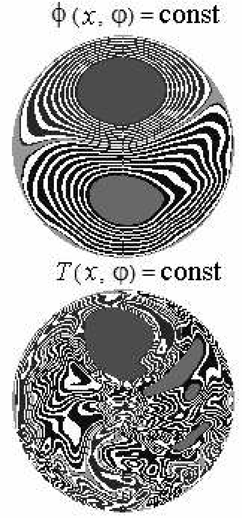
Shear-Flow Effects in Open Traps

A. D. Beklemishev

Budker Institute of Nuclear Physics, Novosibirsk, Russia

The “Vortex Confinement”



V.Pastukhov & N.Chudin, Trans.Fus.Sci.&Tech., 2007

Plasma can be confined in the dead zone of a vortex flow. Such confinement is often observed in simulations as hot spots being associated with large-scale vortices.

My talk is built upon the viewpoint that such “vortex confinement” is the primary mechanism behind improved plasma confinement in presence of sheared flows (at least in open traps.) It is not obvious, since in most cases the vortices are unstable.

Creating and maintaining a wide vortex that contains the hot plasma core is possible. It is relatively cheap in terms of power consumption, and can improve the transverse confinement indeed.

Outline

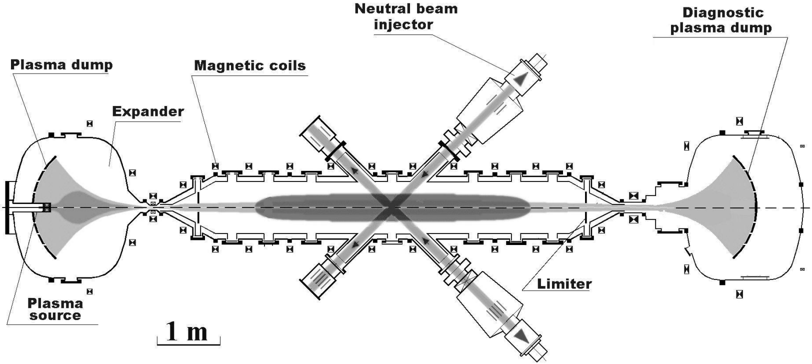
- Reference systems: the GDT and GAMMA-10 open traps;
- Modeling the “Vortex Confinement”:
    - The Kelvin-Helmholtz stability of a “wide” vortex (1P);
    - Vortex drive from the end-plates and its viscous stabilization;
    - Drift modification of the KH stability (2P);
    - Stabilization of the linear drift-interchange in sheared flows;
    - Nonlinear saturation of fluid- and drift-interchange modes;
    - Analytic estimates and comparison to the GDT experiment;
- Pastukhov & Chudin model of the shear-flow-driven ITB in GAMMA-10;
- Conclusion.

Reference systems

There are two suitable reference systems for parameter ordering and comparisons:

- The Gas-Dynamic Trap (Budker INP, Novosibirsk, Russia). The plasma potential is changed by application of voltage to the limiters and the end-plates.
  P.A.Bagryansky et. al., Trans. Fusion Sci. Tech. 51, No.2T, p.340, 2007.
- The Gamma-10 Tandem-Mirror Trap (PRL, Tsukuba, Japan). The plasma potential is modified by an off-axis electron heating.
  T.Cho, et al., Physics of Plasmas 15, 056120, 2008.

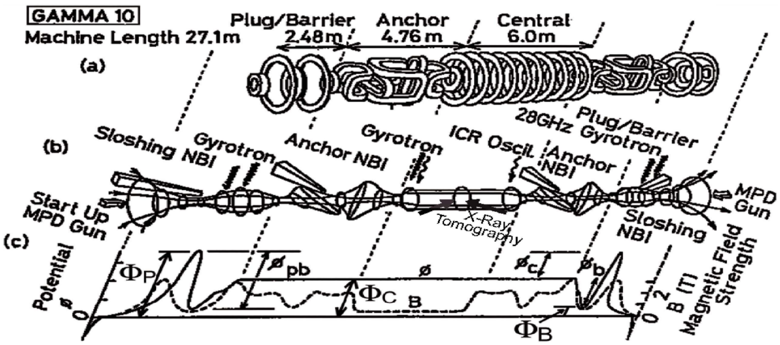
The Gas-Dynamic Trap (GDT)



Plasma parameters: $L \sim 7/12m$, $r \sim 15cm$, $k = B_{max}/B_{min} = 30$, $T_w \sim 180eV$, $T_{hi} \sim 15keV$, $n \sim 2 \cdot 10^{13} cm^{-3}$, $\beta \sim 60\%$.

Since 2003 routinely operates with an average magnetic hill.

The Gamma-10 Tandem Mirror

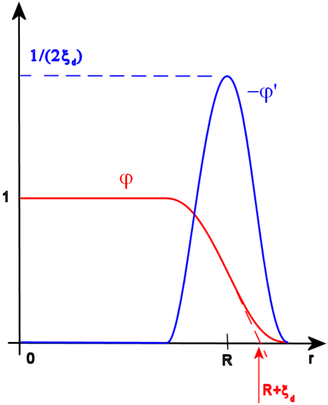


Plasma parameters: $L_c \sim 6/27m,\ \ r_c \sim 14cm,\ \ k = B_{max}/B_{min} \approx 5,$
$T_e \sim 800eV,\ \ T_{hi} \sim 4keV,\ \ n \sim 5 \cdot 10^{12} cm^{-3},\ \ \beta \sim 6\%$.
Since 2005 can operate with an "internal transport barrier".

Modeling the "Vortex Confinement"

- The idea of this section is to start from the simplest model, and add to it necessary effects piece by piece, so as to establish a clear link to existing theories at each stage;
- Drift-ordered fluid equations are derived using *Simakov & Catto, PoP 10, 4744, (2003)*;
- All models are 2D with incorporated 3D-effects. Simplification is due to electrostatic ordering and the absence of the magnetic shear.

2D ideal fluid vortex



Differential layer width:

$\xi_d \equiv 1/(2|\nabla\varphi|_{max})$

- Assume uniform field, temperature and density;
- Quasineutrality ($\mathrm{div}\mathbf{j} = 0$) requires divergence of the ion polarization drift to be zero, and hence

$$\frac{d}{dt}\Delta\varphi = 0,$$

where $\mathbf{v} = \frac{c}{B}\mathbf{b} \times \nabla\varphi.$

- Consider specific, axially symmetric initial states of the "wide-vortex" type (left);
- We have to cope with *two* inflection points on the velocity profile!

