Collisional Gyrokinetic Turbulence with GYSELA

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General outline:

❶ Presentation of the tool: GYSELA

– calculation of the equilibrium radial electric field

❷ Bohm *v.s.* gyro-Bohm transport scaling

– Influence of the size of the device $\rho_\star$ & distance to threshold

❸ Influence of collisions [Abel, Belli, Garbet, Gustafson, Hellsten. . .]

➠ accurate recovery of neoclassical results

➠ interplay collisions / turbulence

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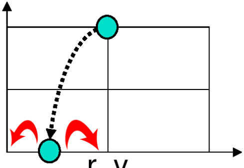
A foreword on GYSELA – the equations

GYSELA ≡ GYrokinetic SEmi–LAgrangian

- follow trajectories backwards
- on fixed grid (weak noise, moderate dissip.)



Guiding-centre

$$\boxed{\partial_t f + \mathbf{v}_E \cdot \nabla_\perp f + \mathbf{v}_D \cdot \nabla_\perp f + v_\parallel \nabla_\parallel f + \dot{v}_\parallel \, \partial_{v_\parallel} f = 0 \quad or \quad \mathcal{C}(f)}$$

Particles

$$\boxed{-\frac{1}{n_0(r)} \nabla_\perp \cdot \left[\frac{n_0}{B\,\omega_c} \nabla_\perp \phi \right] + \frac{e}{T_e(r)} \left(\phi - \langle \phi \rangle \right) = \frac{2\pi B}{m n_0} \iint d\mu \, dv_\parallel J_0 \left(f - f_{init} \right)}$$

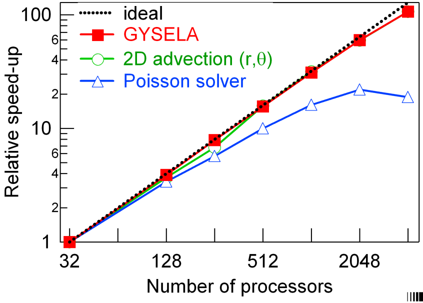
↯ coordinates: $(r, \theta, \varphi, v_\parallel, \mu)$; electrostatic ; adiabatic electrons

[Grandgirard '07]

Numerics in GYSELA – parallelisation

Challenging parallelisation in GYSELA: [Crouseilles '07, Latu '07]

- Eulerian ➠ domain decomposition
- Semi–Lagrangian ➠ loss of locality due to interpolation



Much memory per node
$\sim 10\,\mathrm{Go}$ at $\rho_\star = 2.\,10^{-3}$
$> 10^{10}$ grid points

Efficiency $\equiv$ Speed-up / ideal
93% on 2048 procs.
83% on 4096 procs.

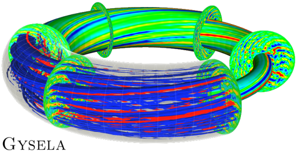
➠ **{MPI + OpenMP}** parallelisation

The GYSELA code – physics

full–f code ($\neq \delta f$):

- Equilibrium & fluctuations: no scale separation assumption
- Non-Maxwellian background equilibrium
- Neoclassical theory

global code ($\neq$ flux-tube):



GYSELA

- Large scale fast transport events
- Profile relaxation ➠ steady state ➠ flux-driven conditions
- Numerical challenge

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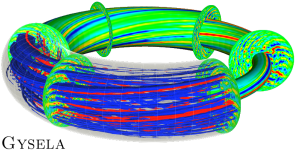

GYSELA

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- Large scale fast transport events
- Profile relaxation ➠ steady state ➠ flux-driven conditions
- Numerical challenge

The delicate calculation of E_r in *full–f* codes

Full–f: a background Maxwellian is not assumed

at short times. . . [Idomura '03 ; Angelino '06, Dif-Pradalier '08(a)]

- non-equilibrium initial state ➠ large-scale transient E_r
- cure: canonical equilibrium ➠ correct E_r, $j_{||}$ . . . right from the start
- routine use in GYSELA (also in ORB5)

. . . at long times [Dif-Pradalier '08(b)]

