FAST ION TRANSPORT AND PLASMA ROTATION IN ION-CYCLotron HEATED PLASMAS

V.S. Chan, S.C. Chiu,* T.K. Mau, † Y.A. Omelchenko,§ Y. Sentoku

General Atomics, P.O. Box 85608, San Diego, CA 92186-5608
*Sunrise R&M, Inc., 8585 Hopseed Lane, San Diego, California 92129
†University of California, San Diego, California
§Dynamics Research Corp., 11681-1 Compass Point Drive N., San Diego, California 92126

Abstract. Minority ion cyclotron heating can produce energetic ions with banana orbits that are finite compared with the minor radius of a tokamak. The radial transport of the fast ions in the presence of Coulomb collisions results in a radial current and a corresponding J×B torque density on the bulk plasma. Collisions between the minority ions and majority ions provide an additional frictional torque that adds to or opposes the magnetic torque. Using a code that follows the particle drift trajectories in a tokamak geometry under the influence of rf fields and collisions, modeled with an rf quasi-linear operator and a Monte-Carlo operator respectively, it is shown that a finite central rotation velocity can result even when the volume integrated torque density is small. This is consistent with the results of [1] for the case of a symmetric toroidal wavenumber (n_φ) spectrum. A physical picture emerges explaining the co- and counter-rotation with low- and high-field resonance, respectively, as a consequence of finite orbit width. For an asymmetric spectrum, it is found that when n_φ is in the co-current direction, the rf produces a net co-direction torque leading to co-rotation for both low- and high-field side resonance. With negative n_φ, the rotation reverses to the counter-current direction. An analysis of antenna coupling has identified conditions when a positive n_φ power spectrum is favored, which might partially explain the observations in the C–Mod and JET tokamak.

INTRODUCTION

Ion cyclotron resonance heating (ICRH) has been observed to alter the poloidal and toroidal rotation of the bulk plasma and induce a radial electric field in many experiments [2]. Understanding the mechanism causing the changes in plasma rotation observed in ion cyclotron wave heated discharges is very important for developing reliable radio frequency (rf) techniques for plasma control. Recently, rotation velocities of up to 1.3×10^5 m/s in the co-current direction have been observed in H–mode discharges with no direct momentum input on Alcator C–Mod with ICRF heating alone [3]. The presence of a substantial core radial electric field as inferred from measurements suggests inward movement of ions possibly induced by ICRF. When the ICRF is injected to the plasma off-axis on the high field side, it results in the density rising abruptly and becoming peaked [4]. The central toroidal rotation
velocity decreases and changes sign from co-direction to counter-direction as the density rises. The influence of ICRF on the toroidal rotation profile is also studied in the JET tokamak [5]. JET plasmas with ICRF heating only show distinct structures in the toroidal rotation profile that peaks off-axis when the cyclotron resonance layer is far off-axis. The rotation is dominantly co-current in agreement with the Alcator C–Mod result. A more central deposition of the ICRF power and operation in the H–mode both lead to more centrally peaked profiles. Additionally, MHD modes have a strong influence on the rotation profile on JET.

Since similar rotation behavior is observed during Ohmic H–mode on Alcator C–Mod, it has been argued that ICRF is not a direct cause of the change in rotation [6]. Another theory proposes turbulence-induced inward momentum pinch as an explanation [7]. Yet, the physical mechanisms as originally discussed by Chang [8] and Perkins [1] remain appealing and have been validated by more detailed theoretical studies [9,10]. The thrust of our continuing research is to include more realistic refinements to the model with the aim of explaining as many of the experimentally observed features as possible and to suggest ideas for future experiments. This paper focuses on the possible cause of an asymmetric power spectrum propagating inside the plasma that favors the co-direction and its impact on the plasma rotation under changing conditions. The paper starts with a review of the physics in the simulation code and reproduces the results proposed by Perkins [1] and Chang [8]. A discussion of an antenna coupling analysis follows, which demonstrates the possibility of a toroidally asymmetric power spectrum launched into the plasma under certain edge plasma conditions. Simulation results of the ICRF induced plasma rotation are presented for an asymmetric power spectrum, and correlations with experiments are identified. We conclude with a summary and future directions.

