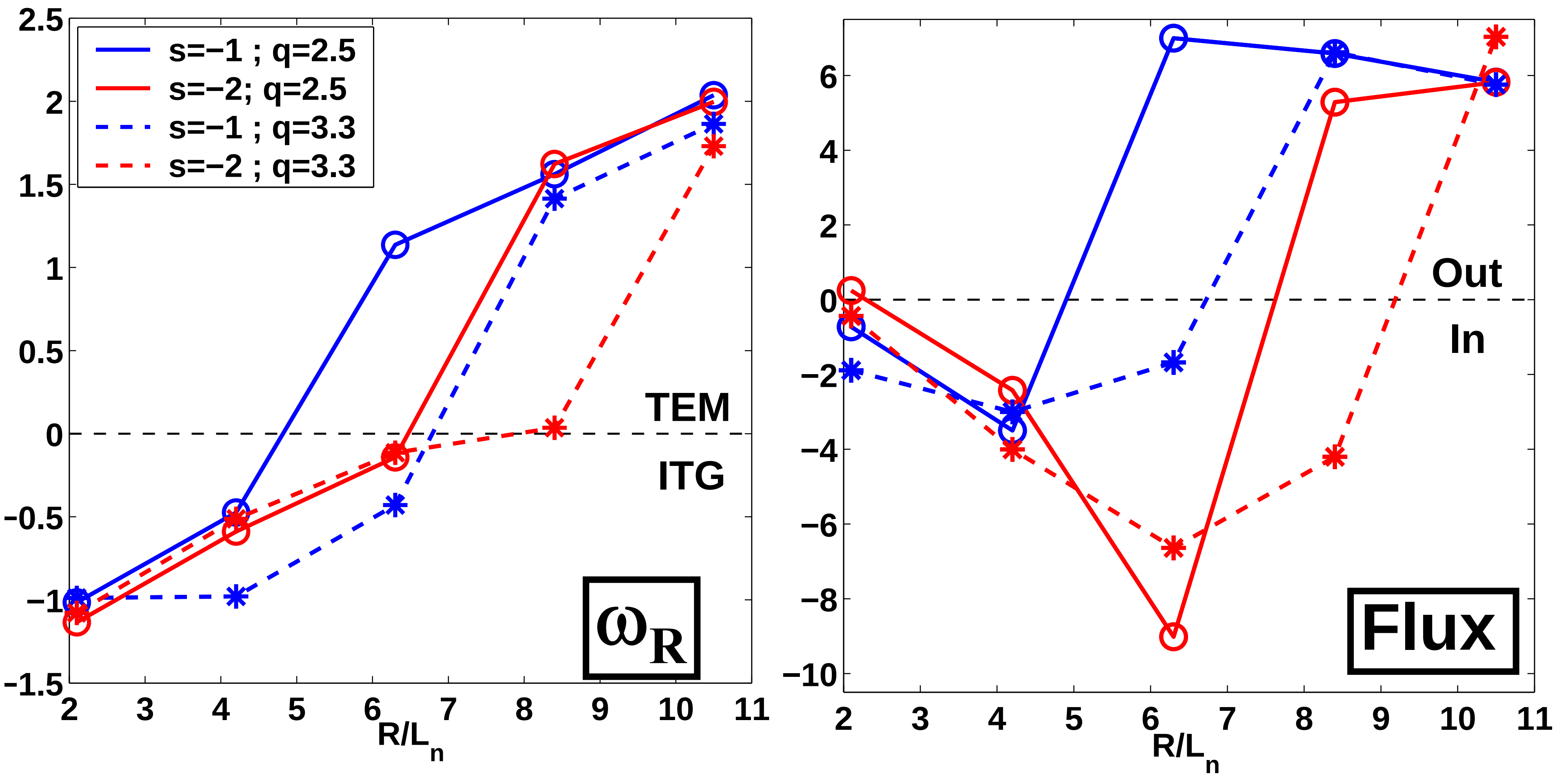
- **Thermodiffusion** (blue) is dominant

- C_P contribution (red) is **negligible**, both because:

  L_{Te}/R is **small**

  ω_d is **small** ($s < 0$)