Global: profile relaxation ➠ steady state ➠ 'flux-driven' conditions

- same turbulence develops (χ_{turb}, λ_c, τ_c, $j_{||}$, ϕ_{00}, ϕ_{RMS})

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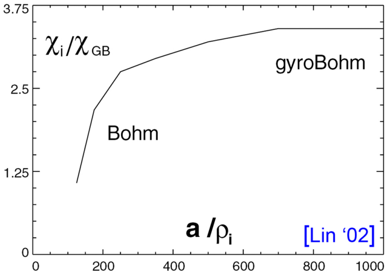
Turbulent scaling with $\rho_\star$

$$\boxed{\omega_c \tau_E \propto \rho_\star^{-2.71} \beta^{-1.24} \nu_\star^{-0.28}}$$

Consistent with gyroBohm scaling ($\lambda_c \sim \rho_i$)

➠ gyroBohm:
- far from instability threshold
- small $\rho_\star$

➠ Bohm:
- close to instability threshold (lin. physics, streamers)
- large $\rho_\star$



Open issues: departure from gyroBohm scaling

❶ Distance towards instability threshold

❷ Transition threshold in $\rho_\star$

❸ Physical mechanism of this transition

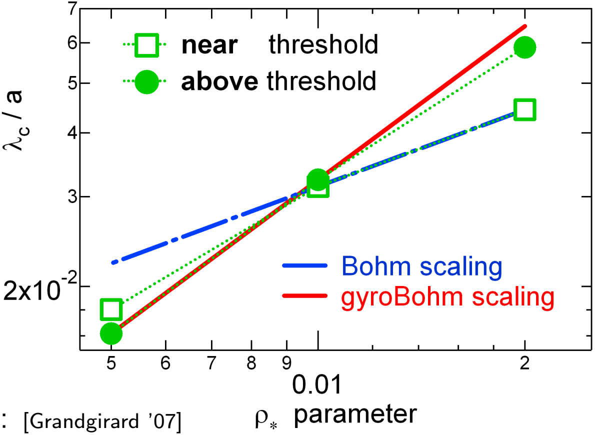
global code

➠ non diffusive large-scale transport ($\lambda_c \sim \sqrt{a\rho_i}$)

➠ turbulence spreading [Garbet '94, Hahm '04]

➠ shear flow stabilisation [Garbet '96, Waltz '02]

- gyroBohm: above threshold & low values of $\rho_\star$
- Bohm: near threshold & high values of $\rho_\star$



more details: [Grandgirard '07]

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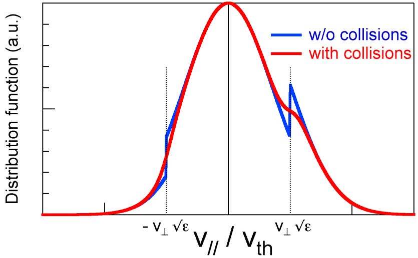
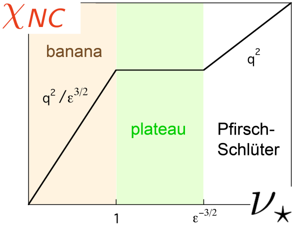
Neoclassical theory $\equiv$ describe the effects of binary Coulomb collisions in an inhomogeneous magnetic field in the presence of trapping



trapped $\langle v_\theta \rangle = 0$ (sym.) $\neq$ passing

➠ regularisation: boundary layer $v_\parallel$

modelisation : **friction force**



Small causes...

$\tau_{NC} \gg \tau_{turb}$; $\chi_{NC} \ll \chi_{turb} \sim \chi_{exp}$

...with strong possible effects on turb. transport

❶ linear damping of residual Zonal Flows (ZF)

❷ value of poloidal rotation v_θ ➠ value for E_r?

Neoclassical theory with GYSELA

(*i*) model the boundary layer ➠ diffusion

(*ii*) dominant transport: ion–ion collisions

Collisions with a full–f code:

(*iii*) full distribution function f must relax $\longrightarrow f_{Maxwell}$

Collisions with GYSELA:

(*iv*) pitch-angle & energy variables ➠ not adapted GYSELA: $(v_{\|}, \mu)$

(*v*) adiabatic electrons: conservation of momentum is not granted

Complex general framework: **mini. model ➠ max. physics ?**

exact NC transport at low computational cost [Garbet, this conference]

Goal: **interaction gyrokinetic turb. $\leftrightarrow$ neoclassical theory**

❶ Which choice for the collision operator?

