MHD in non-inductive tokamak plasmas: simulations and comparison to experiments on Tore Supra

P. Maget

Aknowlegments to
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Motivations

- **Non-inductive tokamak scenario: continuous energy production**
  - Unlimited plasma discharge
  - Plasma current driven by
    - Self-generated bootstrap current (~15% on Tore Supra, more than 50% needed for ITER)
    - Auxilliary systems (~85% provided by Lower Hybrid (LH) waves on Tore Supra)

- **Non-inductive discharges tend to have hollow current profiles**
  - On Tore Supra: hollow current profile driven by LH deposition
  - On ITER: hollow current profile driven by off-axis bootstrap

- **Magneto-Hydro-Dynamic stability of hollow current profiles**
  - At moderate $\beta = \mu_0 p/B^2$: (Double-)Tearing instabilities
  - At high $\beta$: pressure driven modes (Neoclassical Tearing, Interchange, Ballooning, kink)
Motivations

- Impact of MHD instabilities on Tore Supra non-inductive pulses
  - Loss of wave-driven fast electrons
    - Less current drive
    - Risks for plasma facing components (water leaks...)
  - Degradation of energy confinement

- This has motivated studies aiming at
  - Understanding what determines MHD-stable domain
  - Understanding how MHD modes impact energy confinement
  - Understanding how Tore Supra case extrapolates to higher-\(\beta\)

Tools
- MHD diagnostics \((\delta T_e, \delta B)\)
- Numerical codes for linear and non-linear studies
Outline

- Numerical tools
- Impact of the Double-Tearing Mode
- Fully developed non-linear regimes
- Issue on n=1 mode stability
- Conclusion and perspectives
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Numerical tools

- **Tool for non-linear MHD**: XTOR [Lütjens ‘08] (full MHD, toroidal)

\[ \begin{align*}
\rho (\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v}) & = \mathbf{J} \times \mathbf{B} - \nabla p + \mu_0 (\mathbf{V}_{\text{src}} - V_\varphi) + \nu \nabla^2 \mathbf{v} \\
\partial_t p + \mathbf{v} \cdot \nabla p & = -\Gamma \rho \nabla \cdot \mathbf{v} + \nabla \cdot \chi_\perp \nabla p + \cdots \\
& \quad + \mathbf{B} \cdot \nabla \left[ \chi_{||} (\mathbf{B} \cdot \nabla p)/B^2 \right] + H \\
\partial_t \mathbf{B} & = -\nabla \times \mathbf{E} \\
\mathbf{E} + \mathbf{V} \times \mathbf{B} & = \eta (\mathbf{J} - \mathbf{J}_{\text{NI}}) \\
\mathbf{J}_{\text{NI}} & = \mathbf{J}_{\text{cd}} + \mathbf{J}_{\text{boot}} \\
\mathbf{J}_{\text{cd}} & = (\mathbf{J} - \mathbf{J}_{\text{bs}})_{t=0}
\end{align*} \]

- **Toroidal rotation**

- **Parallel transport**

- **Non-inductive sources**

- **Standard MHD model adapted for Tore Supra experiments**

  - Non-inductive current sources
  - Anisotropic Heat transport
  - Toroidal rotation source (as generated by ripple)
Numerical tools

- **Re-Scaling of dimensionless coefficients**
  - Lundquist number \((S = \tau_R/\tau_A) \sim 10^9\) in experiments
  - Hardly tractable in numerical simulations for resistive MHD modes
    - Fine radial discretisation \((\Delta r \sim S^{-2/5})\)
    - Long time evolution \((T \sim S^{3/5})\)
  - Using lower \(S\) implies rescaling other parameters
    - **Option MHD / pressure dynamics**
      - Correct pressure dynamics during mode growth
      - \(\tau_{\text{MHD}}/\tau_E = (\tau_{\text{MHD}}/\tau_E)^{\text{exp}} : S^{3/5}\chi_{\perp} = (S^{3/5}\chi_{\perp})^{\text{exp}}\)
    - **Option current / pressure dynamics**
      - Correct pressure dynamics during equilibrium evolution
      - \(\tau_R/\tau_E = (\tau_R/\tau_E)^{\text{exp}} : S\chi_{\perp} = (S\chi_{\perp})^{\text{exp}}\)
  - Typically, we use \(S = 5 \times 10^6 - 5 \times 10^7\)