Normalization

It is convenient to use dimensionless units φ/ϕ, $\mathbf{r}/R$, t/τ, where ϕ is the initial vortex amplitude, R is the layer radius, and

$$\tau = \frac{BR^2}{c\phi} = \frac{R^2}{\rho_i v_{Ti}} \left(\frac{T}{e\phi} \right) .$$

Then the vortex dynamics is described by

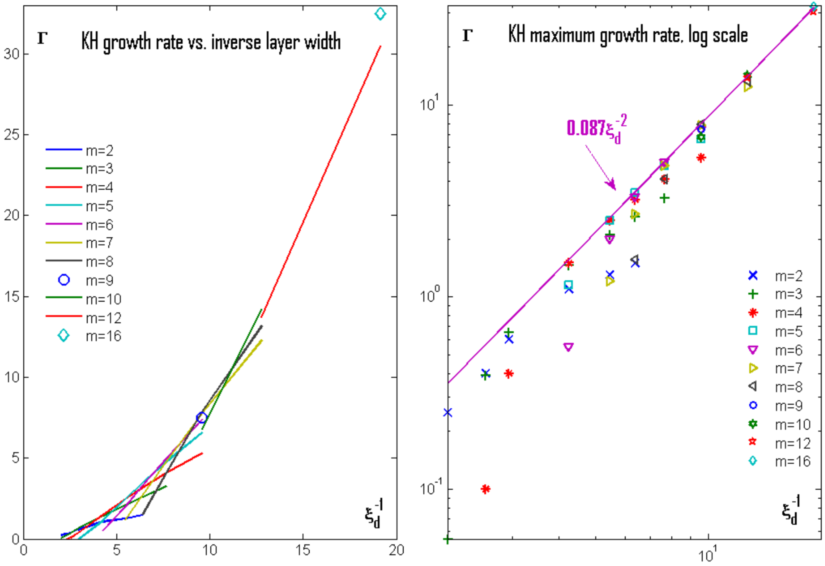
$$\boxed{\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} = 0,}$$

where

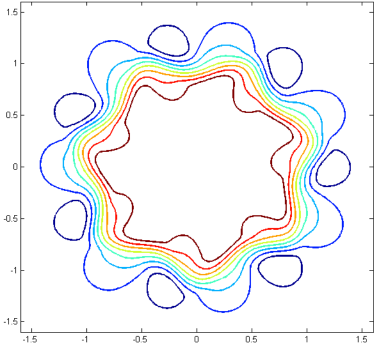
$$\{\varphi, A\} \equiv \frac{\partial\varphi}{\partial x}\frac{\partial A}{\partial y} - \frac{\partial A}{\partial x}\frac{\partial\varphi}{\partial y}.$$

For the GDT parameters, $\phi \sim T_e/e \sim 150\,V$, $\quad \tau \sim 30\ \mu s$.

The KH linear growth rates



The Kelvin-Helmholtz instability



The $m = 7$ linear-stage mode ($\varphi = const$ flow-lines). Looks like an alternating sequence of small vortices (as it should).

- As expected, for any layer width the KH instability exists and is fatal (destroys the initial structure);
- The growth rate scales with the layer width, ξ, and the mode number, m, as

$$\Gamma_m \approx b \cdot m/\xi - a \cdot m^2,$$

but $\Gamma_{max} \approx C/\xi^2$.

The constants a and b depend on the profile (weakly).

Axial currents and collisional viscosity

On open field lines that end on solid conductors it is prudent to take into account the axial currents, i.e., any possible imbalance of the ion and electron currents that flow to the wall along the field lines through obstacles like the Debye sheath, magnetic mirrors, ambipolar plugs, etc. Using these currents it is possible to drive the vortex. Dissipation can assist in its stabilization.

Currents to the wall

The quasineutrality condition is now $\mathrm{div} j_p + \mathrm{div} j_{\|} = 0$. Integrating it along the field line, we get the current to the wall
$j_{\|}(wall) = \int \mathrm{div} j_p d\ell = J_{i0} \left(1 - \exp[e(\varphi - \varphi_{fl} - \varphi_{wall})/T_e]\right) .$
In most cases the deviation from the floating balance is small:

$$\boxed{\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} = H \cdot (\varphi - \varphi_w(\mathbf{r})).}$$

Here

$$H = J_{i0} \frac{e}{T_e} \frac{B^2}{\rho c^2 L} \tau R^2 = \frac{\tau}{\tau_{\|}} \left(\frac{R}{\rho_i}\right)^2 = \frac{R^4}{\rho_i^3 L k} \left(\frac{T}{e\phi}\right),$$

where $\tau_{\|}$ is the axial particle confinement time, k is the effective mirror ratio, $\varphi_w (= \varphi_{fl} + \varphi_{wall}$ is the equilibrium plasma potential.

The plasma-wall coupling

The plasma-wall coupling (or "line-tying"), H, depends strongly on the magnetic field, while being independent of density:

$$H \propto B^3 T^{-1/2}!$$

For the GDT ($\tau_{\|} \approx 1ms$, $B = 0.25T$, and $[100eV$ at $10cm] \div [50eV$ at $14cm]$)

$$H \approx 3.5 \div 40.$$

For GAMMA-10
($\tau_{\|} \approx 10ms$, $B = 0.4T$, and $750eV$ at $12cm$)

$$H \sim 1.$$

For tokamak SOL $H \sim 10^2 \div 10^4$. It is not likely to be small even for fusion-class devices.