❷ Neoclassical transport: results
➠ without turbulence ($\nabla T_i < \nabla T_{i\,c}$)

❸ Interplay: neoclassical effects & turbulence
➠ above ITG threshold

Adopted operator: on $v_\parallel$ only

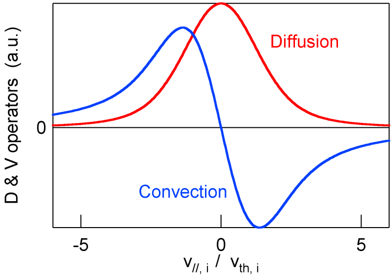
Lorentz operator on $v_\parallel$ only: $\boxed{\frac{\mathrm{d}}{\mathrm{d}t} f = \partial_{v_\parallel}\left(\mathcal{D}\,\partial_{v_\parallel} f - \mathcal{V} f\right)}$

- preferred collisional friction –along $v_\parallel$
- GYSELA: efficient parallelisation on μ (MPI + OpenMP)
  finite differences (Crank–Nicolson)

$$\mathcal{D} = 3\frac{\sqrt{\pi}}{2}\frac{v_T^3\epsilon^{3/2}}{qR_0}\,\nu_\star\,\frac{\Phi(v)-G(v)}{2v}$$

- $\Phi(v) = \frac{2}{\sqrt{\pi}}\int_0^v e^{-x^2}\,dx$
- $G(v) = \frac{\Phi(v)-v\Phi'(v)}{2v^2}$

$$\frac{\mathcal{V}(r,v)}{\mathcal{D}(r,v)} = \frac{v_\parallel}{v_{th}^2} \quad \Rightarrow \quad f \longrightarrow f_{Maxwell}$$



Conservation properties

conservation of: n: exact $\boldsymbol{p}, E$ –approximate

$$mn\,\mathrm{d}_t \boldsymbol{V}_i = en(\boldsymbol{E} + \boldsymbol{V}_i \times \boldsymbol{B}) - \nabla p_i - \nabla \cdot \bar{\bar{\Pi}}_{NC} - mn\,\nu_{\boldsymbol{i}\,\boldsymbol{e}}(\boldsymbol{V_i} - \boldsymbol{V_e})$$

- parallel dissipation $\nu_{\boldsymbol{i}\,\boldsymbol{e}} \Rightarrow \nu_{\boldsymbol{i}\,\boldsymbol{i}}$ $(\ll \nu_{turb})$ selects $\langle v_{\parallel\, i} \rangle = 0$

conservation of momentum & energy is crucial: [Lin '95]
❶ not to overestimate the particle fluxes
❷ not to underestimate the bootstrap current
❸ no influence on the heat transport, flow damping

- Exact $\nabla \cdot \bar{\bar{\Pi}}_{NC} \Rightarrow$ exact banana & plateau regimes [Garbet, this conference]

Not suited for: generation of toroidal momentum

Suited for: interplay turbulence & collisions – **banana+plateau**
collisional enhancement of turbulent transport
turbulent generation of poloidal momentum (ITBs. . .)

Conservation properties

conservation of: n: exact $\boldsymbol{p}, E$: approximate

$$mn\,\mathrm{d}_t \boldsymbol{V}_i = en(\boldsymbol{E} + \boldsymbol{V}_i \times \boldsymbol{B}) - \nabla p_i - \nabla \cdot \bar{\bar{\Pi}}_{NC} - mn\,\nu_{\boldsymbol{i}\,\boldsymbol{i}} \boldsymbol{V}_{\boldsymbol{i}}$$

