# What are the limitations of Braginskii's fluid equations and Hazeltine's drift kinetic equation?

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The short mean-free path fluid equations of Braginskii [1] are known to be missing heat flux gradient terms in the expressions for viscosity. Their absence is a consequence of his ordering the plasma flow velocity comparable to the sound speed and considering the lowest order correction only (in the small gyroradius expansion parameter $\delta$) to the leading order Maxwellian distribution function. Earlier attempts to improve these equations (Mikhailovskii and Tsypin [2,3]) did not resolve the problem entirely since their generalized Laguerre polynomial expansion of the correction to the Maxwellian was truncated too soon, the non-linear contributions from the ion-ion collision operator were neglected, and electrons were not considered. We recently revisited the problem, addressed these shortcomings, and derived a complete closure scheme for short mean-free path ions and electrons [4]. When a subsequent attempt to recover these improved equations from the widely-used drift kinetic equation of Hazeltine [5] failed, we realized that this kinetic equation is only exact through first order in $\delta$. However, second order accuracy is needed to account for the effects of the Reynolds' and gyroviscous stress tensors. We have corrected the drift kinetic equation to account for the $\delta^2$ effects and used the formalism developed to derive expressions for gyroviscosity, perpendicular viscosity, as well as for heat fluxes for plasmas of arbitrary collisionality in terms of a few velocity integrals of the $\delta$ correction to the assumed leading order Maxwellian distribution function. The latter assumption holds for plasmas of arbitrary collisionality confined by closed magnetic flux surfaces in the absence of strong external driving forces [6]. Finally, we have used these expressions for viscosities and heat fluxes to derive a self-consistent hybrid fluid-kinetic description for such plasmas.

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