- **Other parameters**
  - Anisotropic diffusivity: \(\chi_\parallel / \chi_{\perp} = 10^8\)
  - Prandtl number: \(S\nu = 1\)
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Double-Tearing Mode

- Basic linear MHD stability features
  - q-scan by Rescaling of total current
  - Tearing unstable when $q_{\min}$ below 2

**DTM on $q=2/1$**

**DTM on $q=3/2$**

- $\lambda = \gamma A$
- $\lambda_{33986}$: linear growth rate
- $\chi_{//}/\chi_{\perp} = 10^8$
- $S\chi_{\perp} = 100$
- $S = 10^6$
Double-Tearing Mode

- Impact of the Double-Tearing Mode (DTM) on confinement
  - Observation of different situations
    - Off-axis crashes, complete crash (q=2 DTM)

What determines the impact of the DTM?
Double-Tearing Mode

- Unstable DTM experiences full reconnection
  - Full reconnection model [Kadomtsev ’75] extended to non-monotonic q-profiles [Carreras ’79]
    - Helical flux reconnected in multiple-valued region
      \[ \psi^* = \int d\psi_N \left( 1 - \frac{q}{q_{\text{res}}} \right) \]
Double-Tearing Mode

Unstable DTM experiences full reconnection

- Full reconnection model [Kadomtsev ’75] extended to non-monotonic q-profiles [Carreras ’79]
  - Helical flux reconnected in multiple-valued region
    \[ \psi^* = \int d\psi_N (1 - q/q_{\text{res}}) \]

- Core confinement
  - Small impact if off-axis
  - Else, global crash

Off-axis and global reconnections at q=2 consistent with full reconnection model and XTOR simulations
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Non-linear regimes

What is the dynamics of MHD-unstable plasma?

- DTM can produce periodic relaxations
  - The q=2 sawtooth regime [Chang ’96]
  - Example from Tore Supra

q=2 sawtooth regime with global crashes (consistent with full reconnection)
Non-linear regimes

- What is the dynamics of MHD-unstable plasma?
  - DTM can produce periodic relaxations
    - The q=2 sawtooth regime [Chang ’96]
  - In other conditions, saturated regime with MHD activity
    - Confinement degradation
    - Losses of fast electrons (LH-driven)

Transition to a regime with saturated MHD
Non-linear regimes

- Discrimination between periodic relaxations and saturation
  - Experimental database for Tore Supra shows
    - Periodic regime if crash radius small enough

Non-linear regimes and position of inversion radius after MHD mode

\[
\frac{\Delta T_{e0}}{T_{e0}} \quad \text{(periodic relaxations)}
\]

\[
(\sqrt{\Phi})^{\text{crash}}
\]

\[
0 \quad 0.2 \quad 0.4 \quad 0.6
\]

\[
0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8
\]
Non-linear regimes

- Non-linear MHD simulations
  - Here $\frac{\tau_R}{\tau_E} = (\frac{\tau_R}{\tau_E})^{\text{exp}}$ to have consistent pressure / equilibrium dynamics
  - Total current rescaled to increase $q=2$ radius
    - At the same time, goes from Double to Single tearing

Non-linear regimes for different $q=2$ positions

- Periodic relaxations for DTM
- Saturated state when $q=2$ more outside and STM

\[ \sqrt{\Phi_{(q=2)}} = 0.37 \]
\[ \sqrt{\Phi_{(q=2)}} = 0.55 \]
Non-linear regimes

- Fully non-inductive discharges ($q_{\text{min}} \sim 3/2$):
  - Magnetic equilibrium

\[ q(0) = 2.35 \quad q_{\text{min}} = 1.47 \quad q(a) = 8.62 \]
Non-linear regimes

- Fully non-inductive discharges ($q_{\text{min}} \sim 3/2$): two regimes
  - Successfull long discharges
    - No sign of $n=1$ activity
  - But in similar conditions
    - $n=1$ mode can grow and saturate at large size

