Advanced methods in global gyrokinetic full $f$ particle simulation of tokamak transport

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Abstract. A new full $f$ nonlinear gyrokinetic simulation code, named ELMFIRE, has been developed for simulating transport phenomena in tokamak plasmas. The code is based on a gyrokinetic particle-in-cell algorithm, which can consider electrons and ions jointly or separately, as well as arbitrary impurities. The implicit treatment of the ion polarization drift and the use of full $f$ methods allow for simulations of strongly perturbed plasmas including wide orbit effects, steep gradients and rapid dynamic changes. This article presents in more detail the algorithms incorporated into ELMFIRE, as well as benchmarking comparisons to both neoclassical theory and other codes.

Turbulence spectra of FT-2 plasma has been calculated with ELMFIRE, obtaining results consistent with experimental data.

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INTRODUCTION

Plasma turbulence is believed to be one of the main processes leading to enhanced (anomalous) radial transport observed in tokamaks, which spoil the confinement to unacceptable levels. Small scale, low frequency instabilities drive that turbulence and its study is therefore of crucial importance for improving confinement efficiency. The overall interaction of those small scale processes can however be correctly simulated with global simulations that cover all, or significant part, of the whole tokamak. In order to reproduce interesting processes where the particle distribution function deviates from Maxwellian, kinetic methods are needed. As most interesting processes occur at a characteristic frequency much lower than the particle gyration around the magnetic field, the gyrokinetic model was introduced, first for the linear regime [1] and later for nonlinear problems [2, 3]. This method produced considerable savings in computation by ignoring the high frequency phenomena of not relevant importance.

The gyrokinetic method was further simplified with the assumption that particle distributions are close to a known kind (e.g. Maxwellian), introducing the delta $f$ technique [4]. This method allows for further reduction in computation at the expense of limiting its range of accuracy due to its assumption on the particle distribution. With the delta $f$ approach, gyrokinetic plasma simulation has become a standard tool for trans-
port analysis in toroidal magnetic fusion devices, under conditions of weak perturbations [6, 7, 8, 9, 10, 11]. All these methods calculated the electrostatic potential from a complicatedly modified Poisson equation, and lacked from a proper treatment of the electromagnetic perturbations with multiscale structures in both space and time. This problem was solved later [5] with a scheme based on Krylov-Bohloibov averaging method, that includes the polarization drift into the equations of motion. Also considerations for inhomogeneous plasma and electromagnetic fluctuations have been developed [12] for the gyrokinetic model. However in the presence of strong gradients or other phenomena that could perturb the particle distribution function, the delta $f$ still does not produce accurate results, being necessary to use the full $f$ model.

Another simplification commonly used applies to the consideration of electrons. Their inertia may be neglected by considering an adiabatic model, resulting in further computational savings. One of the reasons why the study of kinetic electrons has received little attention is mainly due to computational difficulty as electrons move much faster than ionic species. However lately new techniques are being developed [13] to study their influence on unstable modes, like the ETG.

From the gyrokinetic theory, two main approaches have been followed for computational resolution of the intervening equations: deterministic and particle in cell (PIC) codes. Both approaches can work under both delta and full $f$ models, have been intensively tested [14] producing similar results with still remarkable persistent differences regarding zonal flows [15] which have influence on transport and conductivity.

The code ELMFIRE solves the gyrokinetic full $f$ equations for quasi-neutrality with a stochastic PIC algorithm based on Sosenko’s method developed in [5]. Its main features are described in section 2, as well as some details on its programming implementation and optimization to existing computational resources. In section 3 are presented results of standard comparison benchmarks [7], both in the linear and saturation regimes. Linear tests refer to growing rate and phase evolution of most unstable modes, while in nonlinear case saturation levels of conductivity are studied. Those tests make no comparison to experimental data, however there is enough computational results from different models to serve as quality benchmark.

As ELMFIRE solves the gyrokinetic equations with a Monte Carlo method, its results suffer from statistical noise coming from the fact of using a finite number of test particles. Noise production in PIC codes has been widely studied [17] but however simulations of ETG instability [18] have shown the possibility of noise strongly affecting growth rates. The study of noise influence on results is therefore a key issue that is addressed in section 4.

