PIC simulations of microturbulence in the presence of a magnetic island

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Abstract.

In tokamak experiments, magnetic islands destroy the axial symmetry of the plasma through a helical magnetic-field perturbation. In the reconnected region the confinement is rather poor, due to the fast parallel transport along the field lines. The behaviour of the plasma turbulence in such a situation needs to be investigated, since it has a significant impact on both the transport in the island region and the stability of the island itself.

A first attempt to study this rather rich area is presented in this paper. A (static) island structure is included in the particle-in-cell (PIC) global nonlinear gyrokinetic code ORB5 [1, 2]. The perturbed magnetic field due to the island changes the orbits of the ions and influences the structure of the fluctuating potential. The equations of motion and the equation for the distribution-function perturbation are modified accordingly. The first simulations of the perturbed electric potential, heat flux and mode spectrum due to ITG turbulence (without zonal flows) are presented.

INTRODUCTION

The interplay between turbulence and magnetic islands in tokamaks is an outstanding problem in fusion research, its interest ranging from the basic understanding of the dynamics of plasma instabilities to the prediction of energy confinement. The presence of a radially-extended reconnected region changes the transport properties of the plasma by reducing the temperature gradient and hence the drive of the turbulence, but also by altering the magnetic topology, connecting the regions on the inner and on the outer side of the resonant surface. On the other hand, the development of large-scale MHD instabilities can be affected by turbulence as electromagnetic microinstabilities can provide seed islands for neoclassical tearing modes [3]. Moreover, transport across the island can modify the temperature profile, thus affecting the neoclassical drive [4, 5].

For the computations, this is a multi-scale problem which involves different lengths, from the gyroradius over the radial and poloidal extension of the turbulent structures to the width of the reconnected region. The time scales under consideration range from the gyrofrequency over the toroidal transit frequency and the diamagnetic frequency to the island growth rate and the collision frequency. It is at present impossible to perform simulations in which all these scales are resolved. As the time over which the island evolves is much longer than the time needed for turbulence to develop, a static island of fixed width is considered here. Our study is based on gyrokinetic $\delta f$ PIC simulations, which are particularly suited for this aim, as they do not require to change the coordinate system, which is not field-aligned. The gyrokinetic equation is solved using the code
ORB5 [1, 2]. The island is introduced by modifying the equations of motion assuming that a radial field perturbation is present and taking into account the motion along the perturbed field lines in the evolution equation for the perturbed part of the distribution function $\delta f$.

This paper describes the code ORB5 and the modifications introduced in order to treat a static magnetic perturbation, as well as the first simulations of ITG turbulence (without zonal flows) in the presence of an island.

**THE PIC GYROKINETIC CODE ORB5**

ORB5 is a global collisionless electrostatic $\delta f$ nonlinear gyrokinetic code. It solves the gyrokinetic equations in the formulation of T. S. Hahm [6] using particles and the Poisson equation for the electrostatic potential using finite elements on a fixed grid. The distribution function is split into an analytically-known time-independent part $f_0$ and a perturbation $\delta f$ which is represented numerically by an ensemble of markers, $f = f_0 + \delta f$. These markers evolve in time according to the gyrokinetic equations of motion,

$$\frac{dR}{dt} = v \parallel b + \frac{1}{B^*} \left[ \frac{\mu B + v^2}{\Omega_{ci}} b \times \nabla B - \frac{v^2}{\Omega_{ci}} b \times (b \times \nabla B) - \nabla \phi_g \times b \right],$$

$$\frac{dv_\parallel}{dt} = -\mu \left[ b - \frac{v_\parallel}{B^* \Omega_{ci}} b \times (b \times \nabla B) \right] \cdot \nabla B,$$

$$\frac{d\mu}{dt} = 0,$$

where $R$ is the position of the gyrocentre, $v_\parallel$ the velocity component along the magnetic field, $b$ the unit vector along the magnetic field $B$, $\mu$ the magnetic moment, $\Omega_{ci}$ the cyclotron frequency, $\langle \phi \rangle_g$ the perturbed potential (solution of the Poisson equation) averaged over the gyroparticle, $q_i$ and $m_i$ the particle’s charge and mass, respectively, and $B^* = B + (m_i/q_i)v_\parallel b \cdot \nabla \times b$. Since along the orbits $df/df = 0$, $\delta f$ must obey the equation

$$\frac{d(\delta f)}{dt} = -\frac{df_0}{dt} = -v \cdot \nabla f_0.$$  

In gyrokinetic simulations, only the perturbed $E \times B$-velocity arising from the instability is usually retained on the right-hand side of Eq.(4), since the magnetic drifts can lead to spurious zonal flows [7, 8]. In the simulations presented here, we use the full velocity $v = dR/dt$ to properly include the motion along the perturbed field lines.