Coupling effects

- As $t \to \infty$ the axisymmetric plasma potential tends to its equilibrium value, $\varphi \to \varphi_w$;
- If the wall potential has a radial jump, it drives the flow layer. Its width decreases with time, $\xi \propto 1/\sqrt{Ht}$;
- The linear KH growth rate acquires additional decrement:

$$\gamma_H \approx -H/m^2,$$

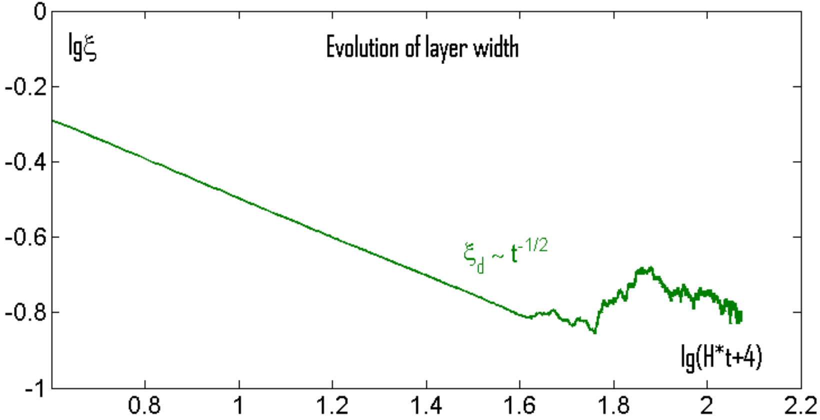
where m is the mode number. Only the low-m modes are damped efficiently;

- For the most unstable mode,

$$\Gamma_{max} - \gamma_H \approx C\xi^{-2} - H\xi^2 > 0,$$

so that the onset of the KH instability if ξ decreases with time is inevitable...

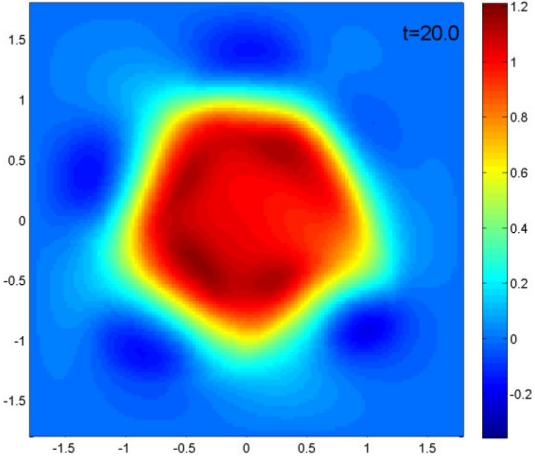
Evolution of the φ profile



The layer width decreases as $1/\sqrt{Ht}$ until the instability threshold is reached... But then it just starts oscillating. The KH instability is no longer destructive!

Nonlinear stage

With the end-wall coupling the Kelvin-Helmholtz instability is no longer destructive. It looks like the inverse cascade is driving the fluctuation energy to small k's, where it is efficiently damped.



movie

Evolution of the normalized potential, φ, for coupling $H = 30$.

Dissipation of vorticity

Collisional viscosity leads to diffusion of vorticity, while there is also some axial loss of vorticity (with ions):

$$\boxed{\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} = H \cdot (\varphi - \varphi_w) + \nu_\perp \Delta^2 \varphi - \nu_\parallel \Delta\varphi.}$$

Here

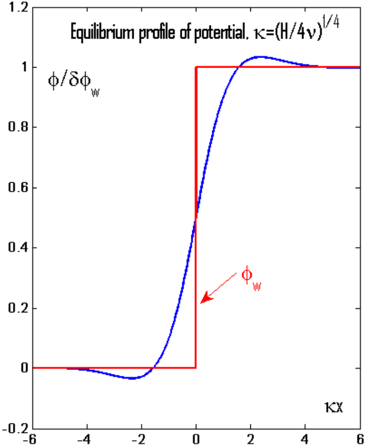
$$\nu_\perp = \frac{\nu_g \tau}{R^2} = \frac{\nu_g}{16 D_B}\left(\frac{T}{e\phi}\right), \qquad \nu_\parallel = \frac{\tau}{\tau_\parallel} = H\left(\frac{\rho_i}{R}\right)^2 .$$

Collisional viscosity in both traps is not too small because of the weak magnetic field: $\nu_{\perp ii} \sim 10^{-3}$, while the Bohm diffusion is 50 times that: $\nu_{\perp Bohm} \approx 0.055$.

The axial confinement is better in GAMMA-10: $\nu_{\parallel GDT} \approx 0.03$, $\nu_{\parallel GAMMA-10} \approx 10^{-3}$.

Influence of viscosity

- Emergence of a stationary smooth axisymmetric state $\varphi \longrightarrow \varphi_s$ as $t \longrightarrow \infty$, $\nu_\perp \Delta^2 \varphi_s = H(\varphi_w - \varphi_s)$;



Solution for the slab geometry:

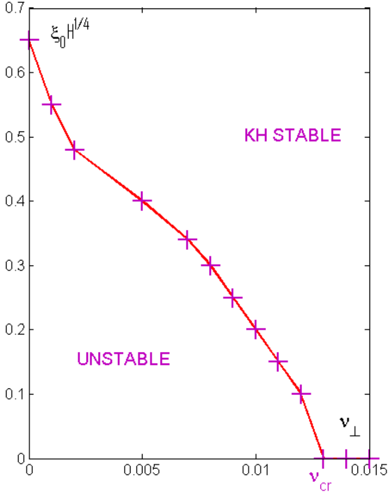
$$\varphi_s = \begin{cases} 0.5e^{\kappa x}\cos(\kappa x), & x < 0 \\ 1 - 0.5e^{-\kappa x}\cos(\kappa x), & x > 0 \end{cases}$$

$$\kappa = (H/4\nu_\perp)^{1/4}.$$

Due to its very weak dependence on $\nu_\perp$, the stationary layer width $\xi_d = \kappa^{-1}$ is not too small,

$$\xi_{GDT} \approx 0.17.$$

KH stabilization



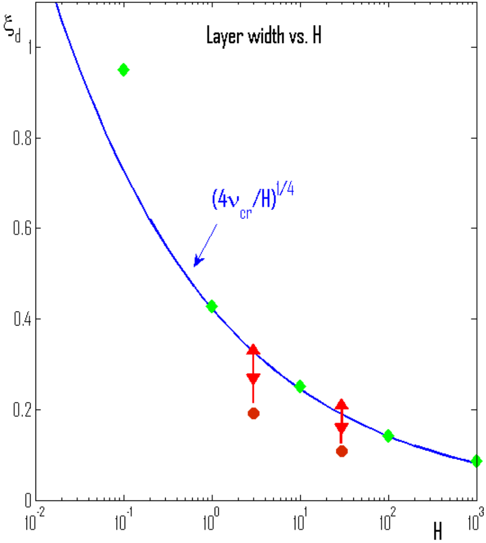
Stability boundary for different widths of the wall potential transition, ξ_0. $\nu_{\|} = 0$.

