Abstract. Nonlinear evolutions of MHD instabilities in the large helical device are studied by means of direct numerical simulations under the vacuum configuration with the magnetic axis position $R = 3.6m$, including effects of its full three-dimensional geometry, flows parallel to the magnetic field lines and the fluid compressibility. The linear growth of the pressure-driven modes and their nonlinear saturations are observed. The linear growth brings about the flows parallel to the magnetic field lines as strong as the perpendicular flows. The fluid compressibility reduces the linear growth rate significantly. In the nonlinear saturation process, a qualitative difference is found in the behaviors of the parallel and perpendicular flows. The plasma appears to approach to a near-equilibrium state, keeping finite amplitudes of the parallel flow. Our numerical results highlight important roles of the parallel flow and compressibility in nonlinear MHD simulations in the large helical device.

Keywords: MHD instability, nonlinear simulation, parallel flow, compressibility

1. INTRODUCTION

Clarifying basic plasma behaviors in nonlinear evolutions of MHD instability is one of the most important subjects of theoretical studies of the Large Helical Device (LHD) [1]. LHD is a helical device with a set of $L = 2/M = 10$ helical coils where $L$ and $M$ are polarity and toroidal field period of the helical coils, respectively. The major radius is 3.9m. Recent LHD experiments have shown good confinement results of $\langle \beta \rangle \geq 4\%$ [2]. However, understanding of the good confinement is not very simple from theoretical points of views, since the vacuum configuration with the magnetic axis position $R_{ax} = 3.6m$ adopted in the experiments is considered linear unstable. Ichiguchi et al. has shown a possible understanding on the good confinement through the local flattening of the pressure by means of their numerical simulations of the three-field reduced equations, which consist of the perturbed poloidal flux function, the perturbed stream function of poloidal flow and the perturbed pressure [3]. The reduced equations for LHD are derived by the use of the stellarator expansion. An assumption in the derivation is that the flow component parallel to the magnetic field line (hereafter we refer to it simply as the parallel flow) remains small if it is initially small. However, it has been found out in the linear

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1 Deceased April 2006
ideal MHD stability analyses for compressible perturbations that the parallel flow is not necessarily small. Furthermore, since an averaging method is adopted together with the helical expansion to derive the reduced equations, the applicability of the equations are limited to relatively large scales. While the reduced simulations are carried out to show some possible understandings on the LHD confinements within the restrictions of the equations, another simulation approach, the direct numerical simulation (DNS) of the nonlinear, fully three-dimensional (3D) Magnetohydrodynamics (MHD) equations has been examined by the authors. The 3D geometry of LHD, the fluid compressibility, the nonlinearity, the parallel flow and the free boundary nature of the plasma boundary are included in the DNS code, the MHD In Non-Orthogonal System (MINOS) code. Here we carry out DNS by the use of the MINOS code to provide further understanding on the plasma confinements in LHD. Special attentions are paid to such effects as the parallel flows and the compressibility, which are omitted from the reduced equations.

This paper is organized as follows. In §2, the numerical codes are reviewed. Boozer coordinate is introduced for the purpose of the Fourier mode analysis. Basic properties of the initial equilibrium used in this paper are shown. In §3, linear and nonlinear behaviors are studied. Linear behaviors obtained by the MINOS simulation are examined for a run with low dissipative coefficients. Nonlinear behaviors are studied by the simulations with relatively large viscosity. The concluding remarks are shown in the last section.

