Zonal Flows: Comparisons of Basic Experiments and Theories

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Abstract. Zonal flows driven by the ITG (ion temperature gradient) drift modes has been performed in the Columbia Linear Machine [R. Scarmozzino, A.K. Sen, and G.A. Navratil, Phys.Rev.Lett. 57, 1729 (1986)]. The difficult problem of detection of zonal flows has been solved via a novel diagnostic using the paradigm of FM (frequency modulation) in radio transmission. Using this and Discrete Short Time Fourier Transform, we find a power spectrum peak at ITG (‘carrier’) frequency of ~120kHz and FM sidebands at frequency of ~2kHz, which has all the signatures of a zonal flow. The results have only partial agreement with most theoretical estimates including the one proposed here.

Keywords: Zonal flows, nonlinear coupling.

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INTRODUCTION

A fundamental open physics question in plasma physics is transport scaling. The role of zonal flows in transport regulation and transport barriers is an important critical concept. Zonal flows (ZF) are believed to be spontaneously excited by drift wave turbulence via Reynolds stress from small scale fluctuations to large scale flow. Both analytic theories [1-4] and gyro-kinetic simulation [5-6] indicate its existence. Zonal flows are poloidally and toroidally symmetric \((k_\theta \approx 0, k_r \approx 0)\) and radially inhomogeneous \((k_r \neq 0)\) flow structures in toroidal plasmas. The theories also predict low frequency (nearly zero) for ZF. The experiments in tokamaks to detect ZF are very difficult and fairly inconclusive. G.R.Tynan et al. study ZF in the DIII-D tokamak [7]. They used bicoherence analysis of signals from Langmuir probes and obtained correlation between bicoherence and L-H transition and estimated the frequency of ZF to be about 50kHz, which is clearly too high. Shats et al [8] in a very good basic experiment report on a ZF-like structure driven by radial non-ambipolar current, instead of the usual Reynolds’s stress. This fact in conjunction with the absence of any evidence of \(k_\theta = 0\), imply a similar but not exact physics basis of ZF. Lastly, Fujisawa et al [9] detect ZF using the excellent diagnostics of two radially close heavy ion beam probes (HIBP). However, their estimates of ZF parameters are based on numerical filtering which is somewhat unreliable. Furthermore, the poloidal symmetry is obtained only via circumstantial evidence. The difficulty of direct measurements and the lack of definitive experimental results in tokamaks motivated our basic physics study of ZF in the Columbia Linear Machine (CLM) [10]. In CLM
experiments we use slab ITG (ion temperature gradient) drift waves to study this phenomenon.

**COLUMBIA LINEAR MACHINE**

A steady-state collisionless cylindrical plasma column in a uniform axial magnetic field is created in CLM (see Fig.1). The core of the plasma is effectively heated by RF in the parallel direction so that an ITG mode is excited [10]. Figure 2 shows the typical average power spectra of density fluctuations. The mode with frequency $f \sim 70kHz$ has been identified as $E \times B$ mode with azimuthal mode number $m = 1, k_\parallel = 0$. This mode is believed to be a rotationally driven Rayleigh-Taylor type instability, driven by $E \times B$ rotation of the plasma column [11]. The mode with frequency $f \sim 120 kHz$ has been identified as an ITG slab mode with azimuthal mode number $m = 2, k_\parallel \sim 2\pi/400cm^{-1}$ and propagates in the ion diamagnetic direction and has been definitively identified [12].

![Diagram](image)