The ELMFIRE code has been used for analyzing the turbulence spectra in tokamak plasmas [19]. The dynamics of turbulence can be described more adequately using Fourier space and determining the growing rates and interaction of different modes. Results are shown in section 5. Finally conclusions show the key points regarding the development of ELMFIRE and its applications.
ELMFIRE, ITS EQUATIONS AND MAIN FEATURES

In reference [5], the gyrokinetic Poisson equation was written in the form

\[
\nabla^2 \Phi + \frac{q^2}{mB_0} \int \left[ (\Phi - \langle \Phi \rangle) \frac{\partial \langle f \rangle}{\partial \mu} + \frac{m}{q\Omega} \langle f \rangle \nabla^2 \langle \Phi \rangle \right] \, dv = -\frac{1}{\varepsilon_0} \left( q\bar{n}_i(\vec{r}) - en_e(\vec{r}) \right)
\]

which differs from the original method of reference [2] in that the polarization drift is taken into account in the ion density while in more common approaches it is separated from the change in ion density and introduced explicitly to the Poisson equation. Here, \( \Phi(x) \) is the electrostatic potential at the position \( x \), \( \langle f \rangle \) is the distribution function of ion guiding centers, \( \mu = v^2_\perp / 2B \) is the magnetic moment, \( v_\perp \) the ion perpendicular velocity component, \( dv \) denotes the velocity phase space differential, \( q, m \) are the ion charge and the mass, respectively, \( \Omega \) is the ion Larmor frequency, \( B = B\hat{b} \) is the magnetic field with the unit vector \( \hat{b} \), \( n_e \) is electron density and the ion density \( \bar{n}_i \) is obtained from the gyroaverage around the ion guiding-center coordinates which are advanced with the polarization drift included. The gyroaverage over the ion Larmor rotation is denoted by \( \langle \cdot \rangle \). In the partial derivative with respect to \( \mu \) the guiding-center coordinate and the parallel velocity are kept fixed.

In order to solve equation (1) one needs either to assume some average ion distribution \( \langle f \rangle \), or, as in reference [21], construct it from the full \( f \) particle simulation together with the solution of the gyrokinetic equation for the potential \( \Phi \). Here, instead, we use the method presented in reference [20], where the polarization drift \( v_p = (1/\Omega B)\partial \langle E \rangle / \partial t \) is implemented directly in the guiding-center equations giving us a true average particle motion and proper diagnostics of particle transport. Here, \( E \) is the electric field. This equation is valid at least to order \( (k_\perp \rho_i)^2 \), depending on implementation, and determines the accuracy of the present method in the \( k_\perp \)-spectrum. Here, \( \rho_i \) is ion Larmor radius and \( k_\theta \) the poloidal wave vector. In this method the perturbation in ion density \( \bar{n}_i \) by \( v_p \) is directly calculated from the particle orbits during simulation. This direct calculation is made implicit in \( E \) by evaluating the change in the ion polarization density \( \delta \bar{n}_p \) at each time step by ion polarization shift using unknown \( E \) at the end of the time step as explained in reference [20]. For electrons, finite Larmor radius effects are neglected so that no polarization drift is applied for them.

In the simulation, the particles are initialized according to assumed density and temperature profiles and, typically, Maxwellian velocity distribution is initialized for the particle ensemble. A zero initial potential is often assumed. The so called quiescent initialization [23] of ions on numerically pre-evaluated collisionless orbits is adopted.

After initializing ions on collisionless orbits, for each ion, one electron is initialized at the same location to ensure quasi-neutrality. This is done at the gyro-orbit of each ion since the ion densities and matrix coefficients are sampled from the gyro-orbit, not from the guiding center. However, one should note that collisions and neoclassical effects with poloidal non-uniformity in pressure make it very difficult to initialize the plasma without causing a numerical transient. Therefore, strong restructuring of \( f(x, v, t) \) at the start of simulation follows almost always in practice. After initialization, ions are advanced with a gyrophase-averaged electrostatic potential and electrons are under the influence of a bare potential in torus with circular poloidal cross section with no Shafranov shift.
The guiding-center motion involves \( \mathbf{E} \times \mathbf{B} \), gradient and curvature drifts, collisions and polarization drifts.