- KH can be stabilized by both the viscosity, $\nu_\perp$, and the line-tying, H.
- Line-tying acts only on long-wave modes, while the effect of viscosity has the same k-dependence as $\Gamma_{KH} \approx Ck^2$,

$$\Gamma_{eff} \approx Ck^2 - \nu_\perp k^2 - Hk^{-2}.$$

- The critical viscosity, $\nu_{cr} \approx 0.013 \pm 0.001$, is almost independent of H.

Turbulent profile broadening



If the hydrodynamic viscosity is below the KH threshold, the turbulence broadens the velocity profile as if the viscosity is at its threshold value!

Green points represent stationary profiles at critical viscosity, red shows simulation data for collisional viscosity, $\nu_\perp \sim 0.1\nu_{cr}$.

Drift modification of KH instability

Typical values of the *ExB*-velocity in open traps are comparable or smaller than the velocity of the diamagnetic drift of ions. Thus the drift effects are very important. Besides modification of the polarization current, we will need an equation for the evolution of ion pressure.

Drift-ordered equations

The vorticity equation is now coupled to the pressure convection equation:

$$\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} - U\nabla \cdot \{\nabla\varphi, P\} =$$

$$= H \cdot (\varphi - \varphi_w) + \nu_\perp \Delta^2 \varphi - \nu_\parallel \Delta\varphi - U\Delta S_p,$$

$$\partial_t P + \{\varphi, P\} = \nu_\perp^p \Delta P - \nu_\parallel^p P + Q_p.$$

where the pressure is normalized to its central value, P_0,

$$U \equiv P_0/(en\phi); \qquad S_p \equiv (\nu_\perp^p - \nu_\perp)\Delta P - (\nu_\parallel^p - \nu_\parallel)P + Q_p \approx Q_p(r)$$

is defined by the balance of source and sink terms, which depends on the model of equilibrium; Q_p is the heating source.

Derivation notes

The influence of pressure on the vorticity dynamics comes (in particular) through divergence of the polarization drift. One intermediate result in *Simakov & Catto, 2003* shows it as

$$\nabla \cdot (en\mathbf{V_{pi}}) \approx -\frac{c}{B\Omega_i}\nabla \cdot (\partial_t + \mathbf{V_E} \cdot \nabla)(en\nabla\varphi + \nabla p),$$

Here $\nabla \cdot (en\nabla\varphi + \nabla p)$ is proportional to *full* vorticity. However, eliminating the term $\partial_t \Delta p$ using the pressure convection equation, we can recover expression for the *ExB*-vorticity only:

$$\nabla \cdot (en\mathbf{V_{pi}}) \approx -\frac{c}{B\Omega_i}\Big(\partial_t \nabla(en\nabla\varphi) + \frac{c}{B}\nabla\{\varphi, en\nabla\varphi\} -$$

$$-\frac{c}{B}\nabla\{\nabla\varphi, p\} + \Delta(\nu_\perp^p \Delta p - \nu_\parallel^p p + q_p)\Big)\,.$$

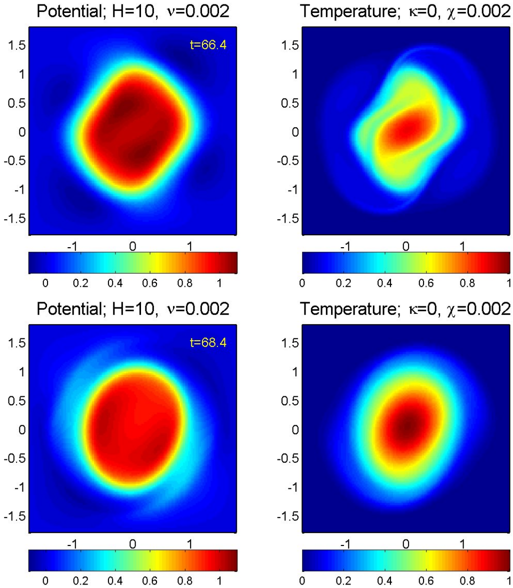
Drift (FLR) effects in KH

The gyro-viscosity (the FLR-effect) can stabilize all modes (except the $m = 0$ and the "rigid" $m = 1$, which do not appear in KH) if it is large enough.

- In both reference traps the ion gyro-viscosity is relatively large, $U_{GDT} \sim 5$, $U_{GAMMA-10} \sim 1$;
- but its influence on the equilibrium width of the flow layer is negligible;
- hence, at realistic collisional viscosity the flow layer becomes so thin, that the KH is destabilized anyway.

Conclusion: If the collisional viscosity is small, radial jumps of the wall potential should be avoided in favor of smoother transitions (at present not realized in the GDT).

Drift (FLR) effects in KH



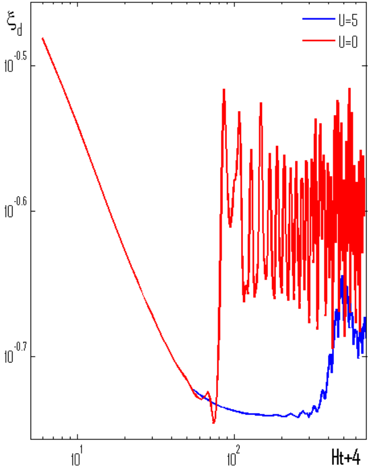
Without FLR, U=0

With FLR, U=5

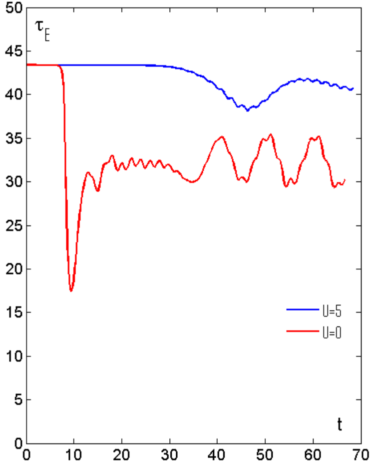
Evolution of the normalized potential and pressure (temperature) for coupling $H = 10$, with and without the drift effects (GDT).