**FIGURE 1.** Layout of the diagnostics in the Columbia Linear Machine

**DIAGNOSTICS FOR THE DETECTION OF ZONAL FLOWS**

We show the mode spectrum of a strong ITG mode with a relatively narrow spectral width of $< 10kHz$ in CLM in Fig. 2. There are two main difficulties in the experimental detection of ZF both in tokamaks and CLM. The first is due to its near zero frequency where it can be embedded in significant 1/f noise. More importantly, because of its low level of poloidal velocity, it represents only a small increment (1 to 3%) of the Doppler shift over the equilibrium Doppler shift $(E_{r0}/rB_z)$ of ITG frequencies. Direct experimental determination of such a minute change in poloidal velocity or the resultant Doppler shift by usual diagnostics is nearly impossible.
Therefore we have developed a novel method based on finding the “signature” of ZF electric field in the spectra of other modes: $E_x B$ and ITG modes both of which suffer significant equilibrium rotational Doppler shift in CLM. In our case we expect that the very low frequency radial electric field $E_{ZF}$ due to ZF (with $k_\theta \approx 0, k_\parallel \approx 0$) will cause some additional time-dependent rotation (additional Doppler shift) due to $E_{ZF} \times B$, which can lead to frequency modulation of ITG mode. This frequency modulation is similar to well-known FM modulation of carrier frequency widely used in radio transmitters and should result in similar sidebands in the frequency spectra. In the CLM’s laboratory frame the frequency of the ITG mode ($m=2$) is

$$\omega(t) = 2\Omega_{E_x B} + 2\Omega_{ZF}(t) - \omega_{ITG},$$

where $\Omega_{E_x B}$ is the Doppler shift due to the equilibrium $E_x B$ rotation, $\Omega_{ZF}$ is the additional Doppler shift due to the ZF, $\omega_{ITG}$ is the ITG mode frequency in plasma frame. It is noted that ZF itself does not suffer a Doppler shift as its $m=0$, however ITG with $m=2$ does suffer an additional Doppler shift due to $E_{ZF} \times B$. We can consider the ZF time dependence as $\Omega_{ZF}(t) = \Omega \cdot \cos(\omega_{ZF} t)$, where $\Omega = |E_{ZF}| / rB$ is the zonal flow amplitude (i.e. rotational frequency) and $\omega_{ZF}$ is the very low intrinsic zonal flow frequency. Then a typical signal from probes corresponding to the ITG mode is given by the following Fourier series:

$$U(t) = U_0 \cos(2\Omega_{E_x B} t + \varepsilon \sin(\omega_{ZF} t) - \omega_{ITG} t) =$$

$$U_0 \sum_{n=-\infty}^{\infty} J_n(\varepsilon) \cos(2\Omega_{E_x B} t + n\omega_{ZF} t - \omega_{ITG} t)$$

(1),

where $\varepsilon = 2\Omega / \omega_{ZF}$. Therefore, we now focus our effort on the experimental determination of fine structure of ITG mode power spectrum containing sidebands.
Our typical data records of fluctuation of density or floating potential of plasma contained 256K points with digitization frequency of 1MS/s. In the following data collection scenario we utilize discrete short time Fourier transform with 2048 µs FFT window size and 256 µs shift between the window frames resulting in frequency resolution of about ~ 0.49kHz and about 1000 samples of spectra are used for sample averaging. The resulting power spectra of ITG mode were tested for the presence of sidebands according to the criterion below. We selected the spectra with a clear ITG mode and two sideband peaks located symmetrically at both sides and for the amplitudes of sidebands higher than the threshold noise level. We found that up to 25% of total number of FFT frames satisfy this criterion.

![Power Spectrum](null)

**FIGURE 3.** Fine structure of power spectrum of ITG density fluctuations.

The typical result of the sample averaged ITG spectra with this selection is shown in Fig.3 for density fluctuations. In order to exclude the effects of extraneous low frequency drifts in our system, we align in frequency the ITG spectral peaks in the frames. The frequency of the ITG mode in the laboratory frame is about 120kHz and the sideband frequency is \( \omega_{ZF} / 2\pi = 1.92kHz \pm 0.60kHz \). The power spectrum of potential fluctuations has very similar sideband picture. Now the value of \( \varepsilon \approx 1 \) is determined from \( \left[ J_1(\varepsilon)/J_0(\varepsilon)\right]^2 \approx 1/3 \), which is equal to the ratio of the power in the sideband and that in the ITG mode measured in our experiment. Lastly, the ZF amplitude \( \Omega \approx 1kHz \) is determined from the definition of the parameter \( \varepsilon = 2\Omega / \omega_{ZF} \) and the measurement of \( \omega_{ZF} \approx 2kHz \) mentioned above.