Two different coordinate systems are used: Orbits are followed in straight-field-line coordinates [24] and potential is solved in quasi-ballooning coordinates [25]. This is to avoid poor resolution in solving potential structures in straight-field-line coordinates and problems with periodicity condition in ballooning coordinates. Grid is needed only in quasi-ballooning coordinates where the potential is solved and sampling is performed. In radial direction, boundary conditions are as in reference [23] with prompt reinitialization at outer boundary or, in more realistic simulations, pairwise reinitialization of outflowing ions and electrons according to assumed neutral distribution or ionization on randomly directed straight return paths of recycled neutrals can be chosen to maintain the otherwise flattening density profile.

An accurate electron time step is less than \( \Delta t \leq \Delta z / v_{Te} \) with toroidal grid cell size \( \Delta z = 2\pi R / N_z \) in magnetic field direction where \( R \) is the major radius of the tokamak and \( v_{Te} \) is the electron thermal velocity. To ensure stability with the time step \( \Delta t \) reaching the accuracy limit, the electron parallel motion is treated implicitly [26]. Coordinate systems, sampling of terms in equation (1) and implicit treatment of electron parallel motion are explained in detail in reference [21, 22].

**BENCHMARKS: LINEAR AND NON-LINEAR RUNS**

Confidence on a new simulation scheme is improved by benchmarking it to existing published cases and analytical results in convenient limits. Such a project has been initiated within the EFDA Integrated Modeling Project #4, based on the previously successful Cyclone project in the United States. While this project is gathering momentum, some separate analyses have already been performed on the ELMFIRE code to validate the full \( f \) model.

Some of the published Cyclone cross-benchmarking results of the previous undertakings can be seen in reference [7]. We present linear and non-linear benchmarking on this normalized parameter set (with \( a / \rho_s \approx 150 \) unless otherwise stated.

**Linear benchmarking**

The linear benchmarking has been performed for \( R/L_T = 6.9 \), \( R/L_n = 2.2 \), and \( \varepsilon = r(1/2)/R = 0.18 \), where \( r(1/2) = a/2 \) is the mid-radius of the simulation region, \( a \) is the minor radius and \( L_T = |\nabla \ln T|^{-1} \) and \( L_n = |\nabla \ln n|^{-1} \) are the ion temperature and density gradient scale lengths, as the nominal Cyclone base case suggests. The scaled physical parameters (per the dimensionless parameters in publications) used in the benchmarking are \( R = 0.55 \) m, \( a = 0.19775 \) m, \( B_T = 1.1 \) m, \( I_{tot} = 100 \) kA and current density is given by \( j(r) = j_0 (1 - (r/a)^2)^3 \), temperature and density at \( r = a/2 \) are \( T = 100 \) eV and \( n = 5 \cdot 10^{19} \) m\(^{-3} \), and the simulation region extends from \( r_l = 0.3a \) to \( r_r = 0.7a \). Whereas normally the ELMFIRE is run using orbit-initialized particles (the quiescent initialization procedure presented above), in the adiabatic linear runs the particles were initialized on a local Maxwellian distribution, to ease the analysis of the
linear growth. This is done because the quiescent initialization – while it eliminates transient orbit dynamics in non-linear runs – introduces initial perturbations in the density which were considered to complicate linear analysis. While the local Maxwellian initialization appears to require a “waiting phase” (as seen in figure 1) in the beginning of the simulation, there are no damped modes from the initialization in this analysis to hinder the growth rate evaluation. In this analysis the adiabatic electron model was used, and binary collisions were not operated on the protons.

In the linear growth analysis the toroidal modes (i.e., the modes with a toroidal mode number $n$) are evaluated independently for each point, and to this end a spectral filtering scheme is applied to capture only the corresponding poloidal wave numbers. The points given in figure 1a have been tabulated in figure 1b in this way on corresponding $k_{\theta}$ values (with $k_{\parallel} = 0$), while in the simulation a much larger set of poloidal mode numbers are simulated. A current profile with $q$ and $\hat{s}$ profiles most closely matching the Cyclone base case at $r/a = 0.5$ is used, however there is a slight deviation in the edges of the simulation region due to the definition of current profile in ELMFIRE, as opposed to the usual definition of a parabolic $q$ profile.

