Electron Bernstein waves in spherical torus plasmas

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Abstract. Propagation and absorption of the electron Bernstein waves (EBWs) in spherical tokamaks (STs) have been intensively discussed in recent years because the EBWs coupled with an externally launched electromagnetic beam seem to be the only opportunity for microwave plasma heating and current drive in the electron cyclotron (EC) frequency range in the STs. The whole problem of the electron Bernstein heating and current drive (EBWHCD) in spherical plasmas is naturally divided into three major parts: coupling of incident electromagnetic waves (EMWs) to the EBWs near the upper hybrid resonance (UHR) surface, propagation and absorption of the EBWs in the plasma interior and generation of noninductive current driven by the EBWs. The present paper is a brief survey of the most important theoretical and numerical results on the issue of EBWs.

Keywords: mode coupling, electrostatic waves, electromagnetic waves, parametric instabilities

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EXCITATION OF THE EBW

Small aspect ratio tokamaks operate at high plasma density and comparatively low magnetic field at which either incident electromagnetic waves could not propagate near the plasma boundary, or its absorption is inefficient. That is why conventional schemes for microwave plasma heating and current drive are inapplicable for the ST devices. The most promising way to overcome this difficulty is based on linear conversion of the electromagnetic waves, incident from the low magnetic field side, into the electron Bernstein waves having no density cut-off and effectively damped at the nearest electron cyclotron harmonic regardless of its number. The linear mode conversion is a well-known phenomenon in plasma electrodynamics [1, 2] and it is usually described in terms of "interaction" between Wentzel-Kramers-Brillouin (WKB) modes that occurs if roots of a dispersion relation begin to coalesce [2, 3]. Unfortunately, the WKB approximation often fails for the incident EM waves in the ST plasmas due to close spacing of the UHR and cut-off surfaces. A typical example of such plasma configuration is shown in fig.1, where dispersion curves for the WKB solutions are purely illustrative because the width of peripheral propagation layer is much smaller than the vacuum wavelength. In this case another method, put forward long ago [4, 5, 6], can be applied to the mode conversion problem. This approach associates the mode conversion process with the UHR singularity of the cold plasma wave equation without references to the WKB theory. Earlier the problem of EMW→EBW conversion was solved with the use of this method for the case of strongly inhomogeneous plasmas [6]. In recent paper [7] it has been shown that in a plasma slab a full solution of the EMW→EBW conversion problem is reduced to finding a solution to the cold plasma wave equation practically regardless
FIGURE 1. Adopted from [7]. Poloidal cross-section of the spherical Globus-M tokamak (central plasma density $5 \cdot 10^{13} \text{cm}^{-3}$; magnetic field 0.4 T; wave frequency 14 GHz), and illustrative dispersion curves for perpendicular EMW incidence at the equatorial plane.

of the inhomogeneity scale-length.

One-dimensional model

It is assumed in paper [7] that all plasma parameters depend on a single Cartesian co-ordinate $x$ and the magnetic field is perpendicular to the $x$-axis. For a given frequency $\omega$ and perpendicular refractive vector components $N_y = c k_y/\omega$ and $N_z = c k_z/\omega$, plasma eigenmodes can be written as $E(r) = E(x) \exp(i [N_y y + N_z z - \omega t])$, with unknown vectors $E(x)$ to be found and spatial coordinates scaled in the units $c/\omega$. According to [6, 7], outside a thin layer enclosing the UHR any solution of exact hot plasma wave equation for $E(x)$ can be approximately presented in the following form

$$E(x) = E^{(C)}(x) + E^{(B)}(x), \quad (1)$$

where $E^{(C)}$ and $E^{(B)}$ are linear superpositions of the cold plasma and Bernstein eigenmodes, respectively. The WKB approximation is always applicable to the Bernstein waves in ST plasmas, so the second term $E^{(B)}$ in eq.(1) is a WKB solution for the EBW mode.