The perturbed potential is obtained as the solution of the Poisson equation

$$\nabla^2 \phi = 4\pi q_i \left\{ n_e - \int \left[ f + \frac{q_i}{m_i B} (\phi - \langle \phi \rangle) \frac{\partial f}{\partial \mu} \right] \delta (R + \rho - r) d^6Z \right\},$$

where \(R\) is the position of the gyrocentre, \(v_\parallel\) the velocity component along the magnetic field, \(b\) the unit vector along the magnetic field \(B\), \(\mu\) the magnetic moment, \(\Omega_{ci}\) the cyclotron frequency, \(\langle \phi \rangle_g\) the perturbed potential (solution of the Poisson equation) averaged over the gyroparticle, \(q_i\) and \(m_i\) the particle’s charge and mass, respectively, and \(B^* = B + (m_i/q_i)v_\parallel b \cdot \nabla \times b\). Since along the orbits \(df/df = 0\), \(\delta f\) must obey the equation

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\]
where $\rho$ is a vector directed from the gyrocentre to the position of the particle and $\langle \phi \rangle$ is the flux-surface-averaged potential. The charge connected to each marker is assigned point-wise to a spatial mesh (charge assignment) in order to provide the source term for the Poisson equation. The computation of the gyroaveraged density follows an adaptive procedure, in order to have the same number of sampling points per arclength along the gyro-ring. Once the perturbed gyroaveraged charge density associated with each marker has been projected onto the splines, the equation for the components of the potential on this basis reduces to a matrix equation.

It should be added that the same spline basis is used to interpolate the radial magnetic-field perturbation, which is initially assigned on a grid.

**MAGNETIC ISLANDS**

In the presence of a magnetic island, the magnetic field can be represented as

$$\mathbf{B} = \nabla \psi_t \times \nabla \xi / m + \nabla \varphi \times \nabla \Psi_{he},$$

(6)

where $\psi_t$ is the toroidal flux, $\xi = m \theta - n \varphi$ is the helical angle ($\theta$ and $\varphi$ being the poloidal and toroidal angles, respectively, and $m$ and $n$ the poloidal and toroidal number characterising the island) and

$$\Psi_{he} = \psi - \frac{\psi_t}{q_s} + \alpha \cos \xi$$

(7)

is the helical flux ($\psi$ is the poloidal flux and the subscript $s$ denotes that a quantity is calculated at the resonant $(m, n)$ surface). If $\alpha = 0$, it is easy to show that Eqs. (6,7) reduce to the usual representation of the magnetic field. The last term of Eq. (7) describes the field perturbation due to the island, which is therefore $\mathbf{B} = \alpha \nabla \varphi \times \nabla \cos \xi = \ldots$
FIGURE 2. The evolution of the temperature gradient during the run (a) and the heat conductivity (in gyro-Bohm units) due to the $\mathbf{E} \times \mathbf{B}$ transport (b). For $t < 12000$, only the modes $2 \leq n \leq 4$ were allowed.

$\dot{m} \alpha \sin \xi \nabla \theta \times \nabla \varphi$ (here, $\alpha$ is approximated to be a constant). The new field component is directed along $\nabla \psi$. The helical flux introduced in Eq. (7) can be used to label the perturbed magnetic surfaces, as $\mathbf{B} \cdot \nabla \Psi_{he} = 0$. The most important consequence for the particle trajectories resulting from the new magnetic-field topology is that the motion parallel to the field includes now a radial component. This has been included in ORB5 by substituting $\mathbf{b} \rightarrow \mathbf{b} + \mathbf{\tilde{b}}$ (where $\mathbf{\tilde{b}} = \mathbf{B}/B$) in the first term of both Eq. (1) and Eq. (2). As mentioned in the previous section, the term $\tilde{v} = v_{\parallel} \mathbf{\tilde{b}}$ must be added on the right-hand side of Eq. (4).