**SIMULATION OF RF-INDUCED PLASMA ROTATION**

**Theory**

We begin by first identifying a variety of mechanisms that can affect plasma rotation as they appear in the flux-surface averaged toroidal momentum equation

\[
\frac{d}{dt} \langle U_\phi R \rangle = -\frac{1}{c} \langle j_F \cdot \nabla \psi \rangle + \langle R_e \phi \cdot F_C \rangle + \langle R_e \phi \cdot F_W \rangle - \langle R_e \phi \cdot (\nabla \cdot \pi_i) \rangle. \tag{1}
\]

Here, \( U_\phi \) is the flow velocity with the subscript \( \phi \) indicating the toroidal direction, \( F_C \) is the frictional force acting on the electrons and ions by the fast ions, \( F_W \) represents collisionless momentum input by the waves, and \( \langle j_p \cdot \nabla \psi \rangle \sim -\langle j_f \cdot \nabla \psi \rangle \) is used with \( j_p \) being the plasma current and \( j_f \), the fast ion current. This term accounts for any mechanism that produces a radial particle current for fast ions, which imposes a \( J \times B \) torque on the bulk plasma. The last term on the RHS is the toroidal component of the stress tensor and may be related to an empirical momentum confinement time \( \tau_\phi \) by setting
\[
\langle R \hat{c}_\phi \cdot (\nabla \cdot \pi_i) \rangle \approx M_i n_i \frac{\langle U_{\phi R} \rangle}{\tau_\phi} .
\] (2)

Using Eq. (2) in Eq. (1) yields momentum balance on a local flux surface. We can also write
the last term in differential form

\[
\langle R \hat{c}_\phi \cdot (\nabla \cdot \pi_i) \rangle = \frac{d}{dr} \varphi (\chi_m) \frac{d}{dr} M_i n_i U_{\phi} .
\] (3)

which, when combined with Eq. (1), can be solved for the toroidal rotation profile subject to
appropriate boundary conditions and choice for the momentum diffusivity, \(\chi_m\).

Depending on the physical mechanism under consideration, some terms on the RHS of
Eq. (1) may be more important than others. The \(F_w\) term would be dominant if there is direct
momentum input by resonant or non-resonant wave processes. The accretion theory of
“spontaneous” rotation [7] would introduce a momentum source at the boundary, and
unstable electrostatic waves would supply a \(\chi_m\) as well as an inward pinch, leading to inward
momentum transport. For this paper, we shall focus on mechanisms related to ICRH of
minority ion species. Substituting Eq. (3) for Eq. (1), neglecting the third term involving
collisionless momentum input by the waves, and setting the LHS to zero, we arrive at the
steady-state equation for calculating the toroidal rotation velocity. The momentum diffusivity
is assumed to be the same as the heat diffusivity and a Monte-Carlo rf quasi-linear code
ORBIT-RF [9] is used to calculate both the \(J \times B\) (magnetic) torque and the frictional torque.

**Simulation**

At present, ORBIT-RF assumes the ICRF wave amplitude and pattern to be fixed both in
space and time. It retains only the left-handed part of the wave electric field, \(E_+\), since the
right-handed component \(E_-\) exerts a less significant adiabatic force on the resonant particles
and the parallel electric field, \(E_\parallel\) is small. The standard Stix estimate [11] is used to relate the
electric field magnitude to the absorbed rf power. The minority ions are treated as Monte-
Carlo test particles coupled with a background plasma through the pitch-angle scattering and
slowing-down collisions. Particle trajectories are calculated by (i) following their adiabatic
drift motion in a static equilibrium magnetic field, (ii) subjecting them to drag and pitch-
angle scattering collisions, (iii) simulating their resonant stochastic heating by the ICRF wave
field. The resonant particle interaction with the ICRF wave is assumed a random walk pro-
cess in velocity space. In our present model, parallel ICRH heating is neglected due to small
wave \(k_\parallel\). Therefore, each time an ion passes through the ion cyclotron resonance locations

\[
\omega_{rf} - k_\parallel v_\parallel = \Omega(B) ,
\]

\[
k_\parallel = \pm k_\parallel^{rf} ,
\] (4)
only its perpendicular velocity component undergoes a random change, $\Delta v_\perp$. This increment can be readily obtained by calculating its mean value and variance from the quasi-linear equation governing the rf-induced particle diffusion in velocity space (for simplicity $k_\perp = 0$ is assumed).