**Non-linear benchmarking**

The non-linear simulations where the modes are allowed to interact are performed using the same Cyclone base case parameters, except for the parameter $a/\rho_s$, which in these cases is taken to be 300 (where $\rho_s = \sqrt{T_i/m_i}$). This was produced both in adiabatic and kinetic electron cases by scaling the the parameters $R$, $a$ and $I_{tot}$ by a factor of 2, while keeping the other parameters intact. The adiabatic runs were collisionless, but the kinetic electron cases were run with collisions. The kinetic cases are discussed in the next section in more detail.

Evolution of the thermal conductivity $\chi_i(a/2)$ is investigated in two different ways: in time and against the local temperature gradient scale length. The latter is also an important diagnostic, as the change in the local gradient scale length can be utilized in producing $R/L_T$ scans [31]. Both results are shown in figure 2.
FIGURE 2. Evolution of ion heat conductivity $\chi_i$ in time and as a function of the evolving temperature profile. The normalization factor is given by $\chi_{GB} = \rho_i^2 c_s / a$.

FIGURE 3. Simulation of the Cyclone base case parameters with kinetic electrons. Four cases are shown: flux-surface averaged and non-averaged, with collisions and without.

**Neo-classical benchmarking: GAMs and $E_r$**

The neo-classical physics of the ELMFIRE has been investigated by assessing GAM frequencies and Rosenbluth residuals [22], and by comparing the radial electric field $E_r$ obtained from the simulation with the neo-classical estimate (Hazeltine-Hinton field) [32]. An example of a radial electric field, with a comparison to the Hazeltine-Hinton field is given in figure 4. In this case the GAM damping rate (due to the high $q$ value) is lower than the evolution of the neoclassical $E_r$. Both of these tests have succesfully shown that the neoclassical physics has been retained in the model even though the new 3D gyrokinetic electric field has been introduced.

Additionally, the development of neoclassical ion thermal conductivity $\chi_i$ can be seen in the electron kinetic runs of the scaled Cyclone case. This is illustrated in figure 3, where we have presented all combinations of flux surface averaging of and binary collisions. In the flux surface averaged runs only the averaged electric potential $\langle \Phi \rangle$ is allowed to interact with the particles, which corresponds to the one dimensional model.
of reference [23]. The difference in transport levels of the unaveraged collisional and collisionless runs is just as expected from neoclassical theory, so the growth seen in the nominal (unaveraged collisional) case of figure 3 is shown to be neoclassical in origin.

![Figure 4](image)

**FIGURE 4.** Evolution of the radial electric field in the FT-2 parameter set, with analytical Hazeltine-Hinton electric field taking into account collisionality regimes.

