Quasi–Optical Propagation of an EC Gaussian Beam, Absorption and Current Drive in Tokamaks

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Abstract.

The issue of quasi–optical propagation of a Gaussian beam of electron cyclotron waves, and of the power absorption and driven current is reviewed for the case of a general tokamak equilibrium. The propagation of a general astigmatic Gaussian beam is described in terms of a set of quasi–optical rays which describe diffraction effects, within the framework of the complex eikonal approach. The absorbed power and the driven current density are computed along each ray solving the fully relativistic dispersion relation for electron cyclotron wave and by means of the neoclassical response function for the current. Results obtained with the new code GRAY in ITER are presented.

Keywords: Electron Cyclotron waves, Beam propagation, Current drive

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INTRODUCTION

In the present ITER design it is planned to install two electron cyclotron (EC) waves launching system (four upper launchers and one equatorial launcher), that will provide 20 MW of EC power in plasma each. The main goals of the EC system are bulk heating and current drive to increase the plasma pulse length and control of neoclassical tearing modes and eventually of sawteeth by means of localized current drive. Thus, we are now facing the problem of making accurate predictions of ECRH and ECCD in ITER (in particular of the EC current density profile) in order to estimate the needed power.

Lot of work has been devoted in the last few decades to investigate the problems related to the propagation of EC waves, and to the computation of power absorption and driven current in a toroidally confined plasma. Good confidence on the theory of EC waves has been achieved, and many codes have been developed and extensively used (see, e.g., the recent review on codes benchmarking in [1]). However, the existing codes can be run in ITER realistic conditions with some caveats. Although the EC physics in ITER is linear due to the amount of power involved, so that quasilinear effects on the distribution function can be neglected in most conditions, ITER is a large device which will operate at high temperature, so that the approximations related to the different scale lengths involved and to the large temperature need to be accurately revised. Finally, in the present launching design astigmatic focused beams will be injected into the plasma. Since the ECCD profile plays a key role in the MHD stabilization process, it is required to be able to describe the propagation of general Gaussian beams in plasmas.

Here, we present a review of the EC theory implemented in the new code GRAY [2, 3],
that performs the computation of the quasi–optical propagation, the power absorption and driven current of a Gaussian beam of EC waves in a general tokamak equilibrium. The issue of wave propagation is addressed in the framework of the complex eikonal approach [4], in which the beam is modeled by a set of mutually interacting rays to describe diffraction effects. These rays obey to the quasi–optical (QO) ray–tracing equations, that are coupled together through an additional constraint in the form of a partial differential equation. EC wave absorption is computed on each ray of the beam solving either the fully or the weakly dispersion relation of EC waves with the dielectric tensor expansion up to a given order in Larmor radius [5]. Accordingly, the EC driven current is computed using a linear adjoint model, taking into account trapped particle effects and wave polarization.

**QUASI–OPTICAL BEAM PROPAGATION**

Following Ref.[4], the solution of the wave equation for the electric field is taken of the form

$$\mathbf{E}(x,t) = e(x)E_0(x)\exp[-ik_0S(x) + i\omega t]$$  \hspace{1cm} (1)

where the function $S(x)$ is the complex eikonal, $S = S_R(x) + iS_I(x)$, in which the real part $S_R(x)$ is related to the beam propagation as in the geometric optics (GO), and the imaginary part $S_I(x)$ to the beam intensity profile shape. In Eq. (1), $\omega$ is the real frequency, $k_0 = \omega/c$ the wavevector amplitude in vacuum, $e(x)$ the complex polarization versor and $E_0(x)$ the slowly varying wave amplitude.

In geometric optics the ray propagation equations are obtained by means of asymptotic methods at lowest order in the expansion parameter $\delta \ll 1$, with $\delta^2 \sim \lambda/L$, being $\lambda$ the wavelength and $L$ the scale of the medium inhomogeneity. To describe the beam propagation, an additional scale length is introduced involving the beam width $w$, satisfying the condition: $\lambda \ll w \ll L = O(\delta)$. The above scaling defines the quasi–optic (QO) approximation. In quasi–optics (like in geometric optics) the function $k_0S$ is assumed of order $\delta^{-2}$, with $|S_I/S_R| \sim \delta$.