2. DIRECT NUMERICAL SIMULATIONS

The 3D compressible and nonlinear MHD equations consist of the equations of the continuity, the momenta, the pressure and the magnetic field

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{V}), \\
\frac{\partial (\rho \mathbf{V})}{\partial t} &= -\nabla \cdot (\rho \mathbf{V} \mathbf{V}) - \nabla p + \mathbf{J} \times \mathbf{B} + \mu \left[ \nabla^2 \mathbf{V} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{V}) \right], \\
\frac{\partial p}{\partial t} &= -\nabla (p \mathbf{V}) + (\gamma - 1) \times \left[ -p \nabla \cdot \mathbf{V} + \eta \mathbf{J}^2 + \mu \left\{ \omega^2 + \frac{4}{3} (\nabla \cdot \mathbf{V})^2 \right\} + \kappa \nabla^2 T + \kappa_{/\parallel} \nabla^2 T \right], \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J})
\end{align*}
\]

where \(\rho, p, \mathbf{V}, \mathbf{B}\) are the mass density, the pressure, the velocity vector and the magnetic field vector, respectively. The symbols \(\omega = \nabla \times \mathbf{V}\) and \(\mathbf{J} = \nabla \times \mathbf{B}\) represent the vorticity and current density, respectively. A plasma gas is assumed to obey the state equation of an ideal gas, \(p = \rho T\) where \(T\) is the temperature. Equations (1)-(4) are already normalized by representative quantities (see Ref.[5]) so that the resistivity \(\eta\), viscosity \(\mu\), the isotropic portion of the heat conductivity \(\kappa\) and the parallel heat conductivity \(\kappa_{/\parallel}\) are dimensionless parameters.
The 3D geometry of the LHD is expressed through the metrics of the helical-toroidal coordinate \((u^1,u^2,u^3)\) on which the eqs.(1)-(4) are expressed.

\[
\frac{\partial \rho}{\partial t} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^i} \left( \sqrt{g} \rho v^i \right), \\
\frac{\partial \rho v^i}{\partial t} = -g^{ih} \left[ \frac{\partial T^k_i}{\partial u^k} + \{ k \ l \} T^l_h - \{ l \ h \k \} T^k_i + \epsilon_{ikh} f^k b^h \right] + \mu \left[ -\frac{1}{\sqrt{g}} \epsilon_{ikh} \frac{\partial}{\partial u^k} \left( g_{hi} \frac{1}{\sqrt{g}} \epsilon_{imn} \frac{\partial v^n}{\partial u^m} \right) + \frac{4}{3} \frac{\partial}{\partial u^i} \left( \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^m} \sqrt{g} v^m \right) \right] \\
\frac{\partial p}{\partial t} + v^i \frac{\partial p}{\partial u^i} = -\gamma p \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^i} \left( \sqrt{g} v^i \right) + (\gamma - 1) \left[ \rho \frac{\partial}{\partial u^i} \left( \sqrt{g} v^i \right) + \frac{4}{3} \frac{\partial}{\partial u^i} \left( \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^m} \sqrt{g} v^m \right) \right] \\
\frac{\partial b^i}{\partial t} = -\epsilon_{ikh} \frac{1}{\sqrt{g}} \frac{\partial}{\partial u^k} \left( -\sqrt{g} \epsilon_{hm} v^h b^m + \eta j^h \right), \\
\epsilon_{bh} = b^i/\sqrt{b^i b^j}
\]

\(10\)

where the symbol

\[
\left\{ \begin{array}{l} h \\ \{ i \ j \} \end{array} \right\} = \left\{ \begin{array}{l} h \\ \{ i \ j \} \end{array} \right\} = \frac{1}{2} \ g^{hp} \left( \frac{\partial g_{ip}}{\partial u^j} + \frac{\partial g_{jp}}{\partial u^i} + \frac{\partial g_{ij}}{\partial u^p} \right)
\]

is the Christoffel symbol and \(T^k_i = \rho v^i v_k + p \delta^k_i\). The same coordinate system is adopted in the HINT code[8] by which we compute an initial equilibrium. Variables with subscripts represent covariant components of a vector. The contravariant and covariant components of a vector are transformed into one another by using the metrics \(g_{ij}\) or \(g^{ij}\). Symbols \(v^i, \omega^i, b^i\) and \(j^i\) represent \(i\)-th contravariant components of the velocity, vorticity, magnetic field and current vectors, respectively. The vorticity and the current density are given as \(\omega_i = \epsilon_{ijk}(\partial v_j/\partial u_k)/\sqrt{g}\) and \(j_i = \epsilon_{ijk}(\partial b_k/\partial u^k)/\sqrt{g}\), respectively. Detailed expressions of the metrics are seen in Ref.[8].

