Fast Particle Interaction With Waves
In Fusion Plasmas

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Abstract. There are two well-known motivations for theoretical studies of fast particle interaction with waves in magnetic confinement devices. One is the challenge of avoiding strong collective losses of alpha particles and beam ions in future burning plasma experiments. The other one is the compelling need to quantitatively interpret the large amount of experimental data from JET, TFTR, JT-60U, DIII-D, and other machines. Such interpretation involves unique diagnostic opportunities offered by MHD spectroscopy. This report discusses how the present theory responds to the stated challenges and what theoretical and computational advances are required to address the outstanding problems. More specifically, this paper deals with the following topics: predictive capabilities of linear theory and simulations; theory of Alfvén cascades; diagnostic opportunities based on linear and nonlinear properties of unstable modes; interplay of kinetic and fluid nonlinearities; fast chirping phenomena for non-perturbative modes; and global transport of fast particles. Recent results are presented on some of the listed topics, although the main goal is to identify critical issues for future work.

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INTRODUCTORY REMARKS

With ITER on its way to construction [1], one might want to focus this paper on alpha particle-driven instabilities and their impact on burning plasma performance. Yet, fast particle interaction with waves is a significantly broader topic, even with regard to fusion plasmas. In present-day tokamaks, the sources of fast ions are neutral beam injection and rf heating, rather than fusion burn [2-5]. The fast particles in tokamaks can also be electrons (e.g., runaway electrons [6]), although the fast-electron-driven instabilities tend to receive not much attention in recent literature. Besides tokamaks, other magnetic fusion devices (stellarators and mirror machines) also deal with important energetic particle issues [7-9].

The fast particle population usually represents a very small fraction of all plasma particles, even when their contribution to the total plasma energy is significant. For this reason, the fast particles interact primarily with the bulk plasma rather than with each other. Being less collisional than the bulk plasma, these particles typically have a non-Maxwellian distribution, which provides free energy to excite waves in the plasma and makes fast particles susceptible to collective phenomena.

The theoretical description of fast particle instabilities largely relies on a perturbative approach that treats particles and waves as two weakly interacting entities. This implies that the waves are linear eigenmodes of the bulk plasma, so that
they have predetermined frequencies and spatial structure, regardless of fast particle properties. A relevant example is the interaction of energetic ions with toroidicity-induced Alfvén eigenmodes [10]. On the other hand, there are important examples that reveal limitations of the straightforward perturbation theory and call for a more sophisticated analysis. The energetic particle modes [11] in general and fishbone modes [12] in particular fall into that category. One of the technical difficulties, associated with these modes, is that their spatial structure can change significantly when the instability enters a nonlinear phase. The underlying reason is that the mode structure is largely determined by the fast particles even in the linear regime. This feature is also pertinent to Alfvén cascade modes in shear-reversed plasmas [13].

An interesting trait of energetic particle modes is that they can quickly change their frequencies (chirp) during nonlinear evolution. Chirping events are well documented experimentally [14, 15], and their overall pattern can be reproduced in idealized theoretical models [16-19]. The next step in their theoretical description must involve more specific modeling that would allow quantitative interpretation of particular experiments. This is a challenging but hopefully feasible task that will likely lead to unique diagnostic applications.

Regardless of specific details, the problem of wave-particle interactions in fusion machines exhibits several general features that suggest a path to relevant theory, as discussed next.

The intrinsic time-scales of fast particle instabilities are typically much shorter than the global time-scales of particle or energy confinement. For this reason, linear stability analysis of a given configuration represents only a very small (and not always the most significant) part of the global picture. An essential part of the story (not covered by linear analysis) is how the time-averaged characteristics of the system are affected by collective effects in the presence of particle sources and slow (collisional) relaxation processes. In order to resolve this issue, the theory has to characterize the long-time behavior of a driven system, rather than be limited to fast initial growth of unstable perturbations in a pre-set equilibrium configuration without sources and sinks. This situation points to the necessity of nonlinear theory, which obviously complicates the picture. On the other hand, the indicated difference in time-scales suggests that, in a weakly driven system, the instability should occur in a near-threshold regime, for which the nonlinear stage is often tractable analytically [20].