At lowest order, in a weakly dissipative medium the complex eikonal function $S$ obeys to $D(x, \tilde{k}, \omega) = 0$, where $D(x, \tilde{k}, \omega)$ corresponds to the real part of the dispersion relation computed for $\tilde{k} = k_0\nabla S$ complex, i.e., for $\tilde{k} = k + ik' = k_0(\nabla S_R + i\nabla S_I)$. Expanding the above relation up to order $\delta^2$, with $|k'/k| \sim O(\delta)$, and upon separation into real and imaginary part, one obtains a set of coupled first-order partial differential equations for $S_R$ and $S_I$.

$$D_R(x, \tilde{k}, \omega) = D(x, k, \omega) - \frac{1}{2}k'k' : \frac{\partial^2 D}{\partial k \partial k} = 0$$ \hspace{1cm} (2)

$$D_I(x, \tilde{k}, \omega) = k' \frac{\partial D}{\partial k} = 0$$ \hspace{1cm} (3)

Equation (2) can be solved with the method of characteristics. From Eq. (3), one has that $S_I$ is conserved along the trajectories of (2) up to order $\delta^2$. Up to this order, $D_R$ can be used instead of $D$ in Eq. (3), which then reads $k' \cdot \partial D_R/\partial k = 0$. Taking for $D(x, k, \omega)$, the eigenvalue of the dispersion tensor corresponding to the mode under consideration
\[ D(x, k, \omega) = k^2 - k_0^2 \nabla^2(x, k, \omega) = 0, \]

the QO ray equations can be conveniently written in the following form

\[ \frac{d\mathbf{x}}{ds} = \frac{\partial \Lambda}{\partial \mathbf{N}} \bigg|_{\Lambda=0} \]
\[ \frac{d\mathbf{N}}{ds} = -\frac{\partial \Lambda}{\partial \mathbf{x}} \bigg|_{\Lambda=0} \]
\[ \frac{\partial \Lambda}{\partial \mathbf{N}} \cdot \nabla S_I = 0 \]

where \( \mathbf{N} = kc/\omega \) is the refractive index vector, \( s \) the trajectory arclength, and the function \( \Lambda \equiv c^2/\omega^2 D_R(x, k, \omega) \) the QO dispersion relation

\[ \Lambda(x, \mathbf{N}, \omega) = N^2 - N^2_T(x, k, \omega) - |\nabla S_I|^2 + \frac{1}{2} \nabla S_I \nabla S_I : \frac{\partial^2 N^2_T}{\partial \mathbf{N} \partial \mathbf{N}} = 0 \]  

The last two terms are not present in the GO dispersion relation, and describe diffraction. The last term in Eq.(7) appearing only in the case of a spatially dispersive medium, and being zero otherwise. In a cold magnetized plasma, the refractive index of the ordinary and extraordinary mode can be expressed as \( N_o = N_o(x, N||, \omega) \), being \( N|| = \mathbf{N} \cdot \mathbf{b} \) the parallel refractive index, and \( \mathbf{b}(x) = \mathbf{B}(x)/B \) the magnetic versor. In this case, the expression for \( \Lambda \) takes the simple form

\[ \Lambda = N^2 - N^2_T(x, N||, \omega) - |\nabla S_I|^2 + \frac{1}{2} (\mathbf{b} \cdot \nabla S_I)^2 \frac{\partial^2 N^2_T}{\partial \mathbf{N} \partial \mathbf{N}} = 0 \]

Within the QO approximation, a beam is modeled by means of a set of mutually interacting rays satisfying the equations (4-6) with suitable initial conditions. Equations (4,5) describe the quasi–optical ray propagation, and are formally similar to the corresponding geometric optics ray equations, except for the dispersion relation (8) that depends in addition on the unknown vector \( \nabla S_I \). Closure of the system requires the self–consistent solution of the new equation (6), which states that the imaginary part of the eikonal, \( S_I(x) \), is constant along the ray trajectories, and that its gradient \( \nabla S_I \) is orthogonal to the ray paths. The initial conditions can be specified on a given surface in vacuum, once the eikonal function \( S(x) \) is known. By choosing the initial ray position on such a surface, the initial values of the refractive index vector \( \mathbf{N} \equiv \nabla S_R \) and of \( \nabla S_I \) are easily obtained.