For confirmation of the above concept and procedure, we now apply it to the \( \text{ExB} \) mode (Fig. 2) by defining the signal due to this mode as
where the prime indicates quantities relevant to the ExB mode. We calculated sideband frequency $\omega_{ZF}$ from the ExB mode data in a similar manner to obtain $\omega_{ZF} / 2\pi = 2.29kHz \pm 1.39kHz$. This is in very good agreement with sideband frequency $\omega_{ZF}$ found above from ITG mode data. It is emphasized that the sidebands discussed above are due to our diagnostic paradigm of FM modulation of ITG mode by low frequency ZF.

Figure 3 shows the low frequency part of density fluctuations in the power spectrum using the selection criteria mentioned above. We can observe a peak near $2kHz$, which is consistent with the ZF frequency in our sideband picture.

**Figure 4.** Low frequency part of power spectra of density fluctuations.

**OTHER SIGNATURES OF ZONAL FLOWS**

We now discuss the data for wave numbers of ZF. First it is noted that for both ion acoustic modes with frequency $f < 20kHz$ [13] and $2kHz$ ZF in our present experiments the azimuthal number $m=0$ have been found. Next another signature of ZF $k_{||}=0$ is confirmed by the measurement of $k_{||}$ in region around $\omega_{ZF}$ frequency. The parallel wave number is determined from the axial phase shift measured via cross-correlation of two Langmuir probes, displaced axially by $33cm$ [13]. The resulting phase shift for the low frequency part of measured spectra, which satisfies the above mentioned ZF criteria is shown in Fig. 4. This figure indicates that $k_{||} \approx 0$ at the frequency $\omega_{ZF} / 2\pi \sim 2kHz$, which is perfectly consistent with $\omega_{ZF}$ obtained from the sideband analysis above. Figure 4 also shows the phase shift for the unselected data,
which indicates ion acoustic modes with a phase velocity $C_s \sim 4 \times 10^6 \text{cm/s}$ [13,14] determined from the reciprocal of the slope of the plot.

![Graph](image)

**FIGURE 5.** Measurement of parallel wave number of Zonal flows.

We now present the data for the radial structure of ZF and the resulting flow shear. Using the procedure discussed after Eq. (1), we determine both the intrinsic ZF frequency $\omega_{ZF}$ and the amplitude of ZF $\Omega (\sim |E_{ZF}|)$ at several radial points and obtain radial profiles of the latter. Then we calculate the radial shear of the corresponding azimuthal flow ($\sim \partial (r\Omega)/\partial r$). Figure 5 shows the radial profiles of density fluctuation of ITG mode, the radially resolved amplitude of the electric field $|E_{ZF}| (\sim \Omega)$ caused by ZF and the corresponding flow shear. When the ITG fluctuation level is low ($< 1.5\%$), the error bar in our analysis is too large and the correlation ratio between the two different azimuthally-placed probes is very small. Therefore, we cannot show the ZF data for $r < 1.4cm$ and $r > 2.5cm$. ZF shear is about $1.5 \text{kHz} (\sim 1/10 \gamma_{ITG})$ and this is roughly consistent with the nonlinear mode coupling theory (see below) which predicts $(\Delta f)_{\text{Shear}} \sim 0.7 \text{kHz}$.
Finally we consider isotopic effects on ZF generation. This is an important question, as some theories suggest that the isotopic effects on ZF and its shear can translate into isotopic effects on transport leading to the breaking of gyro-Bohm scaling. We use hydrogen and deuterium plasmas and focus on maintaining the most important parameters rigorously constant for both gases. The radial profiles of fluctuation level of ITG modes and ZF shears $\rho(\Omega)/\Omega$ for different gases are shown in Fig.6. The ZF shears are almost identical for both Hydrogen and Deuterium. Therefore, our experiments do not confirm the theoretical premise of breaking of gyro-Bohm scaling via isotopic effects on ZF shear.