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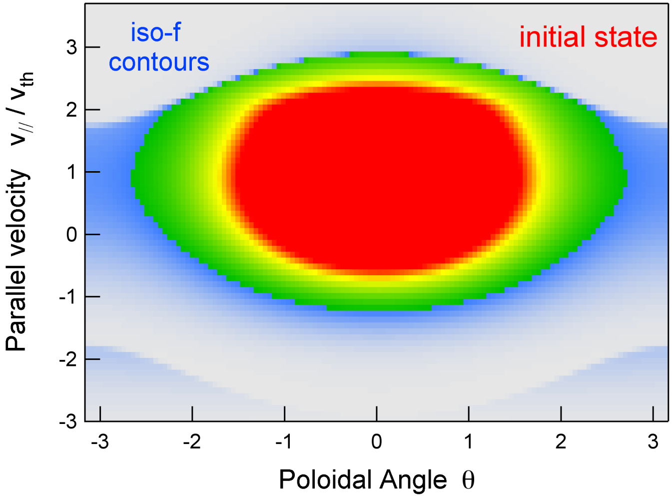
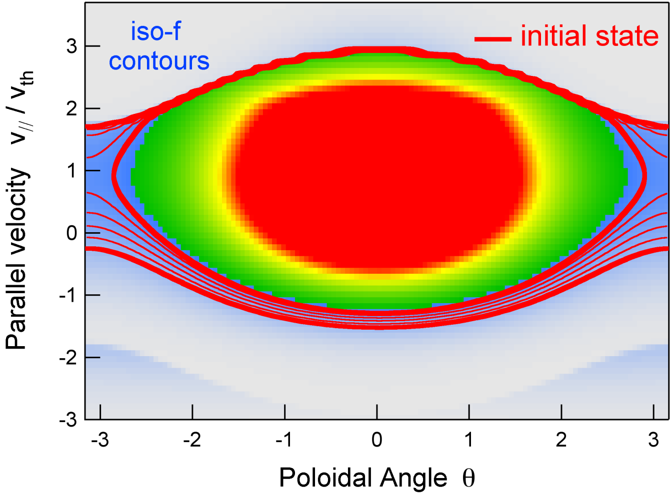
Collisions broaden the loss cone boundary layer



Collisions broaden the loss cone boundary layer



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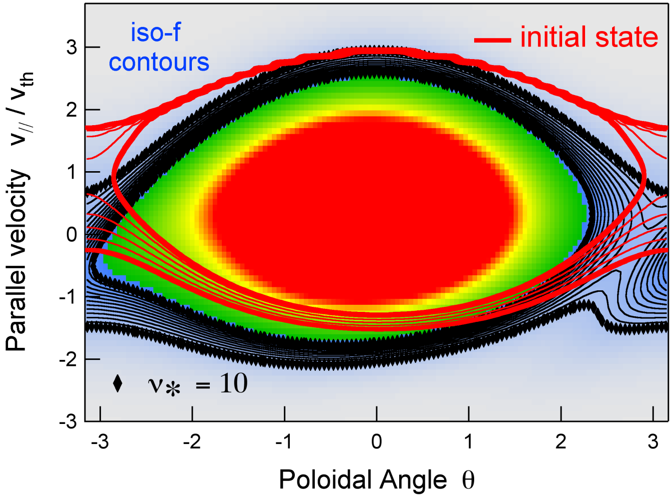
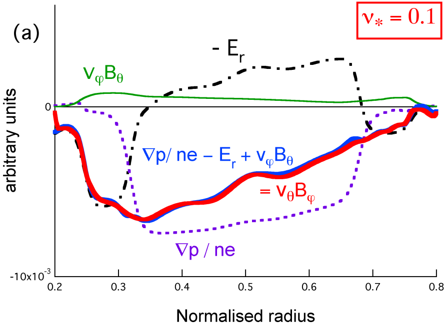
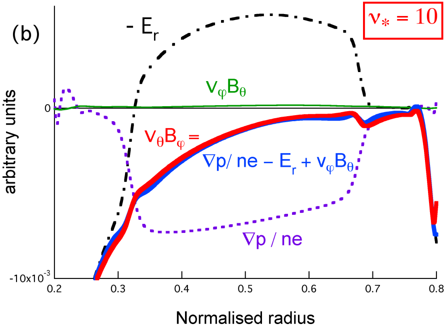