Several key quantities are computed that include the volume-integrated torque, $T$

$$T(\psi) = \int_0^w d\psi' \int \frac{d\ell 2\pi R}{\nabla \psi'} \tau,$$  \hfill (5)

which is the sum of a volume-integrated magnetic torque [first term on RHS of Eq. (1)], $T_M$

$$T_M(\psi) = \frac{e}{c} \int_0^w d\psi' N(\psi'),$$  \hfill (6)

where $N(\psi)$ is the rate of change of the fast ion density inside the volume defined by $\psi$, and a volume-integrated frictional torque [second term on RHS of Eq. (1)], $T_c$

$$T_c(\psi) = \pm \frac{e}{c} \sum_{\text{test particles}} \left( \rho \cdot g \right)_{\psi'},$$  \hfill (7)

where $g = BzR$.

Volume integral of $T$ yields the angular rotation velocity for the bulk ion

$$\Omega(\psi) = \int_\psi^{\psi_{\text{max}}} d\psi' \frac{T(\psi')}{\nabla' \left( \frac{\nabla R^2}{\chi_M(\nabla \psi')^2} \right)}.$$

\hfill (8)

**Comparison with Other Theories**

One of the main findings in [1] is the plasma rotates in the co-current direction on-axis when the cyclotron resonance is on the low-field side (LFS), and in the counter-direction on-axis when the resonance is on the high-field side (HFS). When parameters are chosen to minimize the net torque to reproduce the same situation, our simulation finds a counter-rotation velocity on-axis with the resonance at the HFS, in agreement with [1]. The integrated magnetic torque is positive on the inside and negative on the outside, consistent with fast ions scattered inward and outward, compensated by equal and opposite return currents in the bulk plasma. The frictional torque is negative on the inside and positive on the outside. Our analysis shows that two mechanisms are responsible. When a banana fast ion is scattered inward, the corresponding rate of change of the precessional velocity is negative; hence, a negative frictional force is imparted by the fast ions on the bulk plasma. The opposite is true for a banana orbit scattered outward. Secondly, HFS resonance tends to energize barely trapped particles that are readily detrapped by collisions. This process creates more counter-passing fast ions on the inside and co-passing fast ions on the outside that also contributes to
the shape of the frictional torque. The net torque obtained by combining the magnetic and frictional torque is negative and does not approach zero for \( r/a = 1 \) because of the stochastic rf diffusion suggested by [8].

When the cyclotron resonance is moved to the LFS, the rotation velocity on-axis changes to the co-direction, again in agreement with [1]. The magnetic torque profile is similar to the HFS case, however, the frictional torque is quite different. Specifically, the frictional torque is positive inside the resonance. This suggests that some mechanism other than the precessional force (counter-direction) is dominant inside the resonant layer. For LFS resonance, the rf heats deeply trapped particles that are not easily detrapped by collisions. From finite drift orbit analysis [12], particles scattered inward can either remain trapped or become co-passing (so called “potato” orbits). At high enough energy, the co-passing orbits are favored resulting in a co-current frictional torque on the bulk. Similar conclusion has been reached by [10].

Our analysis supports qualitatively the toroidal rotation features that are predicted by the equations describing the complex interactions among the ICRF waves, Coulomb collisions and finite orbit energetic ions in tokamak geometry. However, the predictions are inconsistent with the experiments in a number of aspects. First and foremost, the experiments observe co-current rotation both for LFS and HFS off-axis heating. Although in Alcator C–Mod the rotation directions changes when the internal density transport barrier is formed. In addition, JET observes rotational profiles that peak off-axis when the resonance is far off-axis, but peak centrally when the resonance is near the center of the plasma. JET also reports only minor differences when the antenna phasing changes from symmetric to asymmetric favoring either the co- or counter-direction. It is clear that additional physics is needed to reconcile with experiment if ICRF is indeed responsible for affecting the plasma rotation. One aspect that has not been considered is the possibility of a toroidally asymmetric power spectrum launched into the plasma with a symmetric antenna phasing and the consequence of that on the rotation.