Equations (5)-(9) are discretized in the \((u^1,u^2,u^3)\)-space. The spatial derivatives in the equations are approximated by the 8th order compact finite difference scheme[9]. By adopting the high order scheme, the contamination of the heat conductivity to the perpendicular direction through the truncation errors generated in the computation of the parallel heat conductivity term is suppressed. In the time direction, the 4th order Runge-Kutta-Gill method is adopted.

An initial, net toroidal current free MHD equilibrium is provided by the HINT computation. The vacuum configuration is set as \(R_{ax} = 3.6\) m. The initial pressure profile is assumed to be

\[
p(\psi_0) = p_0 (1 - \psi_0)^2
\]
FIGURE 1. Basic profiles of the initial equilibrium used in this article. The $m/n = 0/0$ mode of the pressure, the rotational transform $\iota/2\pi$ and the stability index $D_I$ are plotted with the solid, dashed and dotted lines, respectively.

where $\psi_0$ is the normalized toroidal flux. The central pressure value $p_0$ is determined to specify the $\beta$-value on the magnetic axis ($\beta_0$). Note that the $\psi_0$ is slightly different from the toroidal flux $\psi$ of the Boozer coordinate, which is introduced later. Since the HINT code is based on the relaxation method, finally obtained MHD equilibrium has slightly different pressure profile and $\beta_0$ from initially prescribed ones. In the present case, the resultant MHD equilibrium has $\beta_0 = 3.7\%$, where the average $\beta$ is $\langle \beta \rangle \sim 1.2\%$. In Fig.1, the mean pressure $P_{00}$ normalized by the magnetic pressure $B_2^2/2$ at the magnetic axis, the rotational transform $\iota/2\pi$ and $D_I$ with positive value (multiplied by $1/20$) are plotted as the function of the normalized minor radius defined as $\sqrt{\psi}$. The $D_I$ plot indicates that pressure-driven modes with their rational surfaces at $\sqrt{\psi} < 0.6$ ($\sqrt{\psi} > 0.6$) are linearly unstable (stable). For example, a pressure-driven mode with the single Fourier mode $m/n = 2/1$ is linearly unstable while the single Fourier mode $m/n = 1/1$ is linearly stable, where $m$ and $n$ are the poloidal and the toroidal mode numbers in the Boozer coordinate system, respectively.

The Fourier modes of a perturbation are obtained by performing the Fourier transform in the Boozer coordinate system. The Boozer coordinate system $(\psi, \theta, \zeta)$ is constructed based on the MHD equilibrium obtained by the HINT code, where $\psi$, $\theta$ and $\zeta$ are the normalized toroidal flux, the poloidal and the toroidal angles, respectively. We provide the pressure profile as the function of the normalized toroidal flux $p(\psi)$ together with the $\beta_0$ as the input of the VMEC code[10] and calculate the fixed boundary MHD equilibrium. The resultant MHD equilibrium in the VMEC magnetic coordinate system is transformed into the Boozer magnetic coordinates system. A scalar quantity $A(\psi, \theta, \zeta)$ is Fourier transformed on a specified flux surface of the Boozer coordinate as

$$A_{mn}(\psi) = \oint d\theta \int d\zeta A(\psi, \theta, \zeta) \exp[i(m\theta - n\zeta)]$$

$$= \oint d\theta \int d\zeta A(u^1(\psi, \theta, \zeta), u^2(\psi, \theta, \zeta), u^3(\psi, \theta, \zeta)) \exp[i(m\theta - n\zeta)]$$