$$E^{(B)}(x) = C^{(B)} A(x) \exp\{i \int N^B_x(x')dx'\} \quad (2)$$

Here $A(x)$ and $N^B_x(x)$ are determined unambiguously in the WKB approximation and $C^{(B)}$ is a constant to be found. As to the vector $E^{(C)}$ in eq.(1), it is a solution of the cold plasma wave equation

$$\nabla \times \nabla \times E - \frac{\omega^2}{c^2} \varepsilon E = 0 \quad (3)$$

satisfying relevant boundary conditions on a plasma edge. Here $\varepsilon$ is usual dielectric tensor in cold approximation. For the case of EBWHCD experiments the boundary condition corresponds to the EMWs incident from vacuum and converted into the EBWs. In weakly inhomogeneous plasma when the WKB approximation is applicable to electromagnetic modes, solutions of the eq.(3) are ordinary (O) and extraordinary (X) WKB
modes propagating in both directions along the $x$-axis. In the opposite case of strong plasma inhomogeneity four linearly independent particular solutions of the cold plasma wave equation are not readily associated with the WKB O and X waves, though this mode classification according to sensitivity of the modes to thermal effects remains applicable. Equation (1) is an asymptotic form of the exact hot plasma solution valid outside the mode "interaction" region. From the first glance it seems that finding this asymptotic representation and calculating the outgoing EBW amplitude requires solution of the full-wave hot plasma equation for given boundary conditions. Actually, as it has been shown in [6, 7], a global solution of the form (1) can be found directly from the cold plasma equation (3) without real solving the hot plasma wave equation. The conclusion follows from a comparative analysis of standard solutions of the cold and hot wave equations in the UHR vicinity. This main result of papers [6, 7] can be formulated in the following way. Let us imagine that we found everywhere on the $x$-axis, except a narrow vicinity of the UHR, a single particular solution $E^{(UP)}(x)$ of the cold plasma wave equation (3) regular in the upper complex $x$-plane and satisfying the boundary conditions relevant to the case of EMW incident from vacuum on the plasma layer. The UHR is the singular point of the cold-plasma wave equation and therefore found $E^{(UP)}(x)$ contains a singular part near the UHR. More precisely, the $x$-component $E_x^{(UP)}(x)$ of the solution has a pole at the UHR point. Denoting a residue of the $E_x^{(UP)}(x)$ at the pole as $\Re$, the "full-wave" solution of the one-dimensional EMW—EBW conversion problem takes the form

$$E(x) = E^{(UP)}(x) + \Re A(x) \exp \left( i \left[ \int_{x_0}^{x} N_x^{(B)} dx' + \Phi_0 \right] \right), \quad \sigma x > 0$$

$$E(x) = E^{(UP)}(x), \quad \sigma x < 0$$

Here $\sigma = -1$ for $\omega_{ce} < \omega < 2\omega_{ce}$ and $\sigma = +1$ for $\omega > 2\omega_{ce}$, with the electron cyclotron frequency $\omega_{ce}$ calculated at the UHR. Definitions for parameters $x_0$ and $\Phi_0$, electrostatic amplitude $A(x)$ of the EBW and $x$-component of the EBW refractive vector see in [7]. Applicability condition of the eq.(4) is given by the following restriction to the dimensionless plasma inhomogeneity scale length $l$ at the UHR: $\beta \ll l \ll \beta^{-2}$, where $\beta = v_{Te}/c$, $v_{Te} = \sqrt{2T_e/m_e}$. This restriction arises, in particular, from the requirement of overlapping proper asymptotic expressions for the hot and cold plasma wave solutions in a finite spatial region near the UHR. Obviously, the inequality is easily fulfilled for any realistic plasma configuration.

It is worth to noted that function $E^{(UP)}(x)$ can be also regarded as a continuous on the real $x$-axis solution of the collisionless cold plasma wave equation (3) with a small positive imaginary part added to the $\varepsilon_{xx}$ dielectric tensor component: $\varepsilon_{xx} \rightarrow \varepsilon_{xx} + i\nu = x/l + i\nu$. This solution describes the case of dissipative power absorption in the UHR. Due to the resonance behavior of the dominant wave field component $E_x = \Re/\varepsilon_{xx} \approx \Re/(x + il\nu)$ the absorbed power $P_{UH}$ is independent of collision frequency $\nu$ [4].

$$P_{UH} = \frac{c}{8\pi} \nu \int_{-\infty}^{\infty} |E_x|^2 dx = \frac{c}{8l} |\Re|^2$$

(5)
Direct comparison of the absorbed power $P_{UH}$ with the energy flux $S_x$ carried away by the outgoing plasma wave in eq.(4) shows that these quantities are equal [7]. The equality $P_{UH} = S_x$ has been first established for small-size unmagnetized plasmas [8], then for strongly inhomogeneous magnetized plasmas [6] and for the case of full $X \rightarrow$ EBW conversion [9]. From the results of paper [7] we see that this important relation is actually valid without these limitations in a wide range of the plasma inhomogeneity scale-lengths.