The nonlinear effects arise either from nonlinearity of the fast particle motion in the field of the excited waves or from the nonlinearity of the bulk plasma response to the perturbed fields. The first kind of nonlinearity typically requires kinetic treatment, whereas the second kind can usually be described within a fluid model. In general, the theory has to deal with the interplay between the kinetic and fluid nonlinearities. However, because of the differences in their physics origins, one of these nonlinearities may turn out to be more significant than the other. In either case, it is very important to determine whether the near-threshold instability exhibits soft or hard nonlinear regime (i.e., whether it can or cannot be stabilized by weak nonlinearity), since this distinction not only carries immediate experimental implications but also requires appropriate theoretical and numerical tools to describe the evolution of the system.
When the bulk plasma (fluid) nonlinearity is suppressed, the problem can be formulated in terms of the nonlinear interaction of fast particles with linear waves. In this case, the dominant nonlinear response is associated with wave-particle resonances. For fast ions interacting with an Alfvén mode in a tokamak, the resonance condition is

$$\omega - n\omega_{\phi}(E; P_\phi; \mu) - l\omega_{\phi}(E; P_\phi; \mu) = 0. \quad (1)$$

Here $\omega$ is the mode frequency; $n$ is the toroidal mode number; $\omega_{\phi}(E; P_\phi; \mu)$ are the toroidal and poloidal frequencies of the unperturbed ion motion; $E$, $P_\phi$ and $\mu$ are the ion energy, toroidal angular momentum, and magnetic moment, respectively; and $l$ is an integer. Equation (1) selects a thin surface in a three-dimensional space of the ion constants of motion $(E; P_\phi; \mu)$. The thickness of the resonant surface scales as the square root of the mode amplitude. A single resonance usually involves a relatively small area of phase space within a broader distribution of fast particles, which suggests that multiple resonances are needed to affect the entire fast particle population. Multiple resonances can derive from multiple unstable modes. However, even a single mode can produce multiple resonances if the unperturbed particle orbits are sufficiently complicated. The latter can be seen from Eq. (1), which allows multiple values of $l$ for a given mode.

Depending on whether wave-particle resonances overlap in phase space, one can have different scenarios of fast particle transport. When most of the relevant phase space is covered by the overlapped resonances, particles can be lost via stochastic diffusion [21]. On the other hand, undestroyed KAM surfaces between the resonances can serve as transport barriers for fast particles. This brings an interesting question of whether such barriers can be created on purpose, especially at the plasma edge. Doing so may ensure satisfactory global confinement of fast particles even if there is a local instability in the core.

Isolated resonances tend to be benign in terms of global losses of fast particles, unless the fast particle distribution in phase space is very narrow. Consequently, one can expect the fast ions from neutral beam injection to be more vulnerable than the ICRH-produced ions or fusion alpha particles. In the case of a broad fast particle distribution, there are adequate opportunities for analytical description of an isolated nonlinear wave-particle resonance. However, analytical nonlinear theory does not quite cover the case when the resonant particles modify the dispersion relation for the excited waves. This case calls for non-perturbative nonlinear treatment, which basically amounts to nonlinear studies of energetic particle modes, including those with significant frequency chirping. Mode frequency variation allows the resonance to move across the phase space together with the fast particles that are trapped within the separatrix of the resonance. This scenario can generate convective transport of selected particles, as opposed to global diffusive losses.

The rest of this paper presents an illustrative selection of wave phenomena (both benign and potentially malignant) generated by a fast ion population in a tokamak. The paper is organized as follows. It starts with additional comments on linear theory.
and then presents recent results on Alfvén cascade modes, discusses the nonlinear evolution of fishbones (as an example of interplay between kinetic and fluid resonances), and reports recent developments in modeling the anomalous transport of fast ions.