The eikonal function corresponding to a Gaussian beam in vacuum can be written in the local reference system \((\tilde{x}, \tilde{y}, \tilde{z})\) in which the \( \tilde{z} \) axis is directed along the direction of propagation of the beam and corresponds to the beam center axis as

\[ S_R = \tilde{z} + \frac{\tilde{x}^2}{2R_{cx}} + \frac{\tilde{y}^2}{2R_{cy}}, \quad S_I = -\frac{1}{k_0} \left( \frac{\tilde{x}^2}{w_x^2} + \frac{\tilde{y}^2}{w_y^2} \right) \]  

Equations (9) are valid in the case of simple (or orthogonal) astigmatic Gaussian beams, where \( R_{ci} = (\tilde{z}_i^2 + \tilde{z}_R^2)/\tilde{z}_i \), and \( w_i^2 = w_0^2 (1 + \tilde{z}_i^2/\tilde{z}_R^2) \) are the radius of curvature and the
beam width respectively, with \( w_{0i} \) the beam waist, \( z_{Ri} = k_0 w_{0i}^2 / 2 \) the Rayleigh length, and \( \bar{z} = \bar{z} - z_{0i} \) with \( \bar{z}_{0i} \) the waist location. Differentiation of the expressions (9) with respect to \( x \) yields the components of the refractive index vector \( \mathbf{N} = \nabla S_R \) and of the vector \( \nabla S_I \). Since the expressions of the vectors \( \mathbf{N} \) and \( \nabla S_I \) for a Gaussian beam in vacuum satisfy the complex dispersion relation in vacuum only up to order \( \delta^3 \), a slightly modified prescription is used for the components of the wave refractive index \( \mathbf{N} \). They are obtained from the equations

\[
N^2 - 1 - |\nabla S_I|^2 = 0 \quad \text{and} \quad N \cdot \nabla S_I = 0,
\]

with \( \bar{N}_x/\bar{N}_y = (x/y)(R_{ey}/R_{ex}) \). The difference in the refractive index vector is of order \( \delta^3 \) in the transverse direction and \( \delta^4 \) in the propagation direction.

**CURRENT DRIVE EFFICIENCY**

Following the derivation given in Refs. [6, 7], we investigate the linear interaction regime. At first order in the Chapman-Enskog procedure, the electron distribution function \( f \) satisfies the equation

\[
v_{\parallel} \mathbf{b} \cdot \nabla f - C(f) = -\nabla_{\mathbf{u}} \cdot \mathbf{S}_w(f_M) - \frac{\mathcal{E}}{T} f_M \frac{d}{dl} \ln T
\]

(10)

where \( v_{\parallel} \) is the parallel velocity, \( f_M = cm \exp[-\mu(\gamma - 1)] \) the Maxwellian function, with \( \int f_M(\mathbf{u})d\mathbf{u} = 1 \), \( \gamma = (1 + u^2)^{1/2} \) the relativistic factor, \( \mathbf{u} = p/mc = yv/c \) the normalized momentum, \( \mu = mc^2/T \), \( cm \) the normalization constant, \( \mathcal{E} = mc^2/\gamma \) the particle energy, and \( \tilde{\mathcal{E}} = \int \mathcal{E} f_M d\mathbf{u} \) its mean value. The term \( C(f) \) represents the linearised collisional operator, and \( \mathbf{S}_w = -D \cdot \nabla_{\mathbf{u}} f_M \) is the quasilinear diffusion flux, with \( D \) the quasilinear diffusion tensor.

Multiplying Eq. (10) by \( \mathcal{E} \), integrating over momentum, and averaging over a flux surface yields

\[
n_0 \frac{d\tilde{\mathcal{E}}}{dt} = \langle p_d \rangle
\]

(11)

where \( \langle A \rangle = \int dA/B / \int dl/B \) denotes flux surface averaging, and \( p_d \) is the dissipated power per unit volume by the waves

\[
p_d = -n_0 mc^2 \int d\mathbf{u} \gamma v_{\parallel} \cdot \mathbf{S}_w = n_0 mc^2 \int d\mathbf{u} \cdot \mathbf{S}_w \mathbf{u}/\gamma.
\]

(12)

where \( n_0 \) is the electron density.

Integration of Eq. (10) over momentum yields \( \nabla \cdot \mathbf{b} J_{\parallel} = 0 \), being \( J_{\parallel} = -e n_0 \int v_{\parallel} f(\mathbf{u})d\mathbf{u} \) the parallel current density. From \( \nabla \cdot \mathbf{B} = 0 \), one has \( J_{\parallel}/B = \mathcal{F} \), where \( \mathcal{F} \) is a flux function. Note that the following relation holds \( J_{\parallel}/B = \langle J_{\parallel}/B \rangle \).

Multiplying the relation \( J_{\parallel} = B \mathcal{F} \) by a given function of position \( \mathcal{H}(\mathbf{x}) \), and averaging over the flux surface, one obtains the following expression for \( J_{\parallel} \)

\[
J_{\parallel} = -en_0 \frac{B}{\langle \mathcal{H}/B \rangle} \left\langle \int d\mathbf{u} v_{\parallel} f(\mathbf{x}) \right\rangle
\]

(13)
We introduce the adjoint problem
\[ v_\parallel b \cdot \nabla g + C_l(g) = -v_\parallel \mathcal{H} f_M \cdot \]
where the function \( g \) is a generalized Spitzer-Harm function in toroidal geometry. By means of Eq. (14), using the self-adjoint property of the collision operator and integrating by parts, Eq. (13) becomes
\[ J_\parallel = -e n_0 \frac{B}{\mathcal{H} B} \left\langle \int d\mathbf{u} S_w \cdot \frac{\partial \chi}{\partial \mathbf{u}} \right\rangle \]
where the function \( \chi = g / f_M \) is the neoclassical response function for the current.

Introducing the dimensionless response function \( \bar{\chi} = \chi (\nu_c/c) (B_m / \langle \mathcal{H} B \rangle) \), with \( \nu_c = 4\pi n_0 e^4 \ln \Lambda / (m^2 c^3) \) the collision frequency and \( B_m \) the minimum \( B \) value on the magnetic surface, from Eqs.(12,15) the current drive efficiency \( \mathcal{R} \) can be expressed in terms of integrals in momentum space involving the quasilinear flux \( S_w \) and the response function \( \bar{\chi} \) only
\[ \mathcal{R} \equiv \frac{\langle J_\parallel \rangle}{\langle p_d \rangle} = \frac{e}{m c \nu_c B_m} \frac{\langle \int d\mathbf{u} S_w \cdot \frac{\partial \bar{\chi}}{\partial \mathbf{u}} \rangle}{\langle \int d\mathbf{u} S_w \cdot \frac{\partial \bar{\chi}}{\partial \mathbf{u}} \rangle} \]

To compute the current drive efficiency, the response function \( \chi \) needs to be determined, solving the toroidal Spitzer equation (14). In the low collisionality regime, and at lowest order in the ratio between the collision frequency and the bounce frequency (assumed much less than unity), the function \( \chi \) is zero in the trapping region, defined by the relation \( \xi_m^2 < 1 - B_m / B_M \), where \( \xi_m = u_{\parallel m} / u = \sigma (1 - u_{\perp m}^2 / u^2)^{1/2} \), \( \sigma = |u_{\parallel}| / u_{\parallel} \), and the subscript \( m \) denotes variables computed at the bottom of the magnetic well, and \( B_M \) the maximum \( B \) value on the magnetic surface. The local pitch angle is related to that at the minimum of the magnetic well by \( \xi = u_{\parallel} / u = \sigma \sqrt{1 - B / B_m (1 - \xi_m^2)} \).

At first order in banana limit, the integrability condition yields the equation for \( \bar{\chi} \) in the passing particles region
\[ \left\langle \frac{B c f_M^{-1} C(f_M \bar{\chi}) + \sigma}{B_m v_{\parallel} \nu_c} \right\rangle = 0. \]

In the literature, different approaches have been introduced to find an analytical solution to Eq. (17). With reference to ECCD [8], Cohen derived a simple analytical expression for the response function \( \chi \) making use of two approximations: the bounce-averaged collision operator is expressed in terms of the local operator at the bottom of the magnetic well, and the collision operator is evaluated in the high energy limit. The solution can be written as the product of two functions \( \bar{\chi} = F(u) H(\xi_m) \), with
\[ F(u) = \frac{u^4}{Z+5}, \quad \text{and} \quad H(\xi_m) = \xi_m \left[ 1 - \frac{\xi_T}{\xi_m} P_\alpha(\xi_m) \right], \]

where \( P_\alpha \) is a Legendre function with index \( \alpha \) satisfying \( \alpha(\alpha + 1) = -8/(Z+1) \). In computations a different expression for \( F(u) \) is used, that is a good approximation of
the high energy relativistic solution both for large and small velocities [8]. This model is expected to underestimate the ECCD efficiency in case the energy of the resonant particles be of the order of the thermal energy.

**EC ABSORBED POWER AND DRIVEN CURRENT**

When a single wave with frequency \( \omega \) is considered (although the quasilinear theory is not strictly applicable in this case), the quasilinear diffusion tensor reads [9, 10]

\[
D = D_0 \sum_n \gamma \delta (\gamma - N_\parallel u_\parallel - nY)|\Theta_n|^2 a^* a ,
\]

(19)

where \( D_0 \) is a constant, \( a = (1 - N_\parallel u_\parallel / \gamma) \hat{u}_\perp + N_\parallel u_\perp / \gamma \hat{u}_\perp \), \( Y = \Omega / \omega \), \( \Omega = eB / mc \) the cyclotron frequency, and \( \Theta_n = e_- J_{n-1} (b) + e_+ J_{n+1} (b) + e_\pm u_\parallel / u_\perp J_n (b) \) the polarization term, with \( J_n \) the Bessel function of index \( n \) and argument \( b = N_\perp u_\perp / Y \), and \( e_\pm = (e_+ \pm ie_-) / 2 \). The components \( e_x \), \( e_x \), and \( e_z \) of the polarization vector \( e = E / |E| \) are defined in a local reference system with the \( z \) axis along the magnetic field \( B \) and the \( x \) axis in the \((k, B)\) plane.

When the EC wave propagation is described in terms of single rays, the EC interaction is poloidally localized on the magnetic surface. Thus, the current drive efficiency due to the given ray can be written as

\[
R = e mc \nu c \langle B \rangle B_m \int du P(u) \eta_u(u) ,
\]

(20)

where in the above integrals all the quantities are to be computed on the given magnetic surface at the poloidal location corresponding to the ray intersection.

The functions \( P(u) \) and \( \eta_u(u) \) are the normalized absorbed power density and current drive efficiency per unit momentum \( u \) [11]

\[
P(u) = -\frac{\pi \omega^2}{\omega_\omega^2} \sum_n |\Theta_n|^2 u_\perp^2 \tilde{L} f \delta (\gamma - N_\parallel u_\parallel - nY) , \quad \eta_u(u) = \gamma \tilde{L} \tilde{Z} (u) .
\]

(21)

being \( \gamma \tilde{L} \tilde{g} = (\gamma - N_\parallel u_\parallel) \partial g / \partial u_\perp + N_\parallel \partial g / \partial u_\parallel \) a differential operator in momentum space, and \( \omega_\omega \) the plasma frequency.

At lowest order in the QO expansion parameter, the EC power \( P \) is assumed to evolve along the ray trajectory obeying to the following equation [12, 4]

\[
\frac{dP}{ds} = -\alpha P , \quad \alpha = -2 \frac{\omega}{c} \text{Im}(N_{\perp w}^2) \bigg|_{\Lambda = 0} ,
\]

(22)

where \( \alpha \) is the absorption coefficient, \( N_{\perp w} \) the perpendicular refractive index solution of the relativistic dispersion relation for EC waves \( \Lambda_w = N^2 - N_\parallel^2 - N_{\perp w}^2 = 0 \) [5]. The above equation for the power evolution (22) is formally the same as in the case of standard geometric optics. Diffraction effects are taken into account only through the QO trajectories (that are different from the GO ones), and power absorption and current
drive are computed along each QO ray independently of the adjacent rays. Integration of Eq. (22) yields the transmitted along the ray in terms of the optical depth \( \tau = \int_0^s \alpha(s')ds' \) as \( P(s) = P_0 \exp(-\tau(s)) \), being \( P_0 \) the injected power. At each position along the ray trajectory, the flux surface averaged absorbed power density can be written in terms of the absorption coefficient as

\[
\langle p_d \rangle = P_0 \alpha(s) \exp[-\tau(s)] \frac{\delta s}{\delta V}
\]  

(23)

\( \delta s \) being the ray length between two adjacent magnetic surfaces, and \( \delta V \) the volume.

In a tokamak, where the magnetic field is given by \( B = I(\psi) \nabla \phi + \nabla \phi \times \nabla \psi \), with \( \phi \) the toroidal angle, \( \theta \) a poloidal angle, and \( \psi = \psi_{pol}/2\pi \), the poloidal flux function, and the function \( I(\psi) = RB_\phi \) the poloidal current function, the driven toroidal current \( I_{cd} = \int dS_\phi \cdot J_{cd} \) can be expressed either as a volume integral or as an integral along the wave trajectory making use of Eqs.(20,23)

\[
I_{cd} = \frac{1}{2\pi} \int dV I(\psi) \left( \frac{1}{R^2} \right) \langle \frac{J_{\parallel}}{B} \rangle = \frac{P_0}{2\pi} \int_0^s ds' \left( \frac{1}{R^2} \right) \frac{I(\psi)}{B} \alpha(s') \exp[-\tau(s') \nabla \theta(s')].
\]  

(24)

From the above equation, the evolution of \( I_{cd} \) along the ray trajectory is given by

\[
\frac{dI_{cd}}{ds} = -\nabla \theta(s) \frac{1}{2\pi R_f} \frac{dP}{ds}
\]  

(25)

where \( 1/R_f = \langle 1/R^2 \rangle I(\psi)/\langle B \rangle = \langle B_\phi/R \rangle /\langle B \rangle \).

**THE GRAY CODE**

Within the theoretical framework outlined in the above sections, the GRAY code computes the QO ray trajectories and the EC power absorption and current drive of a Gaussian beam in a toroidal configuration. It has been tested in vacuum and in plasmas, with special reference to ITER scenarios. The code runs also in the geometric optics approximation, e.g., as a standard ray-tracing code. The GRAY computational effort is of the same order of that of a standard ray–tracing code with multiple rays.

The scheme of the integration algorithm is the following. The beam is described by means of \( N_T \) rays with initial conditions in a given plane as described below. Then, the equations (4,5) for the \( N_T \) rays are advanced simultaneously by a given integration step by means of a 4–th order Runge–Kutta scheme. Since the value of the function \( S_f(x) \) is conserved along each trajectory, its gradient can be numerically computed at a given ray position by means of a difference scheme based on the intersection points of the closest rays with the surface obtained mapping the initial plane. Once \( \nabla S_f \) and its derivatives are computed numerically, the ray equations are advanced by a further step, and the scheme is iterated. The numerical algorithm is described in more detail in [2, 3].

The beam initial conditions are expressed in terms of a launching position \((x_0, y_0, z_0)\), corresponding to the center of the beam, and of a “toroidal” and a “poloidal” launching angles \((\beta, \alpha)\), such that the cylindrical components of the initial refractive index vector
FIGURE 1. QO trajectories in vacuum of a divergent (a) and convergent (b) circular Gaussian beam. The beam parameters are \( f = 170 \text{ GHz}, w_{01} = w_{02} = 2 \text{ cm}, \bar{z}_{01} = \bar{z}_{02} = 0 \) (a), and \( \bar{z}_{01} = \bar{z}_{02} = +1 \text{ m} \) (b).

of the reference ray \( N_0 \) reads \( N_{R0} = -\cos \alpha \cos \beta, N_{\varphi 0} = \sin \beta, \) and \( N_{z0} = -\sin \alpha \cos \beta. \) In a reference system, in which the origin corresponds to the launching point, the \( \tilde{z} \) axis is parallel to \( N_0, \) i.e., to the propagation direction, and the axes \( \tilde{x}, \tilde{y} \) are chosen along the axes of the constant amplitude ellipse \( [S_I(\tilde{z} = 0) = \text{const, see Eq.}(9)] \), the initial positions of the \( N_T \) rays are chosen in the plane \( \tilde{z} = 0 \) uniformly distributed in the radius \( \tilde{\rho}, \) and the angle \( \theta, \) being \( \tilde{x}_0 = w_x(0)\tilde{\rho} \cos \theta, \tilde{y}_0 = w_y(0)\tilde{\rho} \sin \theta, \) and \( k_0S_I(\tilde{z} = 0) = -\tilde{\rho}^2. \)

Contrary to standard ray-tracing codes, the \( \text{GRAY} \) code allows to deal with divergent, convergent, and astigmatic beams in a very simple manner, i.e., specifying the beam parameters in vacuum, \( w_{0i}, \) and \( z_{0i}. \) Examples of QO trajectories in vacuum are shown in Fig.1 for a divergent and a convergent Gaussian beam with circular cross section. The numerical integration of the QO ray–tracing equations is quite accurate and describes diffraction effects correctly: the QO trajectories lie on the \( S_I(\tilde{z}) = \text{const} \) surface defined in Eq.(9), and do not cross in a single point in the case of a convergent beam, as it would occur in the GO approximation, in which the trajectories are straight lines.

Power absorption and current drive are computed along all the QO rays of the beam. Each ray carries a fraction of the total power such that a Gaussian power distribution is obtained at \( s = 0, \) at lowest order in \( \delta. \) The absorbed power density and the current density profiles are reconstructed on a uniform flux label array \( \rho_h, \) as \( \langle p_d \rangle = \Delta P_d / \Delta V, \) and \( \langle J_\parallel \rangle = 2\pi R_f \Delta I_{cd} / \Delta V, \) with \( \Delta P_d, \Delta I_{cd} \) the deposited power and driven current within the volume \( \Delta V \) associated to \( \rho_h. \) The total power and current densities are obtained summing up the (weighted) contributions of each ray at given \( \rho_h. \)

EC BEAM PROPAGATION, ECRH AND ECCD IN TOKAMAKS

In the following we present results of injection of ordinary mode EC waves with frequency \( f = 170 \mathrm{GHz} \) in ITER from a top launcher (located at \( R = 6.4848 \text{ m}, z = 4.11 \text{ m} \)), for the ITER Scenario 2. The beam trajectories of a convergent and a divergent beam, and the current density profiles are shown in Fig.2 for the case of oblique injection with toroidal and poloidal angles \( \beta = 22^\circ, \alpha = 63.5^\circ, \) respectively.

In most conditions of oblique top injection in ITER, the optical depth of the EC waves is quite large, and the EC power is absorbed in a relatively thin layer, well before the cold resonance condition is met. The wave–particle interaction occurs mainly at
upshifted frequencies ($\Omega > \omega$), and energy and momentum are transferred from the wave to high energy electrons with preferential parallel velocity. This fact allows “efficient” EC current drive generation even in a thermal plasma. Moreover, the resonant interaction takes place only for circulating high-energy electrons and not for trapped electrons.

For the parameters of Fig. 2, power absorption is computed solving either the fully or the weakly relativistic dispersion relation, while the current drive efficiency is computed in both cases taking into account the full relativistic resonance condition. It is found that the EC driven current is $I_{cd}/P_0 \approx -8.5$ kA/MW in the case of relativistic absorption, and $I_{cd} \approx -8.1$ kA/MW in the case of weakly relativistic absorption, the divergent and the convergent beams generating almost the same ECCD. The relativistic ECCD profile is slightly shifted at larger $\rho_t$ values with respect to the weakly relativistic profile. From Eq. (24), one has that the driven current is computed as an integral along the trajectory in which the current drive efficiency $R$ is weighted by the absorption term $\alpha \exp(-\tau)$. In the fully relativistic case, the resonance condition is met along the trajectory at an outer surface, and for higher particle energies. Both these facts contribute to explain the observed behavior. Finally, we observe that this discrepancy is seen here mainly due to the large value of the ITER major radius and to the large temperature, which both contribute to the sharp ECRH and ECCD localization.

Figures 3, and 4 show the beam widths along the propagation, the $N_\parallel$ spectrum, and the $J$ profiles of a QO convergent beam and a GO beam with the same angular width in the far field, and the same beam width in vacuum at $s = 1.6$ m, i.e., at the EC resonance. In the ECRH region, either the beam width or $N_\parallel$ values of the two GO beams are different form what obtained in the QO case, thus leading to different $J$ profiles, with the QO profile narrower & higher, and wider & lower than the GO profile in the two cases, respectively.

**CONCLUSIONS**

The theoretical framework of quasi–optical propagation of a Gaussian beam of EC waves in a tokamak plasma, and of the EC power absorption and current drive has been
FIGURE 3. Maximum and minimum beam width versus $s$ (left), $N_{i}$ values of outer rays versus $s$ (center) and EC current density profile versus $\rho_{t}$ (right). The solid curve corresponds to QO convergent beam of Fig.2 and the dashed curve to GO trajectories with the same divergence angle in the far field.

FIGURE 4. Same as in Fig.3. The GO beam has the same beam width as the QO beam at $s = 1.6$ m.

reviewed. The propagation of an astigmatic Gaussian beam is described in terms of a set of interacting rays taking into account diffraction effects. Absorption and current drive are computed along each QO ray (as in GO codes with multiple rays) and not only on the reference ray. The new code GRAY performs fast and accurate computations of ECRH and ECCD along the quasi–optical trajectories in a general tokamak equilibrium. Extensive use of GRAY has been made to compute the EC driven current in different ITER Scenarios for Neoclassical Tearing Modes stabilization [13].

REFERENCES