It should be emphasized in conclusion of this section that in a one-dimensional (1D) model of plasma inhomogeneity full solution of the EMW $\rightarrow$ EBW conversion problem is reduced to finding a solution to the cold-plasma wave equation (3). Depending on the plasma inhomogeneity scale-length, this solution describes direct $X \rightarrow$ EBW excitation by EMWs tunneling through the evanescent layer, the $O \rightarrow X \rightarrow$ EBW conversion [4, 10] and various combinations of both processes. All particular cases are treated in the same manner in the framework of a single procedure. The principal limitation of the theory [7] is using a one-dimensional plasma model. Nevertheless, the results of paper [7] provide not only a very convenient and powerful method for numerical modeling the EBWHCD experiments in ST plasmas, but a good guess for a two-dimensional theory as well.

Two-dimensional model

Theoretical analysis of the EMW $\rightarrow$ EBW conversion problem in two-dimensional (2D) tokamak plasma is much more difficult than in a plasma slab, because structure of the singularity in the cold plasma wave equation is qualitatively different in the 2D case [11], when eq.(3) becomes a partial differential equation. This explains why there are still no 2D theory of the EMW $\rightarrow$ EBW conversion problem which would be as complete as the 1D theory given in paper [7]. In recent papers [12, 13] analytical approaches have been developed for the $O \rightarrow X$ conversion process playing most important role in the $O \rightarrow X \rightarrow$ EBW scheme of the EBWs excitation. Both works are devoted to calculation of conversion coefficients between ordinary and extraordinary electromagnetic modes near intersection of the cut-off surfaces for these modes (see fig.2). Figure 3 shows schematically evanescent regions for EMWs bounded by the cut-off surfaces in 1D and 2D cases. Position of the X mode cut-off depends on local parallel refractive index $N_{\parallel}$. Physically, the $O \rightarrow X$ conversion is explained by tunneling incident O mode through the evanescent region, which gives rise to X mode behind the region, and vice versa for the inverse $X \rightarrow O$ conversion process. In the 1D case the cut-off surfaces are parallel planes

FIGURE 2. Dotted line is O mode cut-off, solid line is X mode cut-off.
merging at an optimal $N_{||}$ value, which corresponds to vanishing the evanescent region and total mode conversion of plane electromagnetic waves for this particular $N_{||}$. In a tokamak geometry the cut-off surfaces are not parallel. Therefore they may intersect only along a line (in "toroidal" direction) and the evanescent regions always exist for any $N_{||}$. Nevertheless, solutions of cold eq.(3) in 2D inhomogeneous geometry show that for any $N_{||}$ providing the intersection of the cut-offs, an optimal EMW beam with perfect conversion can be found [12].

Theoretical analysis in both papers [12] and [13] is restricted to a situation of moderate plasma inhomogeneity scale-lengths, when the WKB approximation is applied to the incident and outgoing EM modes outside the "interaction" region enclosing the intersection of the cut-offs. Main assumptions and suggestions adopted in the works are rather similar, though a model for 2D plasma inhomogeneity used in paper [13] is more general than the corresponding model of [12], because it takes into account not only toroidal, but also poloidal magnetic field component, which is obviously not negligible in spherical tokamaks. More general solution of paper [13] is different in details from the corresponding result of [12], but structure of both solutions is similar and is given symbolically by the following formula for each amplitude of a Fourier expansion over cyclic "toroidal" coordinate $F(x,y) = \sum_{n=0}^{\infty} Z_n(x) \phi_n(y)$. Here $\phi_n(y)$ are proportional to Hermite polynomials and $Z_n(x)$ are superpositions of parabolic cylinder functions [14].