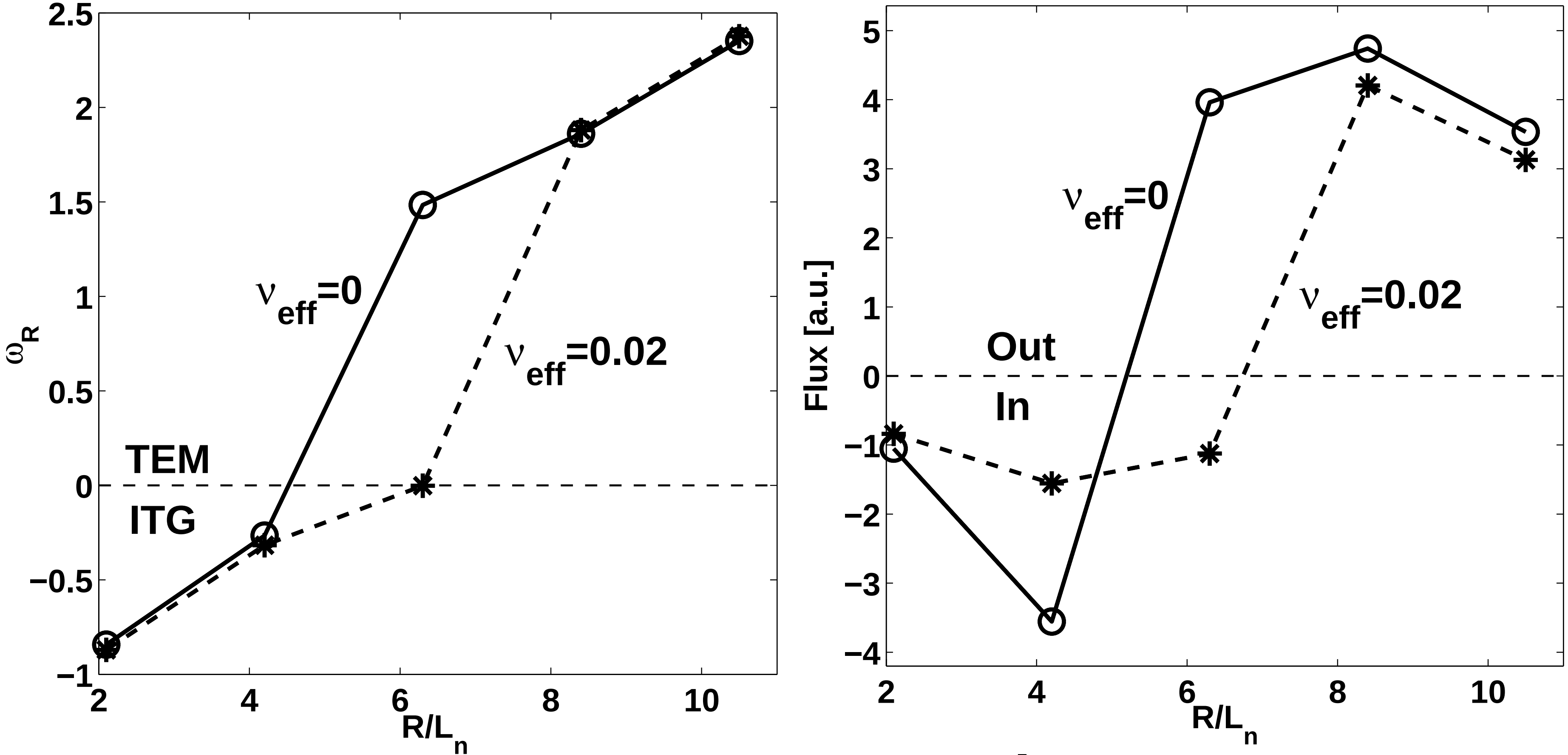
Increase stationary R/L_n through s-α TEM stabilization



- **shear** ↓ → increases **R/L_n** where ITG → TEM
- **q** ↑ → same effect through α_{MHD}
- **Particle flux crosses zero accordingly to mode frequency**