**Example of $n=1$ becoming unstable**

![Graph showing $W_{(2,1)}$ and $T_e(0)$ with time]
Non-linear regimes

- Fully non-inductive discharges ($q_{\text{min}} \approx 3/2$): two regimes
  - Successfull long discharges
    - No sign of $n=1$ activity
  - But in similar conditions
    - $n=1$ mode can grow and saturate at large size

*Example of $n=1$ becoming unstable*

Can non-linear MHD model recover these two regimes?
Non-linear regimes

- Long fully non-inductive discharges: wrong n=1 prediction
  - Non-linear MHD simulations including transport in agreement with global crash: n=1 mode found unstable
  - Successfull discharges (no n=1 mode) cannot be recovered

Example of n=1 becoming unstable
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Issue on n=1 mode

- Condition for recovering n=1 stability in standard MHD
  - Curvature term at high S [Glasser ’75 (GGJ)]
  - No transport

2 tearing branches (2 resonances)

Outer mode stable at $S=S^{\exp}$

(CASTOR code)
Issue on n=1 mode

- Condition for recovering n=1 stability in standard MHD
  - Curvature term at high S \[\text{Glasser '75 (GGJ)}\]
  - No transport

- But transport is expected to play a role (low growth rate)

  \[n=1 \text{ then unstable at } S=S^{\text{exp}}\]

- Consistent with theory expectations \[\text{Lütjens '01}\]

Additional physics should be considered

\[\text{(XTOR code)}\]
Issue on n=1 mode

- Candidate mechanisms for n=1 stabilisation
  - Toroidal rotation (ripple-driven in Tore Supra)
  - Small island physics (outside standard MHD description)

- Rotation effect
  - Rotation in Tore Supra consistent with theory prediction for ripple drive [Trier '08]: Mach number is low (M~0.05)

\[
\rho \left( \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \mu_\varphi (V_{\text{src}} - V_\varphi) + \nu \nabla^2 \mathbf{v}
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Issue on n=1 mode

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- Rotation effect

No effect on (2,1) tearing saturation

The pressure crash still occurs
Diamagnetic rotation: possible candidate

- Not negligible a priori: $\omega^* / \gamma \sim 10$
- Could potentially
  - Limit island growth in the non-linear regime [Scott ’87]
  - Create multiple saturated states [Ottaviani ’04]

Investigations
- 4-field reduced MHD, cylindrical geometry [Hazeltine ’87]
- Preliminary study with XTOR
4-field Reduced MHD [Hazeltine ’87]

- Poloidal flux ($\psi$), electrostatic potential ($\phi$), electron pressure ($p$), parallel ion velocity ($v$)
- Isothermal model

\[
\begin{align*}
\frac{\partial_t}{\partial_t} \psi & = \eta \nabla^2 \psi - \nabla \cdot (\phi + \delta p) + e \\
\frac{\partial_t}{\partial_t} W & = \nu \nabla^2 W + [W, \phi + \delta \tau p] - \nabla \cdot J + \delta \tau \left[ \nabla \cdot p; \nabla \phi \right] \\
\frac{\partial_t}{\partial_t} p & = \kappa \nabla^2 p + [p, \phi] - \beta \nabla \cdot (v - 2 \delta J) + S_p \\
\frac{\partial_t}{\partial_t} v & = \nu \nabla v + [v, \phi] - \frac{1 + \tau}{2} \nabla \cdot p + \delta \tau \beta \frac{1 + \tau}{2} [p, v]
\end{align*}
\]

\[
W = -\nabla^2 \phi \quad J = -\nabla^2 \psi \\
\nabla \equiv \varepsilon \partial \phi - [\psi, .] \\
\omega_e^* = -(m/x) \delta (dp/dx) \text{ with } x = r/a
\]
Issue on n=1 mode

4-field Reduced MHD [Hazeltine ’87]
- Non-linear simulations with n=0,1
- Experimental resistivity $\eta=5\times10^{-9}$, $\nu=\nu//=\eta$
- $\omega^*_e=4\times10^{-3}$ (higher than experimental $\omega^*_e=10^{-3}$)