**INFLUENCE OF NOISE ON THE QUALITY OF RESULTS**

In general, the noise level in the particle simulations reduces as $1/\sqrt{N}$, where $N$ is the average number of simulation particles in each cell. The noise not only makes it difficult to isolate physical fluctuations of plasma density and electrostatic potential from those ensuing from the finite number of simulation particles but also creates unphysical particle and heat flux and demolishes the neoclassical equilibrium. To quantify this effect consider the mixing-length estimate $\delta n/n = 1/k_\perp L_n$ for the nonlinear saturation level of physical fluctuations. In the case of Cyclone Base case benchmark discussed in section 3, $k_\perp \rho_i$ in range $0.2 - 0.3$ was found for the maximum growth of the unstable turbulent spectrum and $L_n/\rho_i = 500$. Hence, $\delta n/n \sim 0.01$ is predicted for the rms fluctuation level of physical fluctuations. As $N \sim 1500$ in the simulation, the $1/\sqrt{N}$ noise is actually somewhat higher for density fluctuation level than expected for physical fluctuations in this case. In spite of this caveat, by appropriate filtering of data either in time or wavenumber, the linear growth rates of the unstable modes were satisfactorily identified behind this noise, as discussed in section 3. Clearly, the noise problem was here introduced by the choice of relatively low temperature ($T = 100$ eV) and weak density gradient ($L_n = 0.5$ m). For steeper density gradient and higher temperature, the physical density fluctuations are much better resolved at saturation according to the mixing-length prediction. According to the adiabatic relation $e\Phi/T = \delta n/n$, these scalings are reflected in potential fluctuations, too, but the noise fluctuations of potential will increase linearly with plasma temperature, too. In figure 5 is shown the rms level of the density and potential fluctuations in the plasma mid-radius for a varying number of simulation particles in the Cyclone Base case of section 3. We find that the density fluctuation level follows the $1/\sqrt{N}$ law, as expected from the previous considerations, and that there is no evolution of the fluctuation level in time supporting the fact that the
noise dominates the fluctuations in the present case. Moreover, the potential fluctuations are about 5 times higher than what the adiabatic relation would predict. The latter can be explained by noting that in kinetic simulations the noise in both ion and electron density fluctuations affects the charge separation of ions and electrons. Although the charge density separation becomes by several orders of magnitudes smaller than either ion or electron density, the overall effect of adding ion and electron noise does not count more than to a twofold enhancement of potential fluctuation level after inversion of the Poisson equation. This is due to the fact that the quasineutrality is enforced to noise, too. Another twofold enhancement to potential fluctuations arises from the statistical sampling of the coefficient matrix of the GK Poisson equation from the ion and electron implicit terms.

FIGURE 5. Density and potential fluctuations as functions of time. Runs used, from higher to lower values, 390, 780 and 1560 particles per cell.

To give a prediction for the contribution of the noise to ion heat conductivity (see figure 3), consider noise potential fluctuations on a magnetic surface having $\tau$ as a decorrelation time for fluctuations. The radial diffusion coefficient can be estimated from $D = \langle dr^2 \rangle / \tau$, where $dr$ is the particle shift from the surface by the radial $ExB$ velocity during the decorrelation time. Taking $dr = \langle \Phi \rangle_{\text{rms}} \tau / B \Delta y$, we find an estimate $D = \langle \Phi \rangle_{\text{rms}}^2 \tau / B^2 \Delta y^2$, where $\Delta y$ denotes the grid cell size in the poloidal direction. The decorrelation time can be estimated from $\tau = \Delta z / v_{Te}$, where $\Delta z$ is the grid cell size along the magnetic field line. Here, the dominant noise is assumed to be created by electron free motion along the magnetic field, and is taken to be decorrelated by the time electrons can pass one cell element along the field line. With the parameters and data of the case in figure 3 for $N = 1500$ we obtain $D = 0.4 \text{ m}^2 / \text{s}$. This should be compared to the value of $\chi \sim 0.2 - 0.4 \text{ m}^2 / \text{s}$ obtained at initial stages of simulation. This level was found to scale as $1/N$ in the present case in the simulations.

Using the mixing-length estimate for the physical level of fluctuations, the radial ion heat conductivity can be estimated from $\chi = (5/2)(L_T / L_n)(v_{Ti}^2 / \Omega_i)(1 / k_L L_n) g$, where $g$ arises from the relation $\delta n = ne\Phi(1 - ig) / T$ between the density and potential fluctuations. Using $g \sim \gamma / \omega \sim 0.5$ from the linear growth rate analysis for the collisionless Cyclone Base case in figure 3, we may estimate $\chi \sim 0.3 \text{ m}^2 / \text{s}$ for the collisional case of figure 3. This is lower than what we predict and find for the ion heat conductivity by noise. This was further confirmed by running for saturation in the absence of collisions. Here, as shown in figure 3, $\chi$ does not evolve from its initial fluctuations and stays at a level of $0.4 \text{ m}^2 / \text{s}$. Therefore, as discussed in section 3, the ion heat conductivity arising
in the collisional case of figure 3 must originate from the neoclassical ion heat conductivity. The heat conductivity by turbulence is not strong enough to be identified with the present parameters. It is easy to see that the diffusivity by the noise according to our estimate scales as $T^{3/2}$ if an adiabatic potential response is assumed, while our estimate of $\chi$ from the physical fluctuations similarly scales as $T^{3/2}$. Therefore, although helping us to identify the physical density fluctuations from noise increasing temperature does not help to resolve the physical ion heat conductivity if the number of simulation particles is too low. However higher temperature implies longer wavelengths for unstable modes and thus a coarser grid can be used in the simulation. This makes higher the number of particles per cell ($N$), and $\langle \Phi \rangle_{\text{rms}}$ is reduced for that reason and also because of the longer $\Delta y$, which implies smaller noise flux. On the other hand, like in the linear growth analysis, it is possible to reduce in diagnostics the level of noise heat flux by time filtering the potential data. If the time window of filtering is larger than $\tau$ but shorter than decorrelation times of physical convection cells, one may estimate the physical heat and particle diffusivities by $E \times B$ drift under the noise.