Cartesian coordinate $x$ is directed along a bisector of the angle between O mode and X mode cut-off surfaces towards the plasma density increase (see fig.3). It is interesting that conversion coefficients, i.e. the ratio of the outgoing and incident power fluxes obtained in both papers in the framework of this expansion, are actually identical despite the aforementioned fact of some local difference in solutions for the electric fields in the "interaction" region. Probably, a kind of a waveguide in the "interaction" region is formed and this explains the insensitivity of the conversion coefficient to the magnetic field details. Another peculiarity of the conversion process found in [12] and then confirmed in [13], is its asymmetry relatively to the conversion direction. Namely, efficiency of the O→X conversion $T_{OX}$ is not equal to the X→O efficiency $T_{XO}$ at the same plasma region. Instead, for tokamak geometry $T_{OX}^{UP} = T_{XO}^{DOWN}$, where $UP$ and $DOWN$ denote the "interaction" regions located symmetrically above and below the equatorial plane (see fig.2). A comparison of $T_{OX}$ conversion coefficients calculated using 1D formulae of [15] and corresponding 2D formulae of [13] is shown in fig.4 for a Gaussian incident beam and typical MAST plasma configurations. It is seen that 2D effects may be quite noticeable.
FIGURE 4. Calculated using formulae [13]. 1D (dash) and 2D (solid) conversion coefficients for a Gaussian incident beam having width $\rho$. Plasma inhomogeneity scale-lengths are typical for H-regime (a), and for L-regime (b) in MAST. Shaded regions correspond to realistic beam widths and frequencies.

Thus, there are two most important features of two-dimensional $O \leftrightarrow X$ conversion: existence of an optimal beam with perfect conversion efficiency and specific symmetry $T^{UP}_{OX} = T^{DOWN}_{XO}$ of the conversion coefficients.

PROPAGATION OF THE EBW

Electrostatic dispersion relation

Most practicable approach to propagation and damping of the EBWs in tokamak plasmas is the ray tracing procedure based on a local electrostatic dispersion relation. The exact relativistic relation for a Maxwellian plasma can be presented in the following form [16].

$$D = 1 + \frac{v^2}{N^2} \left[ 1 - W_1 - W_2 \right] = 0$$

(6)

In this equation $N^2 = N^2_\perp + N^2_\parallel$, $N_\perp$ and $N_\parallel$ are the refractive indices perpendicular and parallel to the local magnetic field direction, $v = \omega^2_{pe}/\omega^2$ and $\mu = m_e c^2/T_e$, where $\omega_{pe}$ is plasma frequency and $T_e$ is electron temperature. Functions $W_1$ and $W_2$ are double integrals over normalized impulses $z_\perp = P_\perp/m_ec$, $z_\parallel = P_\parallel/m_ec$ in the phase space.

$$W_1 = -\frac{\pi q \mu}{2K_2(\mu)} \int_{0}^{+\infty} z_\perp dz_\perp \int_{-\infty}^{+\infty} \gamma \exp\{-\mu \gamma\} J_p(x) N_p(x) dz_\parallel$$

$$W_2 = \frac{\pi q \mu}{2K_2(\mu)} \int_{0}^{+\infty} z_\perp dz_\perp \int_{-\infty}^{+\infty} \gamma \exp\{-\mu \gamma\} \cot(\pi p) J^2_p(x) dz_\parallel$$

(7)

Here $K_2(\mu)$ is the modified Bessel function of the second kind, $J_p$ and $N_p$ are the Bessel and Neumann functions. Definitions for the other parameters and variables are given by $q = \omega/\omega_{ce}$, $p = q(\gamma - N_\parallel z_\parallel)$, $\gamma = \sqrt{1 + z_\perp^2 + z_\parallel^2}$, $x = q N_\perp z_\perp$ where $\omega_{ce}$ is electron cyclotron frequency. The second term $W_2$ of eqs.(6), (7) is much more important than
the first one \( W_1 \). Actually, this term describes absorption of the EBW and makes principal contribution into the Hermitian part of the electrostatic EBW dispersion relation as well. Unfortunately, the exact expressions (7) are too formidable for effective application both in qualitative analysis and numerical simulations, therefore any reasonable simplifications are useful. Recently a new approximate form of the electrostatic dispersion relation has been proposed in [17] for the weak relativistic case and small parallel refractive index. Very similar asymptotical approach to the problem of electrostatic plasma waves was used in an early work [18] for ion Bernstein waves. Finally, in paper [16] convenient approximate formulae for the electron susceptibility (7) have been obtained in the framework of the same asymptotical method for the fully relativistic case of a Maxwellian plasma and arbitrary parallel refractive index. These formulae are valid when the following inequalities \( N_\perp \gg 1, k_\perp \rho_{Te} = qN_{\perp}/\sqrt{\mu} \gg 1, \mu \approx 500/T_e (keV) \gg 1 \) are fulfilled simultaneously. Here \( \rho_{Te} \) is the Larmor radius of electrons at thermal velocity. First two inequalities are quite typical for the EBWs. The last inequality is always valid for tokamak plasmas. Then, \( W_1 \) of eq.(7) is simplified to a one-dimensional integral

