Simulated annealing for computing stationary state of ideal MHD ‡

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Abstract. Dynamics of ideal fluids, including ideal magnetohydrodynamics (MHD), is described by Hamiltonian and non-canonical Poisson brackets [1,2]; namely, the time evolution of the state vector of the system is determined by the functional derivative of the Hamiltonian with an operation by the Poisson bracket. Since the Poisson bracket is an anti-symmetric operator, it conserves the energy of the system. In addition to the energy conservation, the Poisson bracket also generates Casimir invariant. Therefore the system evolves on a surface specified by its energy and the Casimir invariants in the corresponding phase space. Here, let us consider an artificial system derived on the basis of the original ideal fluid system; the Poisson bracket is operated twice in the artificial dynamics. Then the energy of the system changes monotonically while keeping the Casimir invariants unchanged. Since the energy extremum on the surface of constant Casimir invariants gives us the stationary state of the ideal fluids, this method of the artificial dynamics gives us a stationary state characterized by the values of the Casimir invariants. This method is called simulated annealing (SA). SA has been applied for two-dimensional vortical motion of neutral fluids [3–5]. More advanced types of artificial dynamics have been also developed and demonstrated for the neutral fluids [6]. We have applied SA for obtaining MHD equilibrium. The Casimir invariants in this case include magnetic helicity, cross helicity and so on. The advantages of SA for the MHD equilibrium are (1) we can characterize the stationary states by the values of the Casimir invariants systematically, (2) there can exist plasma flow and (3) we do not need to assume nested flux surfaces. We have successfully obtained stationary states of two-dimensional low-beta reduced MHD by SA [7]. We have also developed a method to specify the values of the Casimir invariants before the SA calculation; those values are conserved during the SA calculation [8].

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