Consistency with the force balance equation

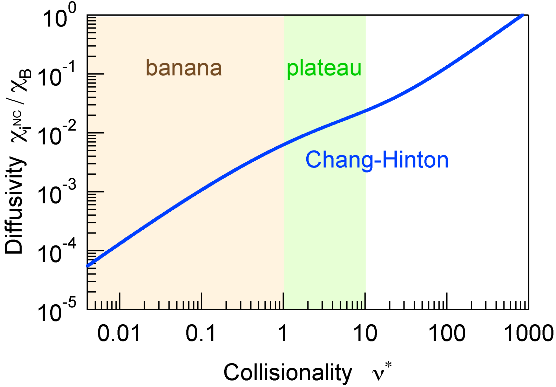
$$\boxed{E_r - v_\varphi B_\theta + v_\theta B_\varphi = \frac{\nabla p}{ne}}$$





Neoclassical results (1/3)

NC thermal transport: $\chi_{NC} = -Q / n \nabla T$



theoretical prediction: [Chang–Hinton '86]

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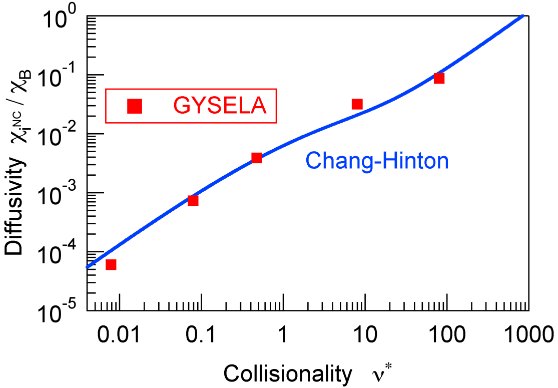

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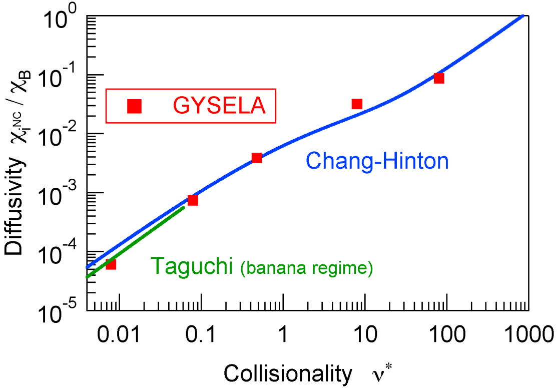
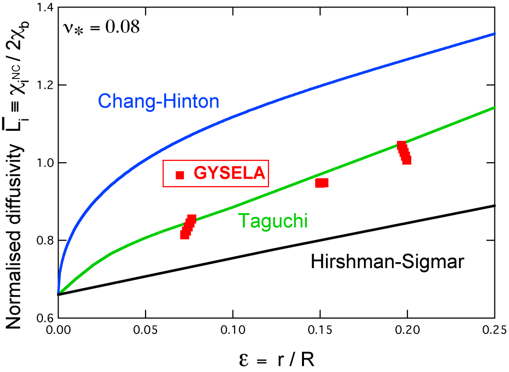


theoretical predictions: [Chang–Hinton '86 ; Taguchi '88]

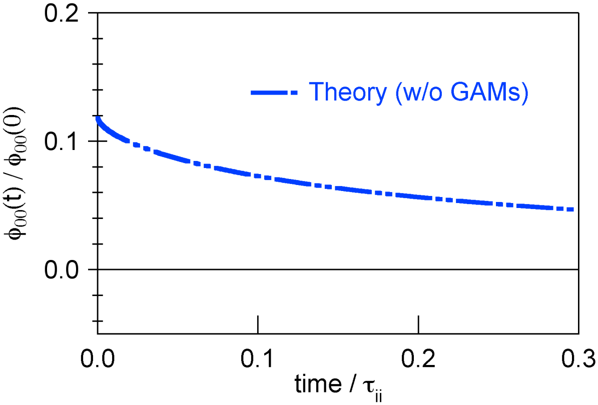
Neoclassical results (1/3)

NC thermal transport –as a function of $\epsilon = r/R$



Neoclassical results (2/3)