\[
W_1 = -\pi q \mu \int_0^{\infty} J_\nu(qN_{\perp}z_{\perp})N_q(qN_{\perp}z_{\perp}) \left(1 + \frac{1}{\mu z_{\perp}}\right) \tau_{\perp}^\frac{1}{2} \exp \left[\mu (1 - \tau)\right] z_{\perp} dz_{\perp}
\]

where \( \tau = \sqrt{1 + z_{\perp}^2} \). The most important part \( W_2 \) of the dispersion relation (6) is approximately given by

\[
W_2 = \sqrt{\frac{\pi \mu}{2N_{\perp}^2}} \exp \left(-\frac{\mu}{2N_{\perp}^2}\right) Q
\]

where complex function \( Q \) of real parameters \( q, \mu, N_{\parallel}, N_{\perp} \) has two equivalent representations [16]. According to one of them \( Q = A_0 + 2 \sum_{m=1}^{\infty} A_m \). Terms \( A_m \) of this converging infinite sum are elementary functions [16], which provides almost instantaneous calculation of a real part Re\( Q \) with high enough accuracy. As to an imaginary part Im\( Q \), this equation is not so convenient in practical calculations, because there are continuous regions of parameters \( q, N_{\parallel}^2 \) where relativistic damping of the EBW, proportional to the Im\( Q \), is absent. Therefore, many terms of the series are required to reach reasonable accuracy for very small values of the Im\( Q \). Fortunately, simple and convenient formulæ for the Im\( Q \) can be obtained [16] using another representation for function \( Q \). In the most interesting case \( N_{\parallel}^2 < 1 \) we have

\[
\text{Im}Q = -\mu q \sqrt{1 - N_{\perp}^2} \sum_{n>\tilde{q}/\sqrt{1 - N_{\perp}^2}} e^{\tilde{\mu}(1-b)} \left\{(a^2 + b^2)I_0(\tilde{\mu}a) - aI_1(\tilde{\mu}a) \left[2b + \frac{1}{\tilde{\mu}}\right]\right\}
\]

where \( n \) is positive integer and \( I_0, I_1 \) are the modified Bessel functions of the first kind. The other variables are defined as follows: \( \tilde{\mu} = \mu \sqrt{1 + 1/N_{\perp}^2}, \quad \tilde{q} = q \sqrt{1 + 1/N_{\perp}^2}, \quad b = n/\tilde{q} \left(1 - N_{\parallel}^2\right), \quad a = |N_{\parallel}| \sqrt{N_{\parallel}^2 - 1 + n^2/\tilde{q}^2}/1 - N_{\perp}^2 \). Figure 5 shows a comparison of exact (7) and approximate (8) \( W_2 \) for perpendicular EBW propagation near the fundamental EC harmonic. Agreement becomes even better for oblique propagation, as
FIGURE 5. Comparison of exact and approximate $W_2$ for $N_\parallel = 0, N_\perp = 30, \mu = 100$ near the fundamental EC harmonic.

FIGURE 6. Comparison of exact and approximate $W_2$ for $N_\parallel = 0.3, N_\perp = 30, \mu = 100$.

it is seen in fig.6. Figure 5 reveals a weak logarithmic singularity in approximate Re$W_2$ in the very vicinity of the EC harmonic. It originates from non-analytical asymptotic expressions for the integrand (7) used in paper [16]. This problem has been recently overcome in [19], where a new approach to the EBW electrostatic dispersion relation has been put forward. Formulae of paper [19] reproduce complex function $W_2$ in detail when the following conditions are fulfilled $\mu \gg 1, \sqrt{\mu} \gg |N_\parallel| q, q \ll \mu$.