Two non-linear regimes
- higher $k_\perp$:
  - large island
  - $\omega \sim \omega_{ExB}$
  - high $\omega_{ExB}$ generation
Issue on n=1 mode

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  - Non-linear simulations with n=0,1
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Two non-linear regimes
- higher $\kappa_\perp$:
  - large island
  - $\omega \sim \omega_{ExB}$
  - high $\omega_{ExB}$ generation
- lower $\kappa_\perp$:
  - small saturated island,
  - $\omega \sim \omega^*$
  - low $\omega_{ExB}$ generation

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Issue on n=1 mode

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- Non-linear simulations with n=0,1
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Two non-linear regimes
- **higher $\kappa_{\perp}$:**
  - large island
  - $\omega\sim\omega_{ExB}$
  - high $\omega_{ExB}$ generation
- **lower $\kappa_{\perp}$:**
  - small saturated island,
  - $\omega\sim\omega^{*}$
  - low $\omega_{ExB}$ generation
Issue on n=1 mode

- 4-field Reduced MHD [Hazeltine '87]
  - Non-linear simulations (n=0,…3):
    - same results
    - Strong island distortion
    - Computation of the large island case crashes early

\[ \omega_e^* = 4 \times 10^{-3}, \quad \tau = 0.4, \quad S = 6 \times 10^7 \]
\[ \kappa_L = 10^{-9}, \quad (t=600000) \]
\[ \kappa_L = 10^{-8}, \quad (t=365000) \]
Issue on $n=1$ mode

- **4-field Reduced MHD [Hazeltine ’87]**
  - **Two different non-linear regimes**
    - Controlled by $\kappa_\perp$ and $\omega^*_e$
    - At low $\kappa_\perp$, pressure perturbation dominated by convection, otherwise dominated by transport
    - Qualitatively satisfying for explaining experimental facts
  - **Transition at** $\kappa_\perp \in [10^{-9}, 4 \times 10^{-9}]$ **for** $\omega^*_e \sim 4 \times \omega^*_e^{\exp}$
    - $\kappa_\perp$ consistent with collisional process
      - $(\kappa_\perp = \eta(1+\tau)/2 = 3.5 \times 10^{-9})$
    - But in experiment, $\kappa_\perp \sim 10^{-7}$: **large island regime**
  - Finally, the question of transport remains central even when diamagnetic effects are considered
Issue on n=1 mode

- Diamagnetic rotation: toroidal effects
  - Can toroidal geometry add significant stabilisation to n=1 mode?
  - Preliminary study with XTOR
  - New version of the code: fully implicit time advance
  - Electron diamagnetic effect in Ohm’s law:

\[
E + V \times B = \eta (J - J_{NI}) - \frac{1}{1 + \tau} \frac{\nabla ||p||}{en}
\]
Diamagnetic rotation: toroidal effects

- Linear study with transport
  - $S^{3/5} \chi_{\perp} = (S^{3/5} \chi_{\perp})^{\text{exp}}$
  - At experimental $S$
    - $\gamma << \omega^*_{e}$
    - $\omega \sim 0.95 \omega^*_{e}$

We expect significant effect of diamagnetic rotation in the full MHD model including transport
Issue on n=1 mode

- Diamagnetic rotation: toroidal effects
  - Non-linear regime with n=0,1
    - S=5x10^7, γ/ω_*~0.16
  - No significant difference compared to ω_*=0 case
  - Consistent with reduced-MHD 4-field model:
    - Small island regime not accessible in presence of realistic transport coefficients
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Conclusion & Perspectives

- Dynamics of MHD modes in non-inductive discharges
  - Double-Tearing: full reconnection
  - Saturation / Periodic relaxations: $q=2$ surface position

- Stability of $n=1$ mode in very long discharges
  - Curvature stabilization at high $S$ without transport
  - But fails in more realistic conditions (finite transport)
  - Two-fluid effect allows small island regime
    - But at collisional transport coefficient level only

- Future work:
  - more comprehensive MHD model needed?
  - Other rationale for transport coefficient implementation?