At the initial time of a DNS, the initial equilibrium is perturbed by giving small random values to the three velocity components in the $(u^1, u^2, u^3)$ space. Since the
random perturbations are not proportional to the linear eigen-functions, a plasma needs certain periods to form the linear eigen-functions in the course of time evolutions. The periods required to form the linear eigen-functions and the consequent amplitudes of the eigen-functions are different among the Fourier modes. We also note that the dissipative terms in eqs. (1)- (4) perturb the initial equilibrium, too. When the dissipative coefficients are relatively large, the pressure is instantaneously perturbed from an equilibriated state \( \mathbf{B} \cdot \nabla p = 0 \). It brings about transient periods for which Fourier modes growth do not obey the expected linear instability.

3. LINEAR AND NONLINEAR PHENOMENA

3.1. Linear behaviors with small dissipative coefficients

Linear behaviors are studied by analyzing numerical results of a DNS with low dissipative coefficients. Here we set \( \eta = 1 \times 10^{-6} \), \( \mu = 1 \times 10^{-6} \), \( \kappa = 1 \times 10^{-6} \) and \( \kappa_{/} = 0 \). We decompose the velocity vector into the orthogonal form with the normal \((V \nabla \psi)\), the parallel \((V_b)\) and the binormal \((V \nabla \psi \times b)\) components as

\[
V = V_\psi \left( \frac{\nabla \psi}{|\nabla \psi|} \right) + V_b \left( \frac{B_{eq}}{|B_{eq}|} \right) + V_{\psi \times b} \left( \frac{\nabla \psi}{|\nabla \psi|} \times \frac{B_{eq}}{|B_{eq}|} \right),
\]

where the subscript \( eq \) represents a quantity in the initial equilibrium. In Fig.2, time evolutions of the amplitudes of some Fourier modes of \((V_\psi)\), \((V_b)\) and \((V_{\psi \times b})\) are shown. Hereafter, when a time series is plotted, the horizontal axis is normalized by the toroidal Alfvén time \( \tau_A \). We also assume that the time unit is the Alfvén time \( \tau_A \) when a specific time is referred in the text. In Fig.2, only the Fourier modes which share the same rational surface \( \iota/2\pi = 0 \) are chosen since the Fourier modes on this rational surface are considered being strongly unstable by themselves. (Notice that the Fourier modes on the rational surfaces with \( D_I < 0 \) can grow simultaneously. In the linear analysis of LHD, one linear eigen-function does not consist of single Fourier mode but needs some other Fourier modes. For example, the \( m/n = 1/1 \) Fourier mode grows together with the \( m/n = 2/1 \) and some other modes because they consist one eigen-function together.) All the Fourier modes in Fig.2 show exponential growth. It turns out later that the linear stage of the time evolution is restricted to \( t \leq 45 \), though some Fourier modes keep their exponential growths beyond that time. The growth rates of the Fourier modes for \( t \leq 45 \) are estimated as 0.176, 0.1965, 0.2155 and 216 for \( n = 1, 2, 3 \) and 4, respectively, and the \( n = 7 \) mode has the largest growth rate. The CAS3D[11] computation gives estimates of the growth rates as 0.1789, 0.239, 0.2613 and 0.2686 for \( n = 1, 2, 3 \) and 4 modes, respectively. It may be difficult to compare the DNS results with the CAS3D computation directly, since there are some effects in DNS such as dissipations (especially provided by the parallel heat conductivity) which do not appear in the CAS3D computation. However, the \( n = 1 \) growth rate in the DNS is almost the same as the CAS3D predicts. Furthermore, the growth rate of the \( n = 3 \) mode is similar to the \( n = 4 \) growth rate both in DNS and CAS3D computations. Thus it is considered that the simulation gives reasonable growth rates compared to the ideal ones. It is also
noticeable in the figure that the Fourier amplitudes of the parallel velocity component are as large as those of the other two components. The growth of the parallel component is due to the linear instability. It is consistent with the linear analysis result by Nakajima[4]. This simulation is terminated before $t = 80$. Nevertheless it may be interesting to point out that the $m/n = 2/1$ mode finally obtains the largest amplitude among the $t/2\pi = 0.5$ resonant Fourier modes, since the $m/n = 2/1$ mode is the most dominant through our earlier works for the full-torus LHD geometry.[6, 7] Although the growth rates are larger for higher Fourier wavenumber when the pressure-driven instability, the saturation level can be larger for lower Fourier wavenumber.