EBW heating and current drive

Simple and accurate approximate formulae of papers [16, 19] have been implemented into a ray-tracing code entering a package of programs intended for numerical simulations of the EBW heating and current drive at Culham Science Centre, UK. First version of the code is described in [20]. The package unites self-consistently a code solving the EMW$\rightarrow$EBW coupling problem with the use of the 1D approach [7], the EBW ray-tracing code and a Fokker-Planck code BANDIT-3D [21] which calculates EBW power deposition and driven plasma current. Typical simulation run-time of this EBWHCD package takes a few minutes on a moderate workstation with a single processor.

Similar package exists for NSTX plasma simulations in Princeton, USA. It consists of a 1D "warm" coupling code GLOSI [22], a ray-tracing code GENRAY [23] and a
Fokker-Planck code CQL3D [24]. The advantage of that package is its possibility to use full-wave relativistic dispersion relation. Naturally, in this computational mode the GENRAY ray-tracing is slow and needs multi-processor calculations. Last year we compared both packages using a special model case and found a satisfactory agreement. Unfortunately, the benchmarking has not been completed and therefore not published. Figure 7 shows an example of a typical simulation result for MAST-U plasma configuration. Incident wave frequency is 18 GHz, antenna position is 70 centimeters above the plasma mid-plane. Driven (Fish-Boozer) current is about 90 kA for 1 MW input power.

Results of modeling by the NSTX package are given in paper [25]. It is shown there that Ohkawa current can be efficiently driven ($\approx 50$ kA/MW) at the NSTX plasma periphery where the large trapped electron fraction precludes conventional Fisch-Boozer current drive. Corresponding graphs are not presented here for lack of room.

**EBWs near the plasma mid-plane**

Numerical modeling shows that electron Bernstein waves propagating close to the tokamak mid-plane near a cyclotron resonance may demonstrate a specific behavior. Actually, there are two different types of ray trajectories in this region depending on geometry of the resonant electron cyclotron surface. Namely, when the resonance surface is concave, the rays approach the resonance oscillating around mid-plane and zero value of $N_\parallel$. Frequency and amplitude of the oscillations slowly vary with radial distance. In the other case of a convex electron cyclotron surface, rays behave aperiodically, without oscillations. For switching between these two regimes, it is enough to change frequency, as it is seen in fig.8. This specific behavior was analyzed in [26]. It has been shown that in the concave case there is a kind of a plasma waveguide, inhomogeneous in the poloidal direction and slowly varying in the radial direction, where the waves propagate in the form of discrete eigenmodes. Simplified Hamiltonian for ray approximation has the following form $K = (\pm \Omega^2 \bar{y}^2 + N^2_\parallel)/2$, where dimensionless frequency $\Omega$ depends on plasma plasma parameters and distance to the ECR resonance. Concave (+) and convex
(-) cases differ by sign. In the concave case there is a region near EC resonance, where ray solutions \( \bar{y} = \left( \frac{2I}{\Omega} \right)^{1/2} \sin \left( \int \frac{\Omega}{\Omega(t)dt} + \alpha \right) \), \( N_{\parallel} = \left( \frac{2I}{\Omega} \right)^{1/2} \cos \left( \int \frac{\Omega}{\Omega(t)dt} + \alpha \right) \) possess an approximate (adiabatic) integral of motion \( I = K/\Omega \). In the convex case both the ray deviation from the mid-plane and \( N_{\parallel} \) grow exponentially close to the ECR surface \( |\bar{y}| \sim |N_{\parallel}| \sim \exp (\Omega \tau) \).

An approximate wave equation \( i \frac{\partial \Psi}{\partial \tau} = \vec{K} \Psi \) for the concave case has been derived and analyzed in [26]. Here \( \vec{K} = (\Omega^2 \bar{y}^2 - \partial^2 / \partial \bar{y}^2) / 2 \). Solution of this Schrödinger equation for the time-dependent quantum harmonic oscillator is a set of eigenmodes traveling in the \( x \) (radial) direction and confined in the \( y \) and \( z \) directions. The WKB approximation is applicable to the large-\( n \) eigenmodes, so for a few first modes the ray method fails. Figure 9 shows share of launched power deposited into eigenmodes for three different incident beam widths. The narrower the beam, the larger the share of low eigenmodes. Right part of fig.9 gives a comparison of mode absorption for the low eigenmodes and corresponding rays. Solid curves stand for rays, dashed - for eigenmodes in MAST plasma configuration, shot number 7723. Arrow shows the ECR relativistic layer boundary. It is seen that low mode number eigenfunctions are damped more strongly than small \( N_{\parallel} \)
rays and lose their energy before reaching the relativistic ECR layer.