However, one should note that both physical and noise radial fluxes of electrons and ions by $E \times B$ drift create net current due to the ion gyroaveraging [33]. This current density is proportional to $k^2 \rho_i^2 \Gamma_e / 4$, where $\Gamma_e$ is the radial particle flux of electrons. $k_\perp$ represents here the dominant part of fluctuations responsible for the flux. This non-ambipolarity has to be incorporated into the neoclassical equilibrium. Its most direct implication is that the radial electric field in the equilibrium becomes dependent on this non-ambipolar current. The latter is negative which makes the radial electric field more positive in equilibrium. Due to the noise contribution in the particle fluxes, the noise can deteriorate the accuracy in obtaining the radial electric field and flow velocities of the plasma. This is in particular the case where the radial noise flux of particles dominates over the corresponding flux of particles by physical fluctuations and over the ion flux by ion-ion collisions. The effect is further strengthened for the noise due to dominant role of short wavelengths in potential spectrum in driving radial particle flux (implying larger $k^2 \rho_i^2$ in the non-ambipolar current formula). In the case of figure 3, the radial electric field was found to be negative and in good agreement with the neoclassical estimate, as the noise flux was smaller than the ion flux due to ion-ion collisions. On the other hand, in its collisionless run, the radial electric field was positive and was determined by the non-ambipolarity of the noise flux. The closer investigation of this effect is beyond the scope of the present paper and will be elaborated in more detail elsewhere.

In summary, the least constraints for the required number of simulation particles to suppress the noise can be found in applications where $L_n$ is not too large and temperature not too small. Other parameters like $L_T / L_n$, characteristics of turbulent spectrum, or grid specifications may equally affect the required number of simulation particles. In the case of FT-2 parameters [21], a good statistics with little role of noise in transport quantities was obtained with $N > 800$. This was due to short $L_m = 0.08$ m and relatively high temperature of $T = 100 - 500$ eV. It therefore appears that full $f$ particle simulations are easiest to apply in applications where the density profile has not too weak gradient. For practical applications, $N = 500 - 5000$ depending on the nonlinear saturation level of turbulence should be enough for sufficiently good statistics using the full $f$ PIC method.
ANALYSIS OF TURBULENCE SPECTRA IN TOROIDAL PLASMAS

In Fourier space, the turbulent dynamics can be described by the linear behavior, following a dispersion law, and the nonlinear wave interactions. The linear growth, prominent on relatively low wavenumbers, acts as a source for the turbulence, which is fed by the nonlinear interactions that produce a cascade to higher wavenumbers, broadening the spectrum. In one-field nonlinear theories the evolution of a mode is given by

\[ \frac{\partial \phi_k}{\partial t} = i\omega_k \phi_k + \sum_{k_1+k_2=k} \Lambda^{k}_{k_1,k_2} \phi_{k_1} \phi_{k_2}, \tag{2} \]

where \( \omega_k \) is the solution of the linear dispersion law, and \( \Lambda^{k}_{k_1,k_2} \) is the mode coupling matrix element. For instance the widely cited Hasegawa-Mima model (see Refs. [27] and [28]) gives \( \Lambda^{k}_{k_1,k_2} = \frac{1}{2} \hat{z} \times (k_1 \times k_2) (k_2^2 - k_1^2) \). The Hasegawa-Mima model assumes a locally homogeneous plasma, with magnetic field \( B = B_0 \hat{z} \), and an adiabatic electron response \( \delta n_e = n_0 \phi / T_e \). The Hasegawa-Mima equation has no sources or sinks. By setting the time derivatives in equation (2) to zero, one can solve the spectrum in stationary conditions.