Collisional damping of {zonal + mean} flows



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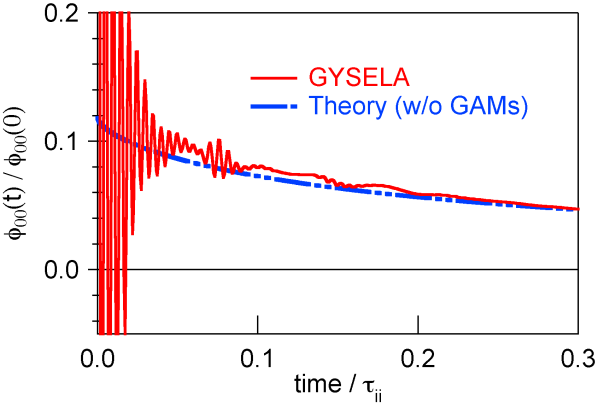

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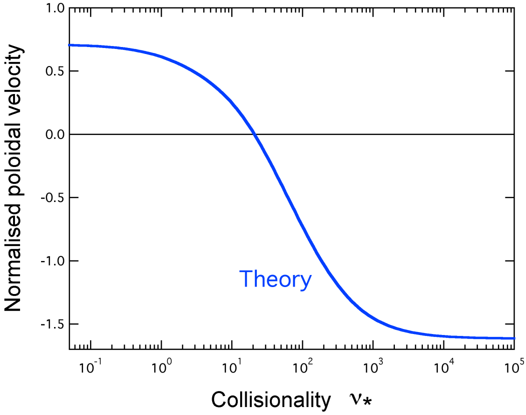
Neoclassical results (3/3)

Ion poloidal rotation $v_{\theta,\ i}$ reverses sign with $\nu_\star$

$\nabla \cdot \bar{\bar{\Pi}}_{NC} \Rightarrow V_\theta = K_1 \frac{\nabla T}{eB}$

exact $\nabla \cdot \bar{\bar{\Pi}}_{NC}$ ➠ v_θ
banana & plateau



theoretical prediction: [Kim '91]

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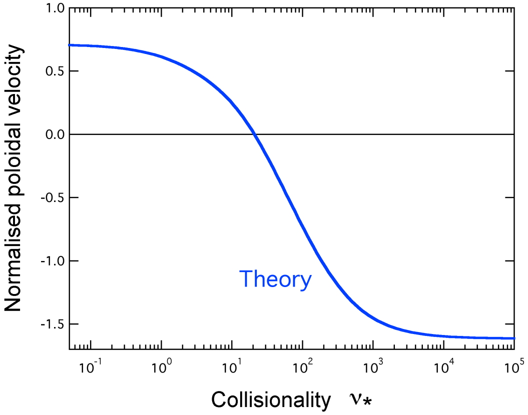

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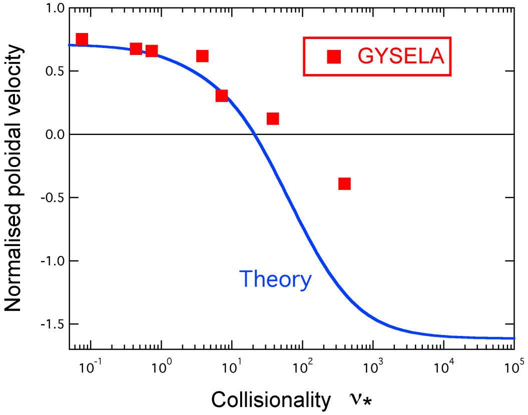

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[Gyrokinetics + neoclassical ➠ heavily demanding (global & *full–f*)]

Coll. & turbulence: crucial players for transport?

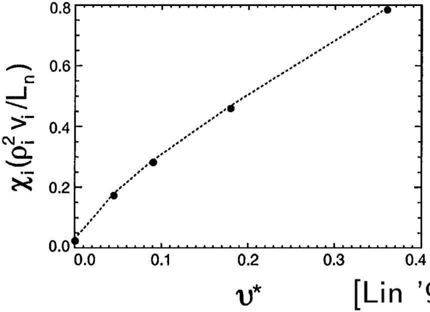
Close to exp. regimes: banana $\nu_\star \ll 0.1$ + turb. strength

❶ Collisional enhancement of turbulent transport

collisions $\nearrow$ ➠ ZF $\searrow$ ➠ turbulence $\nearrow$

strongly dependant on the turbulent regime

$\equiv$ distance to threshold R/L_T



[Lin '99]

FIG. 2. Ion heat conductivity in nonlinear gyrokinetic simulations with $\boxed{R/L_T = 5.3}$ vs the $\boxed{\text{ion-ion collision}}$ frequency.