**UTILITY OF LINEAR THEORY**

Linear stability analysis is commonly viewed as an essential first step in studying wave-particle interactions. However, given its intrinsic limitations, it is appropriate to consider the question of which elements of linear analysis are sufficiently robust to be incorporated into a nonlinear description. Eigenfrequencies (dispersion relations) for weakly damped or weakly unstable perturbations constitute one such element. The eigenfrequencies are normally recognizable in experiments even when the perturbations reach nonlinear levels. It is noteworthy that the eigenfunctions are at times less robust than the eigenfrequencies. Of particular interest in this regard are shear Alfvén perturbations in cold uniform plasmas with straight magnetic field lines. Their dispersion relation allows an arbitrary spatial structure of the mode across the magnetic field because there is no communication between different magnetic surfaces (i.e., transverse components of the group velocity vanish for these perturbations).

The linear growth rates of unstable modes are apparently less robust than the mode frequencies, as they change significantly in the nonlinear stage. They are also notoriously difficult to measure directly in experiments. On the other hand, direct measurements of the damping rates for stable modes are quite feasible [22]. However, the difficulty in comparing these measurements with theory is that the calculated damping rates tend to be sensitive (often exponentially so) to variations of plasma parameters. It is therefore more practical to determine plasma parameters from this comparison, rather than predict instability thresholds quantitatively. A conceivable way around this difficulty would be to use linear codes to examine parametric trends in the damping rates and combine this information with direct measurements for reactor extrapolations.

Ideally, the expected input from linear analysis to nonlinear theory and predictions of plasma performance should include marginally stable distributions of fast particles. Of particular interest are marginal stability conditions for the entire family of relevant perturbations rather than for a limited set of modes. Although both analytical and numerical techniques are reasonably well developed to investigate the stability of individual modes, comprehensive stability assessment still remains a challenging task because of the need to perform a broad parameter scan for devices of interest.

**ALFVÉN CASCADES**

Alfvén cascades are shear Alfvén perturbations in tokamak plasmas with non-monotonic safety factor profiles [13, 23]. They are commonly observed below the toroidal Alfvén eigenmode (TAE) frequency gap. As the safety factor $q$ changes in time, Alfvén cascades characteristically exhibit a quasi-periodic pattern of
predominantly upward frequency sweeping. In low-pressure plasmas, the Alfvén cascade frequency \( \omega_{AC}(t) \) tracks the local dispersion relation for shear Alfvén waves:

\[
\omega_{AC}(t) = \frac{V_A}{R} \left| \frac{m}{q_0(t)} - n \right|
\]

(2)

where \( n \) and \( m \) are the toroidal and poloidal mode numbers, respectively, \( R \) is the major radius, \( q_0 \) is the minimum value of the safety factor, and \( V_A \) is the Alfvén velocity at the location where \( q = q_0 \). The robust relation between \( q_0(t) \) and the Alfvén cascade frequency makes Alfvén cascades a very convenient diagnostic tool for measuring temporal evolution of the safety factor [24]. This technique is routinely used in JET to facilitate the creation of internal transport barriers.

**FIGURE 1.** Left: Magnetic spectrogram showing Alfvén cascades in JET discharge #56940. Alfvén cascades with different toroidal mode numbers reach the same lowest frequency. Right: Alfvén spectral line behavior computed from theory [26], showing the effect of pressure on the mode frequency as a function of safety factor \( q_0 \). The solid curve represents the MHD continuum, and the triangular and the circular points are for \( \beta = 0.005 \) and 0.0015, respectively.

Except in the vicinity of \( q_{TAE} \equiv (2m - 1)/(2n) \), each Alfvén cascade mode consists of one predominant poloidal Fourier component. As \( q_0 \) approaches \( q_{TAE} \), toroidicity-induced coupling modifies the dispersion relation of Eq. (2) and changes the mode structure into a sum of two comparable harmonics \((m \) and \( m - 1 \)). This transition is seen as spectral line bending in Fig. 1 near the TAE gap. The corresponding theory has been developed in Ref. [25].

Observations also reveal that the Alfvén cascade spectral lines sometimes bend at low frequencies, which is another significant deviation from the shear Alfvén wave dispersion relation. There are three underlying reasons for such bending: the geodesic acoustic effect [26], the pressure gradient effect [27], and the non-MHD response of fast ions with large orbits [13].