**ISOTOPIC SCALING OF ZONAL FLOWS**

FIGURE 6. Radial profiles of density fluctuations, ZF field and shear.
A CRUDE MODEL OF ZONAL FLOW GENERATION

In one model zonal flows can be considered to be generated via the self-coupling of the ITG mode, whose amplitude is presumed to be determined by a 3-wave coupling involving ion acoustic modes [14]. Using the Reductive Perturbation method [15,16] ZF is calculated as follows for a given ITG mode amplitude. The slab branch of the ITG mode are governed by the following set of equations:

\[
\frac{\partial T_i}{\partial t} - \frac{2}{3} \frac{\partial n_i}{\partial t} - \nu_{s_i} \left( \frac{2}{3} - \eta_i \right) \frac{\partial \phi}{\partial y} = \rho_s c_s \left( \nabla \phi \times \hat{z} \cdot \nabla \right) \left( T_i - \frac{2}{3} n_i \right) \quad (2),
\]

\[
\frac{\partial \nu_{\parallel i}}{\partial t} = -c_s \frac{\partial \phi}{\partial z} - c_e \frac{\partial P_i}{\partial \hat{z}} \quad (3),
\]

\[
\left( \frac{\partial}{\partial t} - \frac{1 + \eta_i}{\tau} \nu_{s_i} \frac{\partial}{\partial y} - \mu \nabla^2_{\parallel} \right) \rho_s^2 \nabla^2_{\parallel} \phi - c_s \frac{\partial \nu_{\parallel i}}{\partial z} - \frac{\partial n_i}{\partial t} - \nu_{s_i} \frac{\partial \phi}{\partial y} = \rho_s c_s \left( \nabla \phi \times \hat{z} \cdot \nabla \right) \left( \rho_s^2 \nabla^2_{\parallel} \phi - n \right) \quad (4),
\]

\[
\frac{\partial n_i}{\partial t} + \nu_{s_i} \frac{\partial \phi}{\partial y} = \rho_s c_s \hat{z} \times \nabla n \cdot \nabla \phi \quad (5),
\]

where \( \nu_{s_i} = \rho_s c_s \frac{P_i}{L_n} \), \( \eta_i = \frac{L_n}{L_T} \), \( \nu_{\parallel i} = \nu_{s_i} \frac{c_s}{c_v} \), \( \tau = \frac{T_e}{T_i} \), \( \phi = e \bar{\phi} \), \( P_i = n_i T_i \). Using the transformation to slow time variable \( t' \) as: \( x \rightarrow x', \xi = \xi (y - c t') \), \( t' = e^2 t \) where \( e \) is a
small parameter proportional to the amplitude and \( c \) is the phase velocity. Potential \( \phi \) is expanded as \( \phi = \sum_n \epsilon^n \phi^n \) where \( \phi^n = \sum_m \sum_l \xi_{nml} \exp \left( i k_m y + k_z z - \omega_m t \right) \) and \( k_m = \frac{2\pi m}{L} \). Substituting these expansions in Eq. (2)-(5) we obtain in the third order \( \epsilon^3 \):

\[
\frac{c}{3} \frac{\partial T_0^{(2)}}{\partial \bar{z}} + \frac{2}{3} c \frac{\partial n_0^{(2)}}{\partial \bar{z}} - \left( \frac{2}{3} - \eta \right) \rho_s c_s \frac{\partial \phi_0^{(2)}}{\partial \bar{z}} = \rho_s c_s \frac{\partial}{\partial \bar{z}} \left[ \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} \frac{\partial T_1^{(1)}}{\partial \bar{z}} - \frac{2}{3} n_1^{(1)} \right] + \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} \frac{\partial}{\partial \bar{z}} \left( \frac{\partial}{\partial \bar{z}} - \frac{2}{3} n_1^{(1)} \right)
\]

(6)

\[
\frac{\nu_s}{3} \frac{\partial \phi_0^{(2)}}{\partial \bar{z}} - \frac{c}{3} \frac{\partial n_0^{(2)}}{\partial \bar{z}} = \rho_s c_s \left[ \frac{\partial n_1^{(1)}}{\partial \bar{z}} \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} + \frac{\partial n_1^{(1)}}{\partial \bar{z}} \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} - \frac{\partial n_1^{(1)}}{\partial \bar{z}} \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} - \frac{\partial n_1^{(1)}}{\partial \bar{z}} \frac{\partial \phi_1^{(1)}}{\partial \bar{z}} \right]
\]

(8)

The above equations can be reduced to the following differential equation for the ZF potential \( \phi_0^{(2)} \) as:

\[
\frac{d^2 \phi_0^{(2)}}{dx^2} = -\frac{2k_s^2 c_s}{\nu_s (1 + \tau)} \frac{d \phi_0^{(1)}}{dx} \frac{d^2 \phi_0^{(1)}}{dx^2}
\]