Generation of poloidal flow – (1/2)

❷ turbulent generation of poloidal momentum (ITBs...)

Common belief: poloidal flow set by NC theory

$$\partial_t v_\theta = -\nabla \cdot \bar{\bar{\Pi}}_{RS} - \mu_{NC}\left(v_\theta - v_\theta^{NC}\right)$$

$$\Rightarrow \boxed{v_\theta = v_\theta^{NC} + v_\theta^{turb}}$$

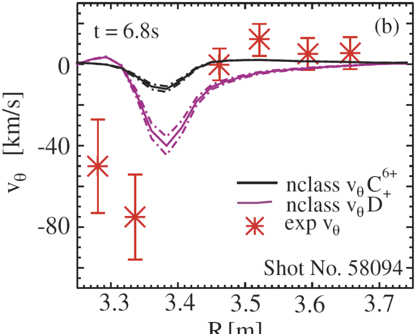
$$E_r - v_\varphi B_\theta + v_\theta B_\varphi = \frac{\nabla p}{ne}$$

$$\Rightarrow \boxed{E_r \neq E_r^{NC}}$$

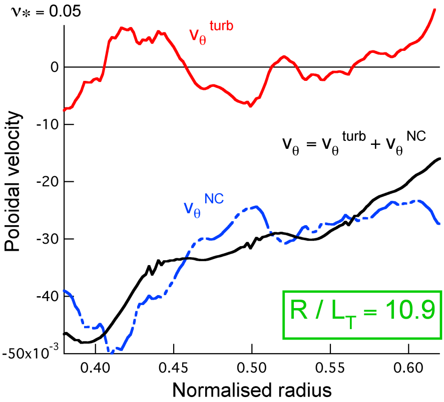
Experiments: $v_\theta,\ E_r \gg v_\theta^{NC},\ E_r^{NC}$

[Crombé '05, McDermott '08]

[Crombé '05]



Generation of poloidal flow – (2/2)



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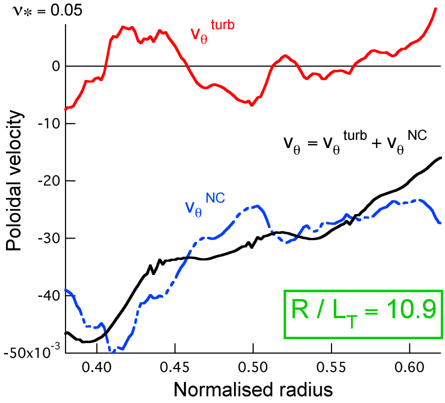

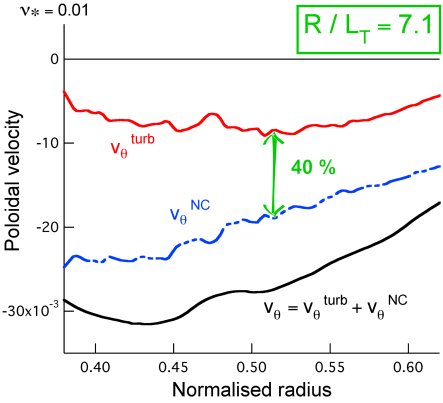

➠ **new result, ongoing interpretation**

Summary of the main results

GYSELA: global & *full–f* addressing **gyrokinetics + NC** theory

- *full–f* codes: careful calculation of the E_r
  - canonical initial equilibrium
- Turbulent scaling with $\rho_\star$ & R/L_T
  - gyroBohm: above threshold & low values of $\rho_\star$
  - Bohm: near threshold & high values of $\rho_\star$
- Simplified collision operator for the banana & plateau regimes
  - exact NC transport, flow damping, poloidal velocity
- With turbulence: turbulent generation of poloidal flow found
  - experimental trends recovered (ongoing exact quantification)
  - strong dependance with distance to threshold