**ANTENNA COUPLING ANALYSIS**

The possibility that a toroidally symmetric antenna array can produce a strongly asymmetric power spectrum is first noted by [13] in the context of fast wave current drive. The source of the difference in the co- and counter-current spectrum lies in the natural asymmetry of the antenna’s radiated power spectrum in the direction perpendicular to the ambient magnetic field. This asymmetry is attributed to the Hall terms in the wave equations in the presence of absorption (collisional or collisionless). In tokamak geometry, the magnetic field in front of the antenna array is tilted at an angle to the toroidal direction, consequently, the up-down asymmetry acquires a toroidal component. The result is that plasma absorption (i.e., antenna loading) is shifted and or skewed toward a preferred
direction, independent of the direction of the magnetic field. In the fast wave current drive situation studied by [13], the preferred direction is the co-current drive direction, i.e., the toroidal power spectrum peaks in the counter-current direction. For ICRF minority heating regime, we are interested in whether the toroidal power spectrum can peak in the co-current direction.

Our analysis is done using a numerical code developed to calculate the loading of a loop antenna in three-dimensional geometry [14]. The code is modified to handle a magnetic field tilted at an angle to the up-down axis of symmetry of the loop antenna in order to model the rotational transform of a tokamak. The near fields of the antenna are matched to the plasma surface impedance calculated from a code developed by [15]. Dissipation in the plasma is included in the surface impedance so the intrinsic asymmetry of the power spectrum perpendicular to the B-field is accounted for. Besides the antenna parameters, the density and temperature pedestal and their gradient lengths in front of the antenna can be separately adjusted to isolate the individual effect on the power spectrum. Our first finding is that the toroidal power spectrum characteristic changes with the pedestal density. Figure 1 shows the power spectra with a pedestal density of (a) $1 \times 10^{13}$ cm$^{-3}$ and (b) $1 \times 10^{14}$ cm$^{-3}$. At low pedestal density, the power spectrum is nearly symmetric, but at high pedestal density, a strong asymmetry in favor of the co-current direction appears. This is consistent with increasing loading with increasing density hence the intrinsic asymmetry due to the Hall term is more pronounced. Changing the central density of the plasma only modifies this characteristic slightly, and changing the direction of the current (poloidal field) reverses the peak of the power spectrum [Fig. 2(a)] as pointed out by [13]. A second finding is the power spectrum is sensitive to the antenna height and edge density gradient. Figure 2 shows the power spectra with a gradient length $L_n$ of (a) 25 cm and (b) 0.8 cm. The spectrum for the more gentle gradient length has a strong asymmetry in the co-current direction, while the spectrum with a strong gradient length has a weaker asymmetry with the peak in the counter-current direction. This is consistent with a higher density in front of the antenna yields a stronger asymmetry in the co-current direction. Finally, Fig. 3 depicts the power spectra for an antenna height $\ell$ of (a) 5 cm and (b) 48 cm, which shows increasing asymmetry with decreasing antenna height. This validates that the poloidal mode loading is responsible for the strong asymmetry. In summary, to produce a power spectrum that peaks in the co-direction with symmetric antenna array phasing, it requires a high pedestal density that extends close to the antenna and an antenna that excites a large number of poloidal modes. It would be useful to check if these conditions are satisfied in experiments.
Fig. 1. The power spectrum characteristic changes with the pedestal density.

Fig. 2. The power spectrum is sensitive to the edge density gradient (antenna height $\ell = 5$ cm).

Fig. 3. The power spectrum is sensitive to the antenna height (edge density gradient length $L_n = 25$ cm).
ORBIT-RF RESULTS WITH ASYMMETRIC SPECTRUM

Unlike JET, only a symmetrically phased antenna array has been used in Alcator C–Mod. It is of interest to investigate the impact of the coupling-induced asymmetry. The consequence of an asymmetric power spectrum on the plasma rotation is studied using the ORBIT-RF code. We first simulate an H–mode-like Alcator C–Mod plasma with a fairly flat density profile (central density = $4 \times 10^{14}$ cm$^{-3}$) and $B_{T0} = 4.5$ T. This corresponds to an ICRF resonance on the HFS at $r/a=0.5$. For a positive $n_\phi = 10$, the ICRF produces a co-current central rotation velocity with the rotation profile as shown in Fig. 4. Figure 5 shows (a) the velocity distribution and (b) the spatial distribution of the energetic minority ions heated by the ICRF. With positive $n_\phi$, both co-moving passing ions and trapped ions are heated. The barely trapped ions can easily be detrapped by collisions that scatter them radially inward and outward. The co-rotation results from the production of more co-moving fast ions that affects the frictional torque on the bulk plasma. We also studied the case of $n_\phi=-10$. A negative $n_\phi$ produces a counter-current rotation under similar parameters.