![FIGURE 2. Time evolutions of the amplitudes of some typical Fourier modes of (a)normal, (b)parallel and (c)binormal components of the velocity vector.](image)

Another linear aspect in the DNS is seen in the fluid compressibility. To see this, we study the kinetic energy budget. The kinetic energy budget is described as

\begin{align}
K & = \frac{1}{2} \left\langle \rho |\mathbf{V}|^2 \right\rangle, \\
\frac{1}{K} \frac{dK}{dt} & = \frac{1}{K} \left\langle \tilde{p} + \left( \mathbf{B}_{eq} \cdot \tilde{\mathbf{B}} + \frac{1}{2} |\mathbf{B}|^2 \right) \right\rangle \left\langle \nabla \cdot \mathbf{V} \right\rangle + \frac{1}{K} \left\langle \left( \tilde{\mathbf{B}} \mathbf{B}_{eq} + \mathbf{B} \tilde{\mathbf{B}}_{eq} + \tilde{\mathbf{B}} \tilde{\mathbf{B}} \right) : \nabla \mathbf{V} \right\rangle \\
& \quad - \frac{1}{K} \left\langle \mu \left\{ \frac{4}{3} (\nabla \cdot \mathbf{V})^2 \right\} \right\rangle,
\end{align}
where the tilde $\tilde{\cdot}$ represents the fluctuation of the quantity, $\tilde{p} = p - p_{eq}$ and $\tilde{B} = B - B_{eq}$. The bracket $\langle \cdot \rangle$ represents the volume average. The first (compressible), second (driving) and third (viscous) terms in the right-hand-side of eq.(15) are referred as T1, T2 and T3, respectively. These terms represent contributions to the instantaneous growth rate by the compressible, magnetic driving and viscous effects. It is well known that the compressible terms in the MHD equations work to stabilize the linear evolution[13]. However, it is not clear how much the compressibility works to the stabilization. In Fig.3, T1, T2 and T3 are plotted. The compressible term $T1$ is negative for $t \simeq 45$ and positive for $t > 45$. Since $T1$ is expected being negative definite in the linear stage, the change of sign of $T1$ at $t \approx 45$ suggests that linear stage is limited for $t \leq 45$. Here we do not study the nonlinear stage because the representation of eq. (15) is arbitrary and makes sense only for the linear period. The magnitude of the compressible term $T1$ is about $70 \sim 80\%$ of the driving term $T2$. It is estimated that the compressible term reduces the linear growth rate to about $1/5$ of that obtained by neglecting the compressible contributions.

In Fig.4(a), the Poincare plots of the magnetic field lines on a vertically-elongated poloidal cross-section at $t = 50$. (b)The pressure fluctuation on the same poloidal cross-section at $t = 50$.