**EBWs in a magnetic well**

Local minimum of tokamak magnetic field which may occur in spherical tokamaks modifies significantly the wave behavior in the ECR vicinity. In contrast with the usual case of increasing magnetic field inward the plasma, the wave vector in a magnetic well goes down close to the EC resonance, which can lead to violation of the WKB approximation validity and partial wave reflection. Figure 10 shows schematically a branch of EBW dispersion relation corresponding this situation. A detailed analysis of the EBW propagation and absorption in a magnetic well has been carried out in [27]. There an approximate wave equation describing main effects is derived. The equation contains only one dimensionless parameter $k_p$, which depends on harmonic number $p$, plasma temperature, magnetic field inhomogeneity scale-length and parallel refractive index [27]. It has been shown in [27] that EBW reflection coefficient near EC resonances in a magnetic well must rapidly decrease with increasing harmonic number. Near the most dangerous second EC harmonic this equation takes the simplest form $U''(\xi) + V(\xi)U(\xi) = 0$; $V(\xi) \equiv k_2^2Z^{-1} - \frac{1}{4}[\ln Z]'$, where $Z$ is plasma dispersion function and $k_2$ is the only parameter of the theory. Solutions of the equation, corresponding to the wave approaching the ECR layer from its high-field side, have been analyzed both in non-relativistic case and perpendicular relativistic propagation. Calculated reflection coefficients are given in fig.10. Main conclusions made in [27] are as follows:

1) There is no conversion of incoming waves incident on the ECR layer from the high-field side into outgoing Bernstein waves.

2) Decreasing of the wave amplitude within the ECR layer is due to the combined effect of the ECR damping and non-propagation. In the WKB approximation, the waves are fully damped in the ECR layer. Reflection from the ECR layer is only due to the
3) Since the WKB approximation validity grows with the harmonic number there is practically no reflection at high $p > 4$ harmonics, and the standard ray tracing procedure can be employed for treating the Bernstein waves, similarly to the case of low-field access to the ECR surface.

**Dangerous parametric decay instability**

OXB conversion near the upper hybrid resonance surface may be accompanied by a parametric decay of the pump upper hybrid (UH) wave into another UH wave and lower hybrid (LH) wave. This instability has been studied in [28] for pump frequencies higher than the second electron cyclotron harmonic frequency, i.e. when turning points for the UH waves are absent. Figure 11 shows schematically the dispersion curves for pump UH wave $k_0$, reflected UH wave $k_1$ and LH wave $k_2$. Arrows denote group velocity directions. It is seen from the graph that a positive feedback loop is possible. This means that an absolute parametric decay instability can be excited when incident power exceeds a corresponding threshold. The minimal power threshold of the instability producing LH wave with $N_z \approx c/5v_T e$ is given by the following formula.

$$
P \frac{W}{\pi \rho^2} \frac{W}{cm^2} = 2 \cdot 10^{-3} \left[ \frac{W}{cm^2/3 T^{1/3} GHz^{1/3} eV^{13/6}} \right] \cdot \frac{f_0^{1/3} T^{11/12} T_e^{5/4} B^{1/3}}{L^{4/3}} ; \quad T \equiv T_e + 4T_i
$$

For MAST experiment parameters $f_0 = 60 GHz$, $T_e \approx T_i = 140 eV$, $B = 0.38 T$, $L = 3 cm$ at UHR, the threshold is equal to $P/(\pi \rho^2) \approx 260 W/cm^2$, which gives $P \approx 80 kW$ for the $10 cm$ beam radius. The instability threshold increases with frequency decrease due to $L \sim f_0^2$, and the threshold decreases with plasma temperature decrease. Thus, for future EBWHCD experiments in MAST-U, the frequency should be chosen from a balance of these two opposite tendencies. Special case of the parametric instability near the fundamental EC harmonic must be also studied.

**CONCLUSIONS**

- Theory of the EBW in torus plasmas is well elaborated in general, though there are still some particular aspects to be studied. Most important among them are further de-
velopment of the 2D conversion theory with corresponding numerical modeling, and in-
vestigation of the absolute parametric instability $UH \rightarrow UH + LH$ at the fundamental EC
frequency.
• At current stage only future experiments can confirm or deny feasibility of the
EBW heating and current drive in ST plasmas.

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