Degradation of confinement with FLR is smaller.

Drift (FLR) effects in KH



Layer width vs. time.



Energy confinement time vs. time.

The interchange instability

All axially-symmetric mirror traps have average magnetic hills on axis, and thus should be interchange-unstable unless somehow stabilized. The GDT is designed with favorable-curvature expanders, while GAMMA-10 has quadrupole min-B cells. It appears that these costly contraptions can be replaced or supplemented with some drive of the shear-flow (e.g., the GDT now operates without curvature in expanders.)

Equations

The drift-interchange modes can be described by the same system as the KH-modes, just add the divergence of the diamagnetic current due to curvature and redefine parameters as field-line averages:

$$\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} - U\nabla \cdot \{\nabla\varphi, P\} =$$

$$= H\cdot(\varphi - \varphi_w) + \nu_\perp \Delta^2 \varphi - \nu_\parallel \Delta\varphi - U\Delta S_p + \varkappa \left\{P, r^2\right\},$$

$$\partial_t P + \{\varphi, P\} = \nu_\perp^p \Delta P - \nu_\parallel^p P + Q_p.$$

Here $\varkappa$ describes the average curvature, $\varkappa_{GDT} \approx 0...3$, r is the radius of the field line in the central cross-section (flux coordinate).

Field-line averaging

Field-line averages can be defined based on $\varphi = const$ and $BR^2 = const$, and hence, $\tau = const$ on each field line. However, redistribution of pressure, density and temperature along the curved field may be complex (and can even drive sheared flows). Treatment of such effects for open traps is done by *Pastukhov* in *Plasma Phys. Rep.* **31**, *577 (2005)*. However, in our current model we neglect them, since the drive by wall potentials (if present) is much stronger.

For low-β paraxial configurations

$$\frac{1}{B(r,z)} \approx \frac{1}{B(0,z)} + \alpha(z)\, r^2,$$

and IF we assume $p(r,z) = p(r)g(z)$, with $\langle g \rangle = 1$, then $P = p/p(0), \quad U = p(0)/en\phi$ as before, and

$$\varkappa = 2U\Omega_i R^2 \tau \langle B(z)\alpha(z)g(z)\rangle.$$

Linear theory without flow

Without the drift effects

$$\omega^2 \Delta\varphi + i\omega\left(H + \nu\Delta^2\right)\varphi - 2\varkappa r P_0' m^2 \varphi = 0,$$

so that in the “local ” approximation for $m > 1$:

$$\omega^2 + 2i\omega A + \Gamma^2 = 0,$$

$$\Gamma^2 = -2\varkappa r^3 P_0' > 0, \qquad 2A_m = Hr^2/m^2 + \nu m^2/r^2.$$

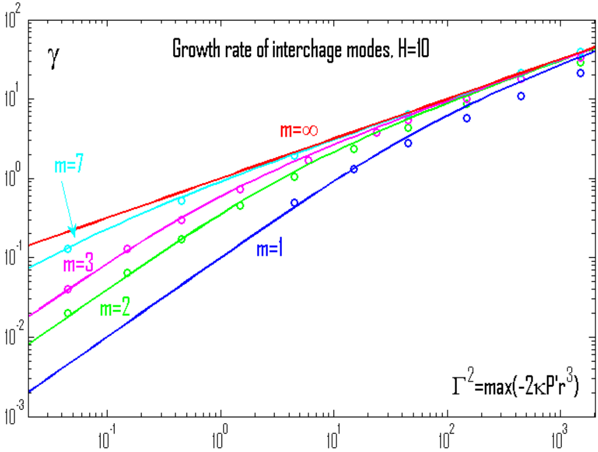
The instability is always present for $\varkappa > 0$,

$$\gamma \le \max_r \left(\sqrt{A_m^2 + \Gamma^2} - A_m\right) < \max_r \Gamma,$$

while viscosity and line-tying only reduce the growth rate.
The $m = 1$ has the growth rate

$$\gamma \le \max_r \left(-2\varkappa r P_0'/H\right).$$

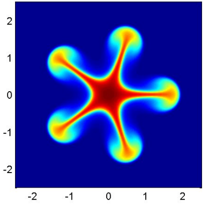
Instability regimes



Growth rate as a function of curvature. With line-tying dominant, $\gamma \propto \kappa m^2$, and with dominant inertia, $\gamma \propto \sqrt{\kappa}$.

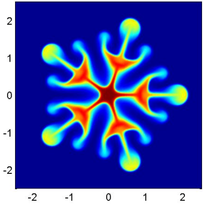
Nonlinear evolution:

P, H=10, κ=3, t=2.61



Inertial mode

P, H=300, κ=3, t=7.4



Resistive mode

Ideal drift-interchange in a sheared flow

Theorem by Timofeev (*Rev. of Plasma Phys., 1992*): *There are no exponentially growing modes except those linked to boundaries or inflection points.*

...It is strange, as even a vanishingly weak flow seems to stabilize a robust interchange. Let's try to see what happens to initial perturbations locally.

We start by writing the usual linearized equation for ideal flute modes($H = 0, \nu = 0$):

$$\frac{1}{r^2}\frac{d}{dr}S(r)\frac{d\psi}{dr} + \frac{1-m^2}{r^2}S(r)\psi + (\omega^2 + m^2\gamma_0^2)\left(\frac{d}{dr}\frac{n_0}{B^2}\right)\psi = 0.$$

Here $\omega_0 = \omega - m\omega_E$, $\psi = \tilde{\varphi}/\omega_0 r$, $S(r) = \omega_0\left(\omega_0 - \omega_*\right)n_0 r^3/B^2$, and γ_0 is the instability growth rate of the standard ideal interchange. If $\omega_* = 0$ this equation is singular at $\omega_0(r) = 0$ due to the hydrodynamic resonance and corresponds to the Timofeev case $n = 2$.