In Fig.4(a), the Poincare plots of the magnetic field lines on a vertically-elongated
poloidal section are shown at $t = 50$. Based on the energy budget analysis in the above, $t = 50$ is already in the nonlinear stage, not in the linear stage. However, since the amplitudes of unstable Fourier modes are still so weak that the observations in the Poincare plots are unclear for $t < 45$. Thus we study at $t = 50$ in Fig.4(a) and (b), respectively. Although it is in the nonlinear stage, linear properties still remain somewhat in the figures. In Fig.4(a), the Poincare plots have stochastic region around the $t/2\pi = 0.5$ rational surface. It turns out later that the most unstable mode is associated with the $t/2\pi = 7/12 \simeq 0.58$ rational surface, not with the $t/2\pi = 0.5$ surface. The magnetic surface which separates the stochastic region and well-confined region is wavy in the outer side of the torus. In Fig.4(b), the pressure fluctuations on a vertically-elongated poloidal cross-section are shown. Seven pairs of positive and negative contours are observed in the region of $t/2\pi = 0.5 \sim 0.6$ region. It makes a good accordance with the observation in Fig.2, in which $n = 7$ Fourier mode has the largest amplitude. The observation is essentially the same as that at $t = 45$, figure of which is not seen here. We easily find that the large fluctuations are mostly seen in the outer side of the torus. It is a typical view of ballooning modes that the fluctuations appear in the outer side of the torus where the magnetic field lines curvature are unfavorable.

![Figures](image)

**FIGURE 5.** Profiles of $m = 11, \ldots, 15/n = 7$ Fourier modes of (a)normal, (b) parallel and (c) binormal components of the velocity at $t = 50$.

In order to study properties of the unstable modes more closely, the Fourier coefficients of $m/n = 11/7, 12/7, 13/7, 14/7$ and $15/7$ modes of the three velocity components at $t = 45$ are plotted as a function of $\sqrt{\psi}$ in Fig.5. All of the plots in Fig.5 are multiplied by $10^4$. It is found in Fig.5(a)(for $V_{\psi\psi}$) that the $m/n = 12/7$ mode has
a single peak around $\sqrt{\psi} \approx 0.45$ where its rational surface of the mode exists and the largest amplitude among the five Fourier coefficients. On the other hand, $m/n = 14/7$ has two peaks and the larger peak exists at $\sqrt{\psi} = 0.55$, far apart from its rational surface $\sqrt{\psi} \approx 0.35$. Thus the most dominant Fourier mode is $m/n = 12/7$, not $14/7$. We also find that the $m/n = 11/7$ and $13/7$ Fourier modes have two peaks just in the neighborhood of the peak of the $12/7$ mode. It has been shown that such a structure is typically seen in eigen-functions of the ballooning mode[12].

### 3.2. Nonlinear phenomena

In the above, we have seen some basic linear properties. Although further observations of that run can be interesting, due to the restrictions of numerical resolutions, we move to analyses of DNS results of a larger viscosity $\mu = 5 \times 10^{-4}$. The other dissipative coefficients are the same as the run in the previous subsection.

![FIGURE 6. Time evolutions of the amplitudes of some typical Fourier modes of (a)$V_\psi$, (b)$V_b$ and (c)$V_\psi \times V_b$ of the velocity vector. (d)A comparison of the growth of $m/n = 1/1$ and $2/1$ Fourier modes of the three velocity components.](image-url)

In Fig.6, the Fourier amplitudes of the three velocity components are shown. Again we see only the Fourier modes which are resonant on the $\varpi = 0.5$ rational surface. Since the viscosity is relatively large, the $n = 1$ mode has the largest growth rate. We also find that the $m/n = 2/1$ mode of $V_b$ in Fig.6(b) grows quite rapidly for $t \leq 70$ and moves to the linear growth after $t \geq 70$. The rapid growth is considered as a consequence of
two effects, (1) the viscous perturbation which we have mentioned earlier because such a non-exponential growth is not observed when $\mu = 1 \times 10^{-6}$, and (2) couplings with noisy Fourier modes, which are not shown in the figure but have some finite amplitudes due to the initial random perturbation. Although these two effects could influence equally the normal and the binormal Fourier modes, only the parallel $m/n = 2/1$ mode experiences the rapid growth and obtain the dominating amplitude. In Fig. 6(d), the $m/n = 1/1$ and $2/2$ mode of the three velocity components are shown. (Readers may also refer to Fig. 6 of Ref.[7] in which the similar plots for a run with $\mu = 2 \times 10^{-3}$ and $\kappa_{1//} = 10^{-3}$ are shown.) It is clearly seen that the amplitudes of the Fourier modes of the parallel component are superior to those of the the other two velocity components. The superiority of the parallel flow is commonly observed in the previous works[6, 7], in which the different initial perturbations are adopted. The parallel velocity component obtains an amplitude comparable to or larger than those of the other two components. Since the parallel flow does not have any direct relation with the pressure-driven (either ballooning or interchange) instability, a plasma can release its kinetic energy safely to the parallel direction without causing a secondary instability associated with the increase of the flow strength.