Varying spectral forms obtained from experiments and nonlinear theories have been presented in the literature. In all, solving the form of a turbulent spectrum is a difficult and generally unsolved problem. Kolmogorov-type power laws \( S(k) \propto k^{-\alpha} \) are usual in turbulence, being \( S(k) = \langle |\hat{\phi}_k|^2 \rangle \) or \( S(k) = \langle |\hat{n}_k|^2 \rangle \) (\( \langle \ldots \rangle \) denotes ensemble averaging). Power laws \( S(k) = C k^{-\alpha_{\text{exp}}} \) have been reported, with \( \alpha_{\text{exp}} \approx 3.5 \) in numerous experiments [29].

Simulation results

Simulations have been conducted with ELMFIRE, reproducing a discharge performed at the FT-2 tokamak [30], using the following parameters taken from the experimental ones:

\[ B = 2.2 \text{ T}, \quad I = 22 \text{ kA}, \quad n = 3.5 \times 10^{19} \text{ m}^{-3}, \quad T_i = 250 \text{ eV}, \quad T_e = 300 \text{ eV} \quad @ \rho = 2 \text{ cm} \]

This is a representative case of transport barrier formation, where one can study the influence of that process on the existing turbulence spectra. Wavenumber and frequency spectra are obtained from the simulation data through fast Fourier transforms of the turbulent quantities, e.g. the density and the electric potential. In a computer simulation, averaging can be done over many adjacent grid points, time steps and wavenumbers, giving statistically reliable results. Another benefit is that we have access to detailed information of the turbulence from all parts of the tokamak. The power spectra presented here are obtained by averaging over 20 neighboring wavenumbers and 25 sample spectra calculated at adjacent time instants for the wavenumber spectra, and over 2 neighboring frequencies and 45 sample spectra for the frequency spectra.
FIGURE 6. a) Density fluctuation spectrum at plasma edge with fitting power law, where $\alpha \approx 3$; b) Density fluctuation profiles spectra before (dotted line), during (dashed line) and after (solid line) the barrier generation at the radius $\rho = 5.1$ cm. c) Evolution of fitting parameter $\alpha$ for wavenumber spectrum of density fluctuations before (dotted line), during (dashed line) and after (solid line) the barrier generation.

CONCLUSIONS

In order to study plasma turbulence and associated transport phenomena, a gyrokinetic full $f$ particle-in-cell code has been developed and implemented for tokamak transport simulations. The code, ELMFIRE, is a 5-D nonlinear global code using quasi ballooning coordinates for resolving the electrostatic potential. The method used in ELMFIRE differs from standard gyrokinetic codes in the sense that the change in charge density, caused by the ion polarization drift, and the particle advancing are recorded separately at each time step. This enables the calculation of the particle mean motion. This implicit gyrokinetic particle solution method makes it possible to simulate transport phenomena involving rapid dynamic changes and steep gradients.

For the adiabatic cyclone base case the code has been successfully validated against linear and nonlinear predictions of the growth rates and frequencies of unstable modes (see figure 1). In figure 2 the comparison between the simulation results of the heat conductivity and the neoclassical heat conductivity is shown. In figure 4 is represented the comparison between the electric field obtained by simulations and the analytical Hazeltine-Hinton electric field. All these examples show a good similarity of the ELMFIRE code with neoclassical physics.

The influence of noise on the quality of the results of ELMFIRE has been investigated and it is found that the average number of simulation particles in each cell should be between 500–5000 in order to get sufficiently good statistics.

From the spectra analysis (see figure 6) we see a suppression of large scales in the region of reduced transport and increased sheared flow while the other regions of the spectrum are virtually unchanged. A variation of exponents of power law spectra with radius has been found, suggesting that the scaling properties of nonlinear wave interactions depend on plasma conditions.
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