Fig. 4. Rotation is in co-direction with $B_T = 4.5$ T (resonance at $r/a = 0.5$) and flat density profile.

Fig. 5. The co-rotation results from the production of more co-moving fast ions that affects the frictional torque on the bulk.
We surmise that the rotational shear (Fig. 4) produced by the ICRF is sufficiently strong to suppress the microturbulence near the resonance layer, leading to the formation of an internal density transport barrier and a strong peaking of the central density. Although the direction of the $n_\phi$ asymmetry is not affected by the change in central density, the dynamics of the energetic ion orbits and its interactions with Coulomb collisions and ICRF are expected to be modified. Changing the simulation conditions to model an internal transport barrier plasma with a central density of $8\times10^{14}$ cm$^{-3}$ and a peaked density profile, ORBIT-RF predicts a strong negative integrated torque primarily due to a large magnetic ($J\times B$) torque, and a counter-current rotation (Fig. 6). Figure 7 compares the spatial fast ion distribution for (a) flat density profile and (b) peaked density profile. One concludes that fast ions are driven preferentially outward for a peaked density profile by the ICRF. The large outward radial current leads to a negative net torque and the reversal of the rotational velocity direction.

![Fig. 6. Rotation changes to counter-direction with peaked density profile typical of internal transport barrier, $n_c = 8 \times 10^{14}$ cm$^{-3}$.](image_url)

![Fig. 7. The fast ions are driven preferentially outward for a peaked density profile. Fast ion spatial distribution for (a) flat density profile and (b) peaked density profile.](image_url)
Finally, we report some preliminary results with finite $n_\phi$ for JET. The parameters chosen are typical of the JET experiment [5]. Figure 8(a) shows a case with a symmetric $n_\phi$ spectrum. It produces a co-rotation with resonance close to axis and a counter-rotation with resonance far off-axis. Figure 8(b) shows an asymmetric spectrum with $n_\phi$ in the co-direction only. The result is drastically different with a co-rotation peaked off-axis. This may be explained by the excitation of co-moving energetic ions on the outside of the resonance due to the Doppler effect. The comparison also points out that the balance (cancellation) between the magnetic and frictional torque is very delicate. The consequence for a high density, high B-field machine like Alcator C–Mod, and a lower density and B-field machine like JET can be very different. Certainly more study is needed to understand this.

![Normalized rotation profile for off-axis LFS resonance with (a) symmetric $n_\phi$ and (b) positive $n_\phi$ in a JET-like plasma ($n_\phi = +10$).](image)

**Fig. 8.** Normalized rotation profile for off-axis LFS resonance with (a) symmetric $n_\phi$ and (b) positive $n_\phi$ in a JET-like plasma ($n_\phi = +10$).

**SUMMARY**

Our study is motivated by the appealing physical prediction that the toroidal rotation velocity in a tokamak can be significantly modified by ICRF heating of minority ions. ORBIT-RF code simulation with a symmetric power spectrum predicts co-current rotation when the resonance is on the LFS and counter-current rotation with the resonance on the HFS. The latter is in contradiction with experimental observations on Alcator C–Mod and JET. This paper calls attention to the possibility of an asymmetric power spectrum with a symmetric antenna array phasing due to an intrinsic antenna loading asymmetric perpendicular to the ambient magnetic field. The conditions for a spectral peak in the co-current direction are identified. Under this occurrence, the ICRF can indeed cause a co-current rotation for HFS resonance with Alcator C–Mod parameters. Moreover, for peak density profile corresponding to an internal transport barrier, the ICRF-induced rotation reverses its direction. Finally, with a finite positive $n_\phi$, a co-rotation peaked off-axis is demonstrated for JET parameters for LFS resonance heating. It remains to be explained why there is no significant difference in the observed rotation on JET with different antenna phasings.
ACKNOWLEDGMENT

This is a report of work supported by the U.S. Department of Energy under Grant No. DE-FG03-95ER54309 and Contract No. DE-AC03-95ER53225. We gratefully acknowledge the use of the ORBIT code by permission of Dr. R.B. White as a starting point of our development, and many illuminating discussions with Dr. F.W. Perkins.

REFERENCES