The preferred direction of Alfvén cascade sweeping indicates the existence of a radial potential well for the upward sweeping eigenmodes, as opposed to a potential hill for the downward sweeping perturbations. The hill-to-well transition appears as a frequency rollover in Alfvén cascade observations in JET plasmas (see Fig. 1). The rollover and the downward sweeping can be interpreted in terms of Alfvén cascade
quasi-modes that arise on the potential hill and stay there transiently prior to damping at the Alfvén continuum resonance. In the limit of large mode numbers \( n \) and \( m \), both Alfvén cascade modes and quasi-modes are governed by the same wave equation

\[
\frac{\partial}{\partial r} \left[ \omega^2 - \omega_G^2 - \frac{V_A^2}{R^2} \left( n - \frac{m}{q} \right)^2 \right] \frac{\partial \Phi}{\partial r} - \frac{m^2}{r^2} \left[ \omega^2 - \omega_G^2 - \frac{V_A^2}{R^2} \left( n - \frac{m}{q} \right)^2 - \omega^2 \right] \Phi = 0, \quad (3)
\]

where \( \omega_G^2 \equiv \frac{2}{MR^2} \left( T_e + \frac{7}{4} T_i \right) \) is the square of the geodesic acoustic frequency and

\[
\omega^2 \equiv -\frac{2}{MR^2} r \frac{d}{dr} \left( T_e + T_i \right) - \frac{\omega_q e B}{m} r \frac{d}{dr} \left( \frac{n_{\text{fast}}}{n_{\text{plasma}}} \right) \]

is an offset arising from the plasma pressure gradient and from the fast ion response in the large orbit limit. Assuming that the quasi-mode is localized near the zero shear point \( r_0 \) such that \( |r - r_0| << r_0 \), one can expand the safety factor \( q(r) \) around \( r_0 \) and look for a radially extended solution of Eq. (3) with \( (r_0 / m) << |r - r_0| << r_0 \). This requires that \( \omega^2 \) be very close to

\[
\omega_0^2 \equiv \omega_G^2 + \frac{V_A^2}{q_0^2 R^2} \left( nq_0 - m \right)^2 + \omega^2. \quad (4)
\]

One can then neglect small radially dependent quantities in the second derivative term of Eq. (3). With these simplifications, Eq. (3) reduces to the non-dimensional form

\[
\frac{\partial^2 \Phi}{\partial z^2} = \left[ \lambda - \eta z^2 - z^4 \right] \Phi, \quad (5)
\]

where \( z \equiv \frac{r - r_0}{r_0} \left[ \frac{V_A^2}{\omega^2 q_0^2 R^2} \left( \frac{mn^2 q_0^2}{2} \right)^{1/6} \right] \) is a dimensionless radial coordinate and \( \eta \) and \( \lambda \) are dimensionless parameters defined as

\[
\eta \equiv (nq_0 - m) \left( \frac{m^2}{mn^2 q_0^2 \omega^2 q_0^2 R^2} \right)^{1/3}, \quad \lambda \equiv \frac{\omega^2 - \omega_G^2}{\omega^2} \left( \frac{\omega q R 2q_0}{V_A q_0^2 R^2} \right)^{2/3} \left( \frac{m^2}{n} \right)^{2/3}. \quad (6)
\]

The upward sweeping Alfvén cascade modes are solutions of Eq. (5) with \( \eta < 0 \), whereas Alfvén cascade quasi-modes represent solutions with \( \eta > 0 \) and with radiative boundary conditions at infinity. Let \( \lambda(\eta) \) be a complex eigenvalue of Eq. (5) for \( \eta > 0 \). Of particular interest is the eigenvalue with the lowest imaginary part, corresponding to the weakest damping. In the limit of large \( \eta \), i.e., when \( q_0 \) is sufficiently far from the \( m/n \) rational surface, Eq. (5) reduces to the Schrödinger
equation for an inverted pendulum. The resulting eigenfunction and eigenvalue are

\[ \Phi = \exp\left(-i \frac{z^2}{2\eta^{1/4}}\right); \quad \lambda = -i\eta^{-1/4}. \]  

(7)