“Local” expansion

For the local treatment, assume that near r_0 the radial profile of the ExB frequency is linear $\omega_E = \omega_{E0} - (r - r_0)\omega'_E$, while ω_* is constant, $m \gg 1$, and expand all coefficients of the problem in $x = m(r - r_0)/r_0$. Then the equation becomes:

$$\left(\hat{s}\psi'_x\right)'_x + \hat{s}\psi + \lambda^2\psi = 0,$$

where $\hat{s} = (\Omega - xu)^2 - \omega_*^2/4$, $u = r_0\omega'_E(r_0)$ is the flow shear, $\lambda \sim \gamma_0$ replaces the interchange instability drive, and Ω is the (Doppler-shifted) mode frequency in the co-rotating reference frame.

Evolution of the initial perturbation

Fourier-transform of equation in Ω and in x after some algebra leads to:

$$\left(\frac{\partial}{\partial\tau} - u\frac{\partial}{\partial k}\right)^2 \bar{\psi} + \left(\frac{\omega_*^2}{4} - \frac{\lambda^2 + u^2}{k^2+1}\right)\bar{\psi} = 0,$$

where $\bar{\psi} \equiv \psi(1+k^2)^{1/2}$. To solve it, we introduce new variables, $\ell = (k+u\tau)/2, \quad s = (k-u\tau)/2,$ and the equation becomes

$$u^2\frac{\partial^2\bar{\psi}}{\partial s^2} - \left(\frac{\lambda^2+u^2}{(\ell+s)^2+1} - \frac{\omega_*^2}{4}\right)\bar{\psi} = 0,$$

where the independent variable ℓ behaves as a parameter! Note that the flow shear now contributes to destabilization (KH), while even a small drift frequency, ω_*, drastically changes the asymptotic at large s, i.e., at $t \to \infty$.

Asymptotic solutions

Choosing the initial perturbation peaked in ℓ at $\ell = 0$ (broad in radial extent), we can deduce the asymptotic behavior:

① $k \ll 1 : \quad \bar{\psi} \sim a_1 \exp\left[\lambda_u s/u\right] + a_2 \exp\left[-\lambda_u s/u\right]$;

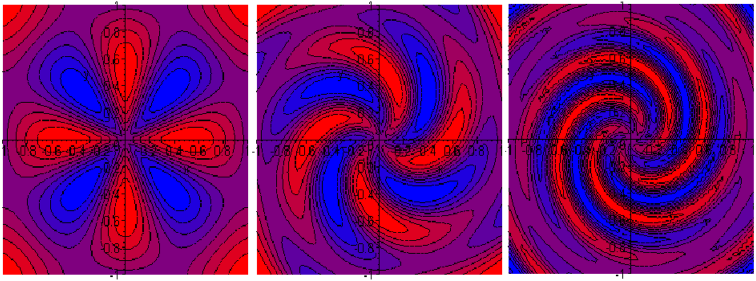
② $1 \ll k \ll k_0 : \quad \bar{\psi} \sim b_1 \left(\ell + s\right)^{\sigma_1} + b_2 \left(\ell + s\right)^{\sigma_2}$;

③ $k \gg k_0 = |2\lambda_u/\omega_*| : \quad \bar{\psi} \sim c_1 \sin(\omega_* s/2u + c_2)$,

where $\lambda_u = (u^2 + \lambda^2 - \omega_*^2/4)^{1/2}$ defines the initial growth rate (if real), and $\sigma_{1,2} = 0.5 \pm \sqrt{1.25 + \lambda^2/u^2}$ are the power-law coefficients (if $k_0 \gg 1$).

For $\tau < 1/u = 1/r_0\omega'_{E0}$ *the amplitude grows exponentially, since the* $k \ll 1$ *limit applies, but then changes to the power law.* Over infinite time it is slower than any exponent, so, the Timofeev's theorem is correct, but the instability persists! It is only stabilized if the FLR (drift) or viscous effects come into play.

Evolution stages



$u\tau=0$ $u\tau=2$ $u\tau=5$

Evolution stages of the $m = 4$ mode in the $(\mathbf{r}, t)$ coordinates.

Linear stage summary

- Evolution of an initial interchange-like perturbation in a sheared flow goes through several stages. Throughout all of them the radial wavenumber linearly grows with time.
- If the flute mode is unstable without the flow, then it will also grow exponentially during the first stage, with the growth rate that is even higher due to the KH terms. Duration of this stage is inversely proportional to the flow shear.
- The second stage starts when the radial wavenumber of the perturbation becomes larger than the azimuthal one. The growth slows to the power law.
- In the third stage the ion gyroviscosity stops the growth altogether.

Thus, the interchange can be linearly stabilized by a sheared flow only via the drift effects. Otherwise its threshold is even lower due to the KH drive.

Vortex confinement

Assume that the sheared flow is driven by the wall potential and is KH-stable, while the flow layer width is small compared to the radius. IF there is a confinement, the pressure profile should be smooth, unlike the flow potential.

Then, if the radial plasma displacements are small, $\xi_r \ll 1$, the pressure evolution is close to linear, $\tilde{P} \approx -\xi_r P_0'$, and so is the interchange-drive

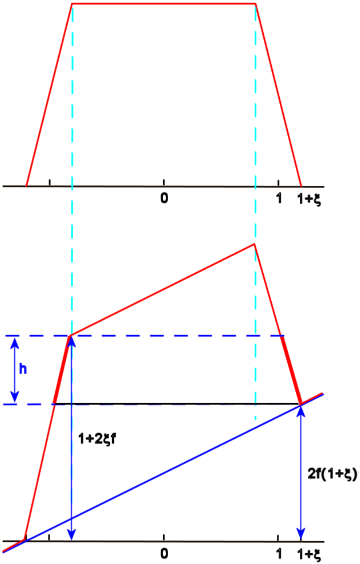
$$f \cdot H \equiv \varkappa\{P, r^2\} \approx -2\varkappa P_0' \partial_\theta \{\xi_r, r\} .$$

It can be regarded as an external "transverse" wind flow acting on the vortex. We can explore the resulting vortex perturbation using the vorticity equation,

$$\partial_t \Delta\varphi + \{\varphi, \Delta\varphi\} = H \cdot (\varphi - \varphi_w(\mathbf{r}) + f(\theta)) + ...,$$

and then find ξ_r self-consistently.

Vortex confinement, $m = 1$



The “rigid” $m = 1$ mode generates a quasi-uniform transverse wind-field, $f \cdot x$ (blue), acting on the driven vortex (red).