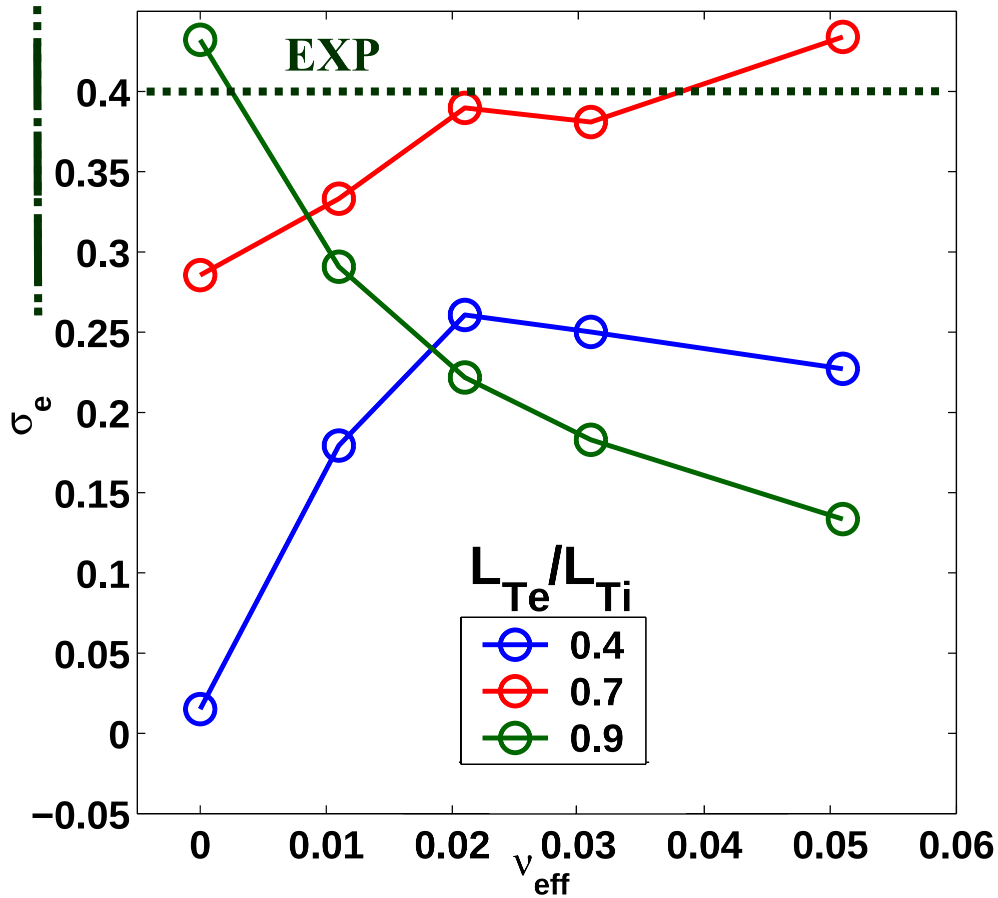
Increase stationary R/L_n through ν_{eff} TEM stabilization

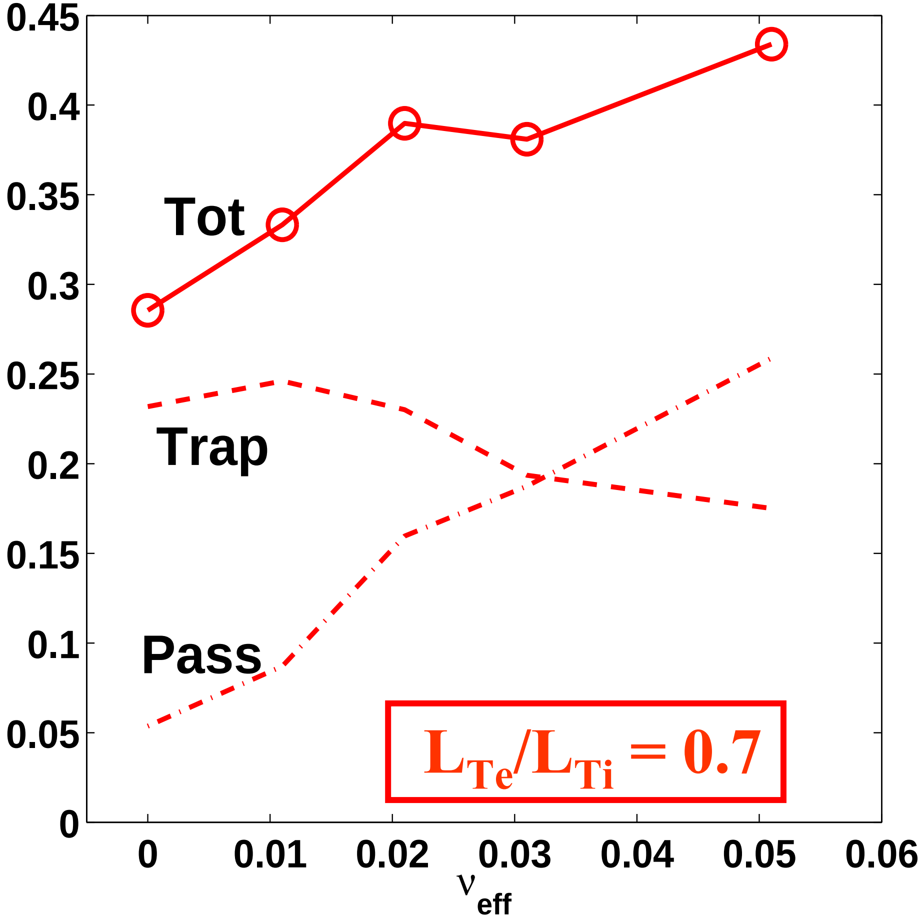
$\varepsilon = 0.11$; $T_e/T_i = 2.5$; $R/L_{Te} = 20$; $R/L_{Ti} = 12$; $Z_{eff} = 1$; $s = -0.7$; $q = 3.3$