In Fig. 7(a), a 3D view of the pressure and stream lines are shown. Since the parallel component of the velocity dominates the velocity magnitude, the stream lines are strongly elongated into the toroidal direction. The stream lines form a spiral in a region where they are in the neighborhood of the mushroom-like pressure structure, since the normal and binormal components of the velocity are superior to the parallel component there. In Fig. 7(b), the $m/n = 0/0$ profile of the pressure is shown at $t = 0$ (initial profile), $t = 220$ (saturation time) and $t = 580$ (the final time). The $\beta_0$ at the final time is reduced to about 5/7 of the initial $\beta_0$. It is relatively a large deformation. However, we find in Fig. 7(c) and (d) that the shapes of the pressure contours change from the mushroom-like shapes at (c)$t = 220$ (nonlinear saturation time) to concentric shapes at (d)$t = 580$, suggesting a recovery of a well-confined state. Such a robust behavior of a plasma is observed in an recent LHD experiment, too[14]. In the experiment, an $m/n = 1/1$ activity is observed. (The difference of the mode between our DNS ($m/n = 2/1$) and the experiment ($m/n = 1/1$) comes from the different rotational transform.) Although the MHD activity brings about a large change of the pressure profile, it does not lead to the disruption.

The Poincare plots of the magnetic field lines are shown in Fig. 8 at (a)$t = 220$ and (b)$t = 580$. At $t = 220$, the Poincare plots show clear mushroom-like structures, which correspond to the structure seen in the pressure contours in Fig. 7(c). At $t = 580$, the Poincare plot is still stochastic in the neighborhood of the initial $\theta = 0.5$ rational surface. However, the magnetic surface are well kept and the observation in the figure supports an understanding that a plasma confinement is going to be recovered. From the $m/n = 0/0$ profile in Fig. 7, it appears that the plasma does not have sufficient free energy to cause the next instability. Thus it is considered that the plasma tends to recover a well confined, near-equilibrium state, keeping finite amplitude of the parallel flow after the nonlinear saturations.
FIGURE 7. (a) Pressure contours on a poloidal section and stream lines in the 3D geometry. (b) The $m/n = 0/0$ pressure profile at $t = 0, 220$ (saturation time) and $t = 580$ (the final time). Pressure contours of a vertically-elongated poloidal cross-section are shown at (c) $t = 220$ and (d) $t = 580$.

FIGURE 8. Poincare plots of the magnetic field lines on a vertically-elongated poloidal cross-section at (a) $t = 220$ and (b) $t = 580$. 
4. CONCLUDING REMARKS

We have seen some numerical results obtained by the DNS of fully 3D compressible MHD equations. A notable point is that the parallel flow becomes as strong as or larger than the other two velocity components. It is also observed that the growth rate is significantly reduced by the fluid compressibility. In the nonlinear stage, the parallel flow stay as a dominant velocity component. The system tends to move to a near equilibrium with the parallel flow keeping it energy. All these observations suggest that a good confinement in the LHD can be brought about the complex contributions of the parallel flow, compressibility and nonlinearity and that the dynamical elements, the parallel flow and the fluid compressibility are woth studying further to understand the good plasma confinement in LHD.

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