(9)

The \( \mathbf{E} \times \mathbf{B} \) rotation \( \nu_0^{(2)} \) and for Gaussian profile the shearing rate \( \nu_s \) due to the ZF flow are then given by

\[
\nu_0^{(2)} = \rho_s c_s \frac{d \phi_0^{(2)}}{dx}, \quad \nu_s = \frac{d \nu_0^{(2)}}{dx} = \frac{4x}{\Delta x^2 (k_s \rho_s)^2} \left( c_s^2 - \frac{\phi_0^{(1)}}{\nu_s (1 + \tau)} \right) e^{-\frac{x^2}{\Delta x^2}}
\]

(10)

The coupling of Zonal flow to ITG mode (back reaction) is described by the following non-linear equation:

\[
\frac{dX_1}{dt} = \gamma_1 X_1 - \frac{V^2}{\gamma_3} X_1^2 - \nu_s X_1
\]

(11)

where \( X_1 = |\phi_0^{(1)}|^2 \), \( V^2 = \left[ A \frac{c_s^2 k_y}{\Omega n_3} \frac{\omega_3}{\sqrt{\omega_2 \omega_3}} \right], \quad A = \left\langle \frac{\phi_0^{(1)}}{\phi_0^{(1)}} \frac{\partial^2 \phi_0^{(1)}}{\partial \bar{z}^2} \right\rangle = \frac{4}{3 \sqrt{3}}
\]

(12)

and \( \Delta x \) is the linear mode width. The third term in Eq. (11) describes the additional shear damping of the drift waves due to the ZF. The solution of Eq. (11) for CLM parameters is \( |\phi_0^{(1)}| \approx 2.6\% \). This is only a 10\% reduction in the saturated level of ITG fluctuations (2.8\%) calculated before in the absence of ZF [14]. With our experimental value of \( |\phi_0^{(1)}| \approx 4.0\% \), we can estimate the rotational frequency of ZF \( \Omega \sim 0.3kHz \) (\(-1/3\) of ext.) and ZF shear \( \sim 0.7kHz \) (\(-1/2\) of ext.) from equation (10). This implies that even though the model is interesting and relevant, but it may not be good enough. However, the 4-wave coupling model [3] (of modulational instability) is not applicable in CLM because it is strictly a toroidal model. Furthermore, the
modulational instability model [3, 4] model relies on two asymmetric side band frequencies. This is not consistent with our symmetrical sideband data shown in Fig. 2. One strong point for the model proposed here is that it indirectly includes a very robust and simple damping (ion acoustic), instead of ion-ion collisions which is relatively very small, but often evoked in other models in the literature. The comparisons of various theoretical models are roughly summarized in Table 1 below.

<table>
<thead>
<tr>
<th>Plasma Phenomena</th>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZF Real Frequency</td>
<td>2kHz</td>
<td>Most theories with $\kappa_{ZF} \sim \kappa_{DW}$: $0$ MI, with $\kappa_{ZF} \sim \kappa_{DW}$: $2.5$ kHz</td>
</tr>
<tr>
<td>ZF Amplitude</td>
<td>$1$kHz</td>
<td>with $v_{i i}$ damping $\phi_{ZF} \sim 10\phi_{ITG}$</td>
</tr>
<tr>
<td>ZF Shear ($\Delta f_{shear}$)</td>
<td>$1.5$kHz $\sim 0.1\gamma_{ITG}$</td>
<td>$0.7$ kHz (RP)</td>
</tr>
<tr>
<td>ZF Growth Rate</td>
<td>Not measurable</td>
<td>$\gamma_{ZF} \sim 19$kHz</td>
</tr>
<tr>
<td>Sideband of MI</td>
<td>Not observed</td>
<td>$17$kHz , -$22$kHz</td>
</tr>
</tbody>
</table>

MI – Modulation Instability [3,4]  
RP – Reductive Perturbation method [15,16]

In conclusion, we have definitively identified ZF with azimuthal (poloidal) and axial (toroidal) symmetry and very low frequency. However, the stabilizing effect of ZF on the parent ITG modes appears to be small and no significant isotopic effects are seen. The comparisons of various theories including the rough model proposed here reveal only partial agreement with experiments as shown in Table 1.

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**REFERENCES**