The flow-lines in the region $h > 0$ will remain closed (and the plasma confined), if the vortex *potential* amplitude is higher than the perturbation amplitude, $1 > 2f$.

Analytic theory

- At large $H/f \gg 1$ and $\nu = 0$ for the slab geometry there is a stationary solution:

$$\varphi_s = -f \cdot y + \begin{cases} (1/3)e^{2\lambda x}, & x < 0 \\ 1 - (2/3)e^{-\lambda x}\cos\left(\sqrt{3}\lambda x\right), & x > 0 \end{cases}$$

where

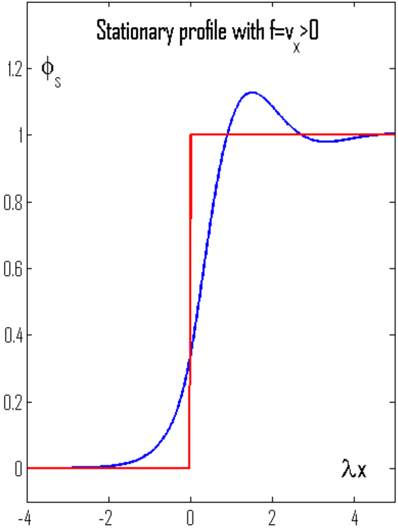
$$\lambda = (H/8f)^{1/3};$$

- At $f/\nu > (H/f)^{1/3}$, i.e, at

$$\nu < H^{-1/3} f^{4/3},$$

the viscous layer width can be neglected;

Stationary profile

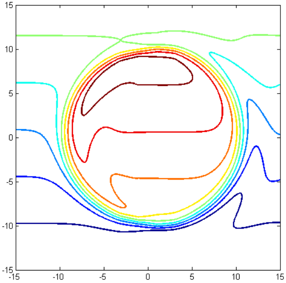


The layer width is

$$\xi_d = \frac{\sqrt{3}}{4\lambda} e^{(\pi/6\sqrt{3})} \approx 1.17 \left(\frac{f}{H}\right)^{\frac{1}{3}},$$

while its center is displaced due to asymmetry by

$$\xi_r \approx \xi_d/3.$$



$m = 1$

Vortex confinement estimates

The vortex confinement criterion is

$$-\frac{2\varkappa P_0'}{H} m\xi_r < 1.$$

Without FLR-effects the mode number will be $m \approx 2\pi/\xi_w$, where $\xi_w \sim 3\xi_r$ is the layer width. Then the condition of existence of the "dead zone" of the flow is

$$\frac{\varkappa}{H} < -\frac{1}{2P_0' m\xi_r} \sim -\frac{3}{4\pi P_0'} \sim 0.2.$$

With FLR only the rigid $m = 1$ is unstable, so that

$$\frac{\varkappa}{H} < -\frac{2H^{1/3}}{P_0'} \sim 2H^{1/3} \gg 0.2.$$

Estimates of displacement

For a stationary rotating structure

$$\frac{d}{dt} \sim -i\left(\Omega - \frac{m}{\xi_w}\right) - f\frac{\partial}{\partial r},$$

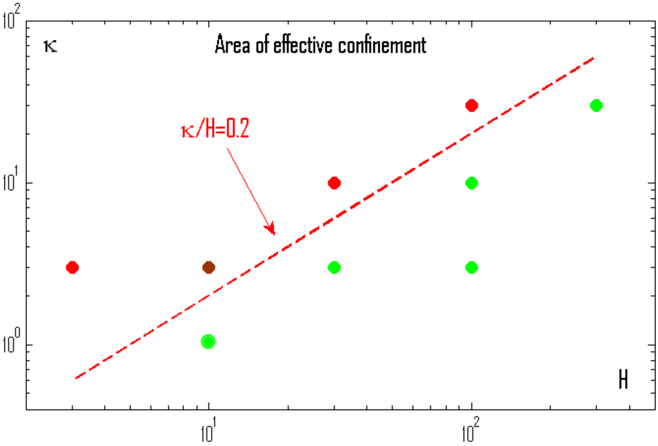
hence $\Omega \approx m/\xi_w$ and

$$\xi_w^3 \sim \frac{f}{H} = -\frac{2\varkappa P_0'}{H^2} m^2 \xi_r,$$

$$\xi_w \sim \left(-\frac{8\pi\varkappa P_0'}{H^2}\right)^{1/4},$$

$$m \sim H^{1/2}\varkappa^{-1/4}.$$

The confinement area (no FLR)



$k = 3, H = 3$

$k = 3, H = 10$

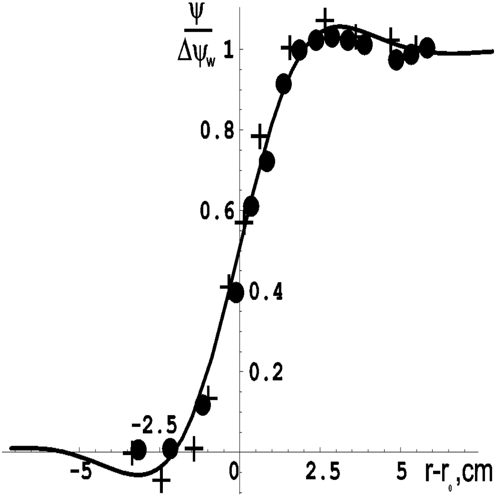
$k = 3, H = 30$

$k = 3, H = 100$

$k = 30, H = 300$

Green dots correspond to confinement, red - to severe losses. 2D computer simulation.

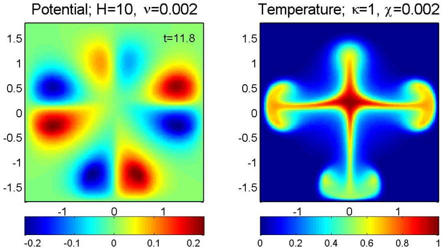
Comparison to GDT experiment



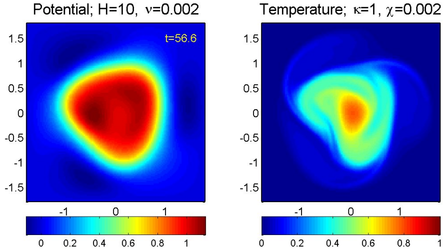
Angle-averaged distribution of the plasma potential for two radial positions of the flow layer, experiment and the analytic estimate.