Fig. 1a is a Poincaré puncture plot obtained by following the orbits of 23 particles along the torus, the electrostatic potential being switched off. The dots obtained as the intersection of the trajectories with the plane $\phi = 0$ show the pattern of the perturbed field lines, which coincides with the contour levels of $\Psi_{he}$. The density profile obtained again in the absence of the electrostatic potential is reported in Fig. 1b. Here and in the following, the flux-surface averages are performed in bins delimited by surfaces of constant $\Psi_{he}$. There is a clear flattening of the density within the island. The contribution of the simulation markers $\delta n = \int d\Omega_p \delta f/V$ (where $d\Omega_p$ is the phase-space element and $V$ is the volume of the cell) is shown by the dashed line.

**TURBULENCE SIMULATIONS IN THE ISLAND REGION**

In this section, the first simulations of microturbulence in the presence of a magnetic island are shown for a set of parameters close to those of the CYCLONE base case [9]. A circular plasma with aspect ratio $R/a = 2.77$ and $\rho_s = 1/160$ has been considered. The density is flat in the simulations, whereas the temperature profile is modelled by a hyperbolic tangent, the position of maximum gradient being located at $s = \sqrt{\psi} = 0.624$, which corresponds approximately also to the position of the island, which has mode numbers $(m, n) = (3, 2)$. In the results presented in Figs. 2 and 3, only a half torus is considered for the development of turbulence, i.e. only even modes are retained.
(however, the mode $n = 0$ is filtered out). ITG simulations without zonal flows are considered ($\langle \phi \rangle = 0$ in Eq. (5)), since the flux surface average of the gyroaveraged potential along the perturbed field lines, arising from the adiabatic electron response, is still not included in ORB5. For $t < 12000$ (in units of $1/\Omega_{ci}$) only the modes with $n = 2$ and $n = 4$ are retained, so that no turbulence develops and the temperature profile can adjust (flatten) according to the presence of the island. Since the ratio between the cyclotron frequency and the toroidal transit frequency $\omega_t = v_T/2\pi qR$ (where $v_T$ is the thermal velocity) is in this case $\Omega_{ci}/\omega_t \approx 4000$, this means that a thermal particle orbits the torus about three times before turbulence is switched on. During this time, the temperature gradient, given here as $R/L_T = R\nabla T/T$, develops a minimum at the island location, as shown in Fig. 2a (the solid curve represents the initial profile). In this simulation, the island (half-)width is about $W = 0.1a$ and the ratio between the thermal banana width and the island width is $w_b/W \approx 0.2$ and there is a moderate overlap between the particle orbits and the island structure which marginally affects the profile flattening [10]. The ratio between Larmor radius and island width is $\rho/W \approx 0.06$.

The time slice $t = 17600$ is taken during the linear growth phase of the turbulence, before the overshoot. As can be seen in Fig. 3a, structures in the perturbed potential (streamers) start to build up on both sides of the island, almost crossing the region around the $X$-point. During the overshoot, the turbulence-driven transport increases, so that the inner and the outer side of the island are connected by the turbulence, cf. Fig. 2b at $t \approx 21000$, at least on the bad-curvature region, where the drive is stronger. On the high-field side, the streamers never cross the island. Fig. 3b shows the potential structures at $t = 32000$, i.e., during the nonlinear decay phase. The island position can again be identified by the breaking of the streamers. The turbulence spectrum shifts to larger poloidal wavelengths.

This is investigated in more detail in Fig. 4. Two runs are compared, where the same initial conditions have been taken, in the first case without any magnetic perturbation, in the second one with an island as in the previous figures (here no turbulence-free phase was considered). The evolution of the toroidal mode numbers ($y$-axes) is plotted against
FIGURE 4. Turbulence spectrum without (a) and with island (b). No odd toroidal mode numbers were present. The same scale is used in both plots.

time (x-axes). It can be seen that the modes that get unstable are approximately the same in both cases, and their temporal evolution is also similar. Their amplitude, however, is much lower when the island is present, in particular around the overshoot. In this pair of simulations, the mode $n = 0$ was kept in the Fourier filtering, as can be seen at the bottom of the two figures.

CONCLUSIONS

Turbulence in tokamak geometry in the presence of a static magnetic island can be investigated by means of the global gyrokinetic PIC code ORB5. The first results show that the island modifies the turbulence by reducing the temperature gradient which drives the turbulence, as one can expect. The heat flux due to the turbulence is also reduced. The amplitude of the turbulent modes decreases with respect to the case without island, in particular close to and immediately after the overshoot. The code can now be applied to the investigation of the nature and of the scaling properties of the turbulent transport in the island region.

REFERENCES