Finite collisionality upshifting of the **ITG → TEM** transition point allows a **higher** stationary **R/L_n** for the **same R/L_{Ti}**

Stationary R/L_n compared to experimental value





- Experimental scenario gives $\sigma_e = L_{Te}/L_n \sim \mathbf{0.4 \pm 0.15}$
- Finite collisionality helps in lowering the needed R/L_{Ti}
- **Passing electrons contribute significantly to total pinch at realistic ν_{eff}**

Conclusions: theory part

- **Linear GK** theory predicts **V/D** → gives stationary $\mathbf{R/L_n}$ if neoclassical transport and sources are negligible
- Two main mechanisms: thermodiffusion ($\mathbf{C_T}$) and $\mathbf{C_P}$
- Passing: $\mathbf{C_T \sim -1/2} \rightarrow \mathbf{N_e \sim T_e^{1/2}}$

  $\mathbf{C_p \propto -\tau\, \omega_R}$ → inward in TEM,

  outward in ITG, small at $\mathbf{T_e/T_i > 1}$
- Trapped: $\mathbf{C_T\, (\omega_R)}$ → inward in ITG

  small/outward in TEM

  $\mathbf{C_p \sim -\omega_d}$ → always inward

Conclusions: monotonic q cases

- Good parameter to describe temperature gradients is $\mathbf{L_{Te}/L_{Ti}}$
- Highest stationary $\mathbf{R/L_n}$ obtained around $\boldsymbol{\omega_R \sim 0}$
- $\mathbf{C_T}$ **inward** and important in ITG, highest $|\mathbf{C_T}|$ near $\boldsymbol{\omega_R \sim 0}$
- $\mathbf{C_P}$ limited to values of $\mathbf{\sim -1}$, more negative in **TEM**
- $\mathbf{C_P}$ provides outward pinch in **ITG** , enhanced by $\mathbf{T_e/T_i < 1}$
- Experimental trends are reproduced **qualitatively** and interpreted: ‘universal’ behavior observed for $\boldsymbol{\nu_{eff}}$ scans
- **Shear** dependence of $\mathbf{R/L_n}$ is found for collisionless cases
- No relevant dependence on **q** is found

Conclusions: eITB scenario

- **eITB** scenario is interpreted as an interplay of:

  - dominant **thermodiffusive** pinch due to both high values of $\mathbf{R/L_{Te}}$ and negative shear → $\mathbf{C_P}$ is negligible

  - increased **ITG-TEM** transition in $\mathbf{R/L_n}$ by **s-α stabilization** effects on **TEM**

  - **finite collisionality → partial stabilization** of **TEM** → allows high values of $\mathbf{R/L_n}$ at low values of $\mathbf{L_{Te}/L_{Ti}}$

  - **Pinch** also carried by **passing** electrons at realistic $\nu_{\mathbf{eff}}$

Conclusions: open issues

Quantitative comparisons of quasi-linear model

Inclusion of neoclassical transport and local sources

Effect of other species/impurities (complex flux study)

Non-linear effects