(*Beklemshev, Bagryanski, Chaschin, OS-2006*)

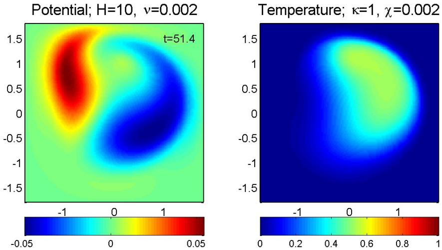
Simulation of the interchange



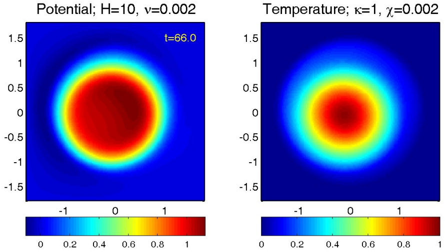
MHD



MHD with flow

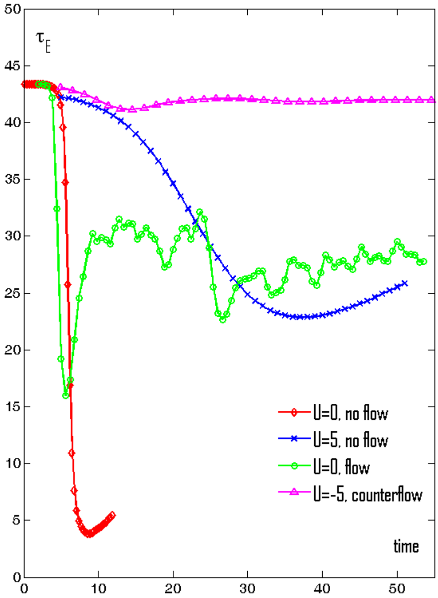


Drift MHD ($U = 5$)



Drift MHD ($U = 5$) with flow

Comparison of confinement



Energy confinement time for the four models of the interchange (GDT parameters).

The vortex confinement combined with the strong FLR effect seems to be quite efficient and is consistent with experiments.

Rotation in the counter direction to the diamagnetic drift is chosen as in the GDT. The co-rotating model is somewhat worse due to developing $m = 2$ KH instability.

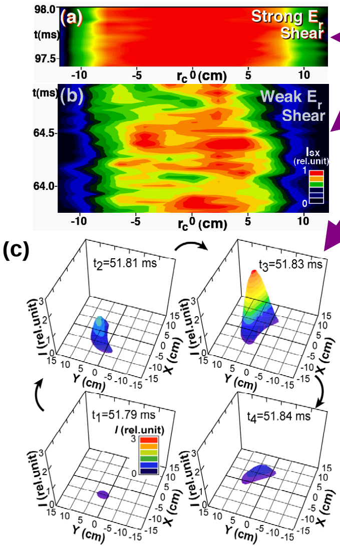
The Pastukhov-Chudin model

Equation for the dynamic vorticity, $\hat{w}$, looks like

$$\partial_t\Big|_{\psi}\,\hat{w} + [\Phi,\,\hat{w}] - [\hat{\rho},\,\langle\frac{v_a^2}{2}\rangle] + \frac{1}{U\gamma}\partial_\psi U\,\partial_\varphi\,(\hat{\rho}_0\,\tilde{T} + \hat{T}_0\,\tilde{\rho}) = U\,Q_w^*$$
$$+\frac{3X_M}{20}\left(\frac{T_i+T_e}{2T_i}\right)\left\{\partial_\psi\left(\hat{\rho}\,\langle r^2\rangle\,\partial_\psi\left(\frac{1}{U^{2/3}\sqrt{T_0}}\,\frac{a^2}{\langle r^2\rangle}\,\hat{w}\right)\right)\right.$$
$$\left. + \frac{a^2}{\langle r^2\rangle}\langle\frac{1}{r^2B^2}\rangle\,\frac{\hat{\rho}}{U^{2/3}\sqrt{T_0}}\,\partial^2_{\varphi\varphi}\hat{w}\right\}.$$

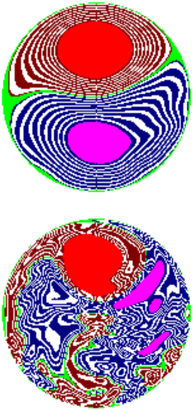
One can see, that the interchange drive is linearized, while in place of the plasma-wall coupling there is vorticity drive (in the RhS) due to complex plasma dynamics in the curved nonuniform field.

Simulation of GAMMA-10 experiment



GAMMA 10 experiments

Simulations



Interpretations

The authors (Pastukhov) state that *additional off-axis electron heating provides a layer of high vorticity that inhibits convection and results in an efficient ITB.*

In terms of the “vortex confinement” it means that to begin with *the hot plasma core of Gamma-10 resides inside of a huge vortex. The vortex itself is not well-confined (can reach the limiter) unless there is off-axis heating that provides the necessary rigidity (the flow acquires position-dependence).*

These are just alternative viewpoints, both can be true simultaneously.

Tokamaks

The plasma-wall coupling is obviously present in tokamak SOL, and it might be interesting to try biasing different parts of limiters and divertor plates, so as to drive the shear flows around the plasma core. However, *this process happens even if not designed.*

The theory of the turbulent drive of the flow-layers and ITBs is much better developed for tokamaks, while open traps lag behind...

Conclusion

- Driving the shear-flow via wall potentials is cheap ($\sim 30kW$, GDT), but may lead to the Kelvin-Helmholtz instability. However, it can be controlled by adjusting potential profiles.
- The gyroviscosity is essential in stabilizing the linear interchange in presence of a sheared flow. The combination of the shear-flow and FLR is sufficient for the nonlinear "vortex" confinement in simulations, which are in good agreement with the GDT data. This finally explains how the GDT manages to operate with an average magnetic hill.
- Application of the "vortex confinement" principle in conjunction with its wall-potential drive is possible even under fusion parameters. Indeed, the flow layer can and should be placed outside of the hot core plasma column, where the associated axial losses are tolerable.