Equation (7) leads to the following expression for the damping rate:

\[ \gamma = \frac{1}{2\eta^{1/4}} \frac{\omega_C^2 + \omega_V^2}{\omega_0} \left( \frac{\omega_C^2}{\omega_0^2 + \omega_V^2} + \frac{V_A}{R\sqrt{\omega_C^2 + \omega_V^2}} \frac{r_0^2 q_0''}{2q_0} \right)^{2/3} \left( \frac{n}{m^2} \right)^{2/3}. \]  

(8)

The opposite limiting case of small \( \eta \) corresponds to perturbations near the rational magnetic surface with \( q_0 = m / n \). In this limit, one can set \( \eta = 0 \) in Eq. (5) to obtain:

\[ \frac{\partial^2 \Phi}{\partial z^2} = (\lambda - z^4)\Phi. \]  

(9)

The least damped quasi-mode solution of Eq. (9) has \( \text{Im} \lambda \approx 0.57 \), with the following dependence of the damping rate on plasma parameters:

\[ \gamma_{\eta=0} = -\frac{0.57}{2} \sqrt{\omega_C^2 + \omega_V^2} \left( \frac{\omega_C^2}{\omega_0^2 + \omega_V^2} + \frac{V_A}{R\sqrt{\omega_C^2 + \omega_V^2}} \frac{r_0^2 q_0''}{2q_0} \right)^{2/3} \left( \frac{1}{mq_0} \right)^{2/3}. \]  

(10)

The characteristic scale-length of this quasi-mode and the distance from the rational surface to the Alfvén continuum resonance are given by

\[ m \frac{r_{\text{continuum}} - r_0}{r_0} \sim \left( mq_0 \frac{2q_0}{r_0^2 q_0''} \frac{\omega_C R}{V_A} \right)^{1/2} \gg m \frac{r_{\text{mode}} - r_0}{r_0} \sim \left( mq_0 \frac{2q_0}{r_0^2 q_0''} \frac{\omega_C R}{V_A} \right)^{1/3} \gg 1. \]  

(11)

These estimates show that it is indeed allowable to use radiative boundary conditions in finding the quasi-mode, and that the quasi-mode lifetime is determined by the time the perturbation stays on the top of the potential hill before it slides down to the Alfvén continuum resonance. The continuum resonance acts as a perfect absorber for the incoming wave.

As seen from Eq. (4), the measured deviation of the Alfvén cascade frequency from the idealized dispersion relation of Eq. (2) can apparently be used to determine the ion temperature, the electron temperature, or fast ion parameters, depending on which contribution dominates in Eq. (4).

Another interesting opportunity for a plasma diagnostic involves nonlinear sidebands of Alfvén cascades. In recent experiments on Alcator C-Mod [28], measurements of density fluctuations with Phase Contrast Imaging (PCI) show a second harmonic of the basic Alfvén cascade signal. This second harmonic
Perturbation can be interpreted as a nonlinear sideband produced by the Alfvén cascade eigenmode via quadratic terms in the magnetohydrodynamic equations [29]. The signal at $2\omega$ is nearly resonant with the $(2m;2n)$ branch of the Alfvén continuum.

The resulting enhancement of the second harmonic is counteracted by the relatively weak non-linearity of the shear Alfvén wave. For shear Alfvén perturbations in a uniform equilibrium magnetic field, the quadratic terms $4\pi \rho (\mathbf{v} \cdot \nabla) \mathbf{v}$ and $(\mathbf{B} \cdot \nabla) \mathbf{B}$ tend to cancel in the momentum balance equation. For this reason, special care is needed to include magnetic curvature effects properly and to evaluate the coupling between shear Alfvén perturbations and compressional perturbations. It turns out that in a low-pressure plasma the non-linear coupling to compressional Alfvén and acoustic perturbations can be neglected in the calculation of the second harmonic density perturbation $\rho_{2\omega}$. As shown in Ref. [29], the ratio of $\rho_{2\omega}$ to the first harmonic density perturbation $\rho_{\omega}$ is roughly $\rho_{2\omega} / \rho_{\omega} \approx m q_0 (|\delta B_{\omega}| / B_0)(R / r_0)$, where $|\delta B_{\omega}|$ is the perturbed magnetic field at the fundamental frequency. The expressions derived for $\rho_{2\omega} / \rho_{\omega}$ can thus potentially be used together with PCI data to determine the perturbed fields inside the plasma, rather than just at the plasma edge as with magnetic probes.

**FISHBONES**

The self-consistent treatment of both kinetic and also MHD nonlinearities has long been a challenging technical issue for numerical modeling. The main difficulty here comes from the need to incorporate an accurate description of the narrow phase-space resonances into global MHD simulations. For linear problems, this difficulty is only a moderate obstacle since the resonant response of the system is often insensitive to the width of the resonance and can be treated in terms of Landau damping. In contrast, nonlinear problems typically require much better resolution for the resonant response, which is prohibitively demanding for any of the existing global codes.

Several attempts have been made to address this issue. Two earlier efforts investigated the fishbone mode—which is an excellent example of a problem with both types of nonlinearities—either by using a nonlinear fast particle pusher with

![Figure 2. Second harmonic of Alfvén cascade is observed in Alcator C-Mod [28] on the PCI diagnostic (left), but not with external magnetics coils (center). The calculated radial profiles of the nonlinear stress ($T^2$) and the radial plasma displacement at the second harmonic ($\Phi_2$) are shown on the right.](image-url)
linear MHD [30] or by using a nonlinear MHD response with a linear description for energetic particles [31]. The full-geometry M3D code has both nonlinear MHD and nonlinear energetic particles, but encounters the resolution difficulty described above [32]. The challenge is to include kinetic phase-space resonances on an equal footing with fluid nonlinearities, while overcoming the resolution issue.

The MHD response of the background plasma generally involves diamagnetic effects associated with thermal ions, creating a distinction between the “diamagnetic” [33] and “precessional” [12] fishbones. For the latter, the diamagnetic effects are negligible, which is here assumed to be the case.

Experimental data on precessional fishbones [34] exhibit a robust pattern with several elements that call for theoretical interpretation. These elements are: (1) explosive initial growth of the fishbone pulse, (2) saturation of the pulse, (3) downward frequency chirping during pulse decay, and (4) recovery between subsequent pulses.

At the onset of the fishbone pulse, the linear drive from fast particles is almost balanced by continuum damping near the \( q = 1 \) resonant surface. To be more accurate, there two closely located damping resonant surfaces in the near-threshold regime [31]: one slightly inside and one slightly outside the \( q = 1 \) surface (see Fig. 3). Their locations are determined by the condition \((V_A / qR)|q - 1| = \omega_{ih}\), where \( \omega_{ih} \) is the mode frequency at the instability threshold. The value of \( \omega_{ih} \) is comparable to the fast particle precessional frequency. As the mode amplitude grows, the nonlinearity of the system becomes increasingly important. As can be seen from back-of-the-envelope estimates, the MHD nonlinearity dominates over the fast particle nonlinearity at a sufficiently early stage of the fishbone pulse. This early stage can be adequately described analytically within a weakly nonlinear approximation, which shows that the MHD nonlinearity plays a destabilizing role, leading to an explosive growth of the pulse. The accelerated growth effectively broadens the two resonances shown in Fig. 3. The weakly nonlinear approximation holds as long as the two neighboring resonances remain well separated. At the limit of its applicability the amplitude of the radial displacement \( \xi \) is on the order of \( \xi \sim r_{q=1} \omega_{ih} R / V_A \ll r_{q=1} \). At this point the fast

![FIGURE 3. Spatial structure of the fishbone perturbation in the near-threshold regime [31]. The real (top, red) and imaginary (bottom, blue) parts of the radial velocity profile show two well-separated resonant sub-layers near the \( q=1 \) surface. The width of each sub-layer is proportional to the mode growth rate. The distance between the sub-layers is proportional to the mode frequency.](image)
particle nonlinearity is still negligible, so the particles continue to drive the mode. One can then expect that the mode will grow somewhat beyond the level of \( \xi \sim r_q(\omega_p R / V_A) \) but the dynamics of this growth will now be different due to the non-perturbative nature of the MHD nonlinearity. It is conceivable that the structure of the MHD resonant layer will resemble a magnetic island of the kind described in Ref. [35] and the fast particles will force this island to grow until there is no free energy left in the fast particle distribution. In other words, the growth of the fishbone pulse can only stop when the fast particle nonlinearity flattens the phase-space distribution of particles near the kinetic resonance. Analytical theory has not yet been able to make credible predictions for the mode saturation level. Numerical simulations demonstrate mode saturation due to fast particle nonlinearity, both with and without MHD nonlinearity in the code. However, numerical viscosity in the global nonlinear MHD code is currently too high to adequately describe the structure of the narrow resonant layer. We thus have an unresolved issue of predicting the mode saturation level in terms of plasma parameters and those of the fast particles.

The above description suggests that the mode saturates when the fast particle drive switches off due to nonlinear modification of the fast particle distribution. Assuming that this modification is irreversible (due to fast particle phase mixing), one can conjecture that the subsequent dynamics of the pulse should be similar to that of a nonlinear pendulum in presence of dissipation. It is therefore natural that the mode frequency changes during the decay of the pulse since the frequency should depend on the mode amplitude. The fact that the frequency goes down can then be viewed as a reversal of the upward chirping predicted by the weakly nonlinear description of the explosive initial growth of the pulse [31].

Another open issue for fishbones is quantitative modeling of recurrent pulses in presence of fast particle sources and sinks. This problem is apparently more demanding computationally than the description of a single pulse because of the multiple time-scales involved.

**TAE-INDUCED LOSSES OF FAST IONS**

A single unstable Alfvén mode tends to be benign in terms of global losses of energetic particles. The fundamental reason for this is that the resonances associated with a single mode occupy only a small fraction of the particle phase space. Many modes are usually needed to achieve resonance overlap in a large part of phase space and thereby produce global diffusion.

It has been theoretically predicted [36] and numerically demonstrated [37-40] that particle trapping causes saturation of toroidal Alfvén eigenmodes (TAE). This enables reduced simulations of TAEs with eigenfunctions and damping rates (in the absence of fast ions) predetermined by linear MHD-calculations. The reduced numerical model reproduces many aspects of the TAE bursts in the Tokamak Fusion Test Reactor experiment [41], such as the timing of bursts, synchronization of bursts for different modes, and the total energy of the confined particles. Another important observation in these simulations is that co-moving particles can support large pressure gradients as they stick out of the plasma where there is no Alfvén perturbation. The simulations
show that counter-moving particles are readily lost to the walls, while the co-moving particles remain confined, even with very strong TAE activity in the plasma core.

**FIGURE 4.** Phase space resonances for beam ions in presence of multiple TAE modes. Left: Isolated resonances for low-amplitude modes ($\delta B/B \sim 10^{-3}$). Right: Strongly overlapped resonances for high-amplitude modes ($\delta B/B \sim 2 \times 10^{-2}$).

However, the calculated saturation amplitude of $\delta B/B \sim 2 \times 10^{-2}$ was substantially higher than the value $\delta B/B \sim 10^{-3}$ inferred from the measured plasma displacement. This discrepancy can be attributed to the absence of nonlinear mode-mode coupling in the simulation. The missing nonlinearity is expected to facilitate saturation of TAEs [42-45], especially at increasing mode amplitude.

**FIGURE 5.** Effect of MHD nonlinearity on toroidal Alfvén mode saturation. The time evolution of the wave energy in a single burst is shown for two simulation runs. Left: Several TAE modes with different mode numbers $n$ are used simultaneously; the spatial structures of the modes are obtained from linear theory. Right: The inclusion of MHD-type nonlinearity reduces the saturation level of the dominant mode ($n=4$) and generates a strong zonal flow ($n=0$).

Recently, comparative simulations with and without MHD nonlinearity have been carried out to investigate mode-coupling effects (e.g., coupling to the $n=0$ magnetic field and zonal flow) [46]. They demonstrate (so far on a short time interval) that the MHD nonlinearity tends to reduce the TAE saturation level. The pattern of synchronized bursts for multiple TAEs is preserved with the MHD nonlinearity [47]. Thus, the inclusion of MHD nonlinearity brings simulation results closer to the experimentally inferred saturation amplitude. These simulations will be continued in order to examine the effects of MHD nonlinearity on a longer (i.e., experimentally relevant